NUMERICAL INVESTIGATION OF MAGNETIC NANOFLUIDS FLOW OVER ROTATING DISK EMBEDDED IN A POROUS MEDIUM

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Abstract: Combined effects of thermal radiation and variable viscosity on a time-dependent boundary layer flow (BLF) of magnetic nanofluids (MNF) over a rotating disk in the presence of the porous medium have been numerically investigated. To carry out the study, Hydrocarbon based magnetic nanofluid containing magnetite Fe3O4 particles of 10 nm with magnetic phase concentration of 10% has been taken. For numerical solutions of the modeled system containing the governing equation of the flow, a Matlab tool ODE45 is employed with shooting technique for the initial guess of the unknown boundary conditions. The flow phenomenon and heat transfer on the plate surface are characterised by various flow parameters such as viscosity variations, unsteady rotation parameter, Prandtl number and radiation parameter. Also, a comparative thermal analysis has been carried out for MNF having three different bases viz. hydrocarbon, fluorocarbon and water. Results reveal that heat transfer rate of hydrocarbon base MNF is 73.4511% faster than water base MNF and 239.7458% faster than fluorocarbon base MNF. This enhanced heat transfer capacity of hydrocarbon base MNF will help in improving the performance of oil and ore extraction drilling systems used in mining industry and other geothermal applications.

Keywords: Depth and Temperature Dependent Viscosity; Magnetic Nanofluids; Thermal Radiation; Boundary Layer Flow.

1. Introduction

Magnetic nanofluids, often referred as ferrofluids, are suspension of nano-sized particles having an average size of range about 3-15 nm of Fe3O4, γ-Fe2O3 or CoFe2O4 in a carrier liquid (e.g. water, kerosene, hydrocarbon, fluorocarbon, toluene, glycol and lubricants etc.) having surfactant (antimony) coating to prevent from agglomeration even when a strong magnetic field gradient is applied. Since last few past decades, wide investigations supported by experiments have been made to understand the behaviour of MNF so that they can be utilized in the fields where enhancement and depression of heat transfer of the thermal devices are paramount to improve the process performance. The study of rheological properties of MNF has a number of wide applications for various industrial
thermal devices, computer storage devices, magnetic sealing, damping and bearing of machines, energy conversion system, a thermal power generating system, nuclear reactors, transportation, electronics as well as biomedicine (drug delivery in disease curing). In all these processes, fluid under use is made to retain its original flow behaviour by keeping the viscosity changes due to temperature or changes due to externally applied field or changes due to the geophysical positioning (depth dependence changes) under control.

The problem of flow over a rotating disk is one of the classical problems of fluid mechanics that has both theoretical and practical values. The infinite rotating disk problem was first discussed by Kármán[1] for an ordinary viscous fluid flow using a similarity transformation for solving the system of non-linear coupled partial differential equations governing the flow. After his pioneering study, the rotating disk problem again came into focus since disk-shaped bodies are often encountered in many real world engineering and industrial applications which require the study of rotating flows. Some of such applications are rotating heat exchangers, rotating disk reactors for bio-fuels production, gas or marine turbine, chemical and automobile industries etc. The steady laminar MHD flow of an electrically conducting fluid over a rotating disk with slip boundary condition is investigated taking into account the variable rheological properties of a fluid by Frusteri and Osalusi [2]. Ram and Sharma[3] deal with the effects of rotation on the ferrofluid due to a rotating disk by applying Neuringer-Rosensweig model.

Disk driven flows with heat transfer through porous media have constituted a major field of study in fluid mechanics. The enhancement of heat transfer from a rotating body has a number of wide applications in case of various types of machinery for example, gas turbine rotors[4] and computer disk drives [5]. Attia[6] studied the effects of the porous medium and temperature-dependent viscosity on the unsteady flow and heat transfer for a viscous laminar incompressible fluid due to an impulsively started rotating the infinite disc. He [7] further extended his work with Darcy model assumption and investigated the steady flow of an incompressible viscous fluid above an infinite rotating disk in the porous medium. Ram and Kumar[8] discussed the effects of field dependent viscosity on ferrofluid flow due to a rotating disk embedded in the porous medium. Rashidi et al.[9] described the approximate analytical solutions of the steady flow over a rotating disk in the porous medium with heat transfer through the homotopy analysis method (HAM). Ellahi [10] examined the effect of the temperature dependent viscosity on boundary-layer flow of a nanofluid in a pipe using the homotopy analysis method. The effects of variable fluid properties like density, viscosity, thermal conductivity etc. on the steady laminar conducting fluid flow due to a porous rotating disk were investigated by Osalusi and Sibanda[11]. Ellahi et al. [12] analyzed the thermal behaviour of mixed convection flow of power law fluid in the presence of copper nanoparticles. Akbar et al.,[13] discussed the flow behaviour of a temperature dependent viscous nanofluid flow under the influence of gravitational force effect using Buonjornio model. Ram and Kumar[14] studied the effects of magnetic field dependent viscosity and viscous dissipation on heat transfer in steady axisymmetric Ferrohydrodynamic (FHD) boundary layer flow of an electrically non-conducting incompressible ferrofluid in the porous medium. They analyzed three-dimensional rotationally symmetric boundary layer flow of field dependent viscous ferrofluid saturating porous medium due to the rotation of an infinite disk maintained at a uniform temperature.

The thermal radiation and heat transfer characteristics of a fluid over a rotating surface has been a subject of great interest from the industrial and energy saving perspectives. These characteristics
have been widely studied in the recent years due to its vast applications in engineering, nuclear reactors, thermal power plants, process industries, solar collectors, drying processes, heat exchangers, geothermal and oil recovery, building construction, etc. Mukhopadhyay [15] has analyzed the effects of thermal radiation on heat and mass transfer on unsteady boundary layer mixed convection flow over a vertical stretching surface in a porous medium with suction. Uddin et al. [16] studied the nonlinear Rosseland radiation effect on BLF of a viscous nanofluid embedded in the porous medium. Khidir [17] investigated the effects of viscous dissipation and Ohmic heating on steady MHD convective flow due to a porous rotating disk taking into account the variable fluid properties (density, viscosity and thermal conductivity) in the presence of Hall current and thermal radiation. These properties are taken to be dependent on temperature. For better accuracy, Successive Linearization Method (SLM) has been successfully applied to different fluid flow problems. The time-dependent flow and the heat transfer of a nanofluid caused by the linear motion of a horizontal flat plate have been analyzed by Ahmadi et al. [18]. The heat transfer behaviour of the micropolar fluid flow in a channel subject to a chemical reaction has been discussed by Fakour et al. [19] and governing equations have been analytically solved using least square method (LSM). The combined hydromagnetic and slip flow of a steady, laminar conducting viscous fluid in the presence of thermal radiation due to an impulsively started rotating porous disk has been discussed by Osalusi [20]. Babu and Sandeep [21] compared oblique and free stream flow cases of a nanofluid in the presence of nonlinear thermal radiation and variable viscosity. Shit and Majee [22] investigated the effects of thermal radiation on the magnetohydrodynamic (MHD) flow and heat transfer over an inclined non-linear stretching sheet. The effects of thermal radiation and heat transfer on the ferromagnetic fluid flow in the presence of a stretching sheet have been investigated by Zeeshan et al. [23]. Water-based nanofluids containing nanoparticles volume fraction of Cu, Ag, CuO, Al₂O₃ and TiO₂ are taken into account in the work of Turkyilmazoglu [24]. He described the flow and heat transfer influenced by the existence of such nano-particles due to a rotating disk. In this direction, many applications of nanofluids in different types of transport phenomena have recently been reported in Refs. [25-29]

Many researchers in MHD and few in FHD have been studied flow behaviour due to the geophysical depth and thermal variation of viscosity, taken one at a time. However, the combined effects of depth and temperature on viscosity have not been popularly analyzed on rotating disk problem yet. The modeled rotating disk problem of our manuscript focuses on the heat transfer and thermal radiation of time-dependent BLF of an incompressible MNF embedded in the porous medium. The problem is designed for flow under the combined influence of depth and temperature dependent viscosity. The plate is subjected to a magnetic field \( \vec{H} \) with components \( (H_r, H_\theta, 0) \) and the plate is maintained at a uniform temperature \( T_w \). The equations governing the time-dependent BLF in component form are non-dimensionalized using the similarity transformations. The resultant non-linear coupled differential equations are then solved numerically by using Runge-Kutta method in MATLAB with a systematic guess of missing boundary conditions. In this model, the investigations are performed for Hydrocarbon based MNF (C1-20B) with an average magnetite particle size of 10 nm and a magnetite phase concentration of 10%. Also, a comparative study has been carried out for MNF of three different bases: Hydrocarbon based magnetic nanofluid (C1-20B), Water based magnetic nanofluid (Taiho W-40) and Fluorocarbon based magnetic nanofluid (FC-72). The physical properties of all three types of magnetic nanofluids [30-32] are tabulated in Table-1.
Table 1: Physical Properties of the MNF: Taiho W-40, FC-72 and C1-20B.

<table>
<thead>
<tr>
<th>Properties</th>
<th>FC-72 (Rini et al. [30])</th>
<th>Taiho W-40 (Snyder et al. [31])</th>
<th>C1-20B (Hong et al. [32])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Fluids</td>
<td>Fluorocarbon (Perfluorohexane C₆F₁₄)</td>
<td>Water (H₂O)</td>
<td>Hydrocarbon (Kerosene)</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>1.68 × 10³</td>
<td>1.4 × 10³</td>
<td>1.25 × 10³</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion (1/K)</td>
<td>1.6 × 10⁻³</td>
<td>0.026 × 10⁻⁴</td>
<td>0.86 × 10⁻³</td>
</tr>
<tr>
<td>Pyromagnetic Coefficient (A/mk)</td>
<td>–</td>
<td>240</td>
<td>80</td>
</tr>
<tr>
<td>Dynamical Viscosity in Zero Magnetic Field (kg/ms)</td>
<td>6.4 × 10⁻⁴</td>
<td>3.99 × 10⁻²</td>
<td>6 × 10⁻³</td>
</tr>
<tr>
<td>Thermal diffusivity (m²/s)</td>
<td>3.084 × 10⁻⁸</td>
<td>64.3 × 10⁻⁸</td>
<td>5 × 10⁻⁸</td>
</tr>
<tr>
<td>Prandtl number (Pr)</td>
<td>12.3</td>
<td>44.3</td>
<td>128</td>
</tr>
</tbody>
</table>

2. Mathematical Formulation of the Problem

The viscosity of MNF is considered to be both depth and temperature dependent [33] given as:

\[ \mu(z, T) = \frac{\mu_{\infty}(1-\alpha z)}{[1+\alpha(T-T_{\infty})]} , \]

where \( \mu_{\infty} \) is the uniform viscosity of a fluid and \( \alpha \geq 0 \) is a constant.

![Figure 1: The Flow Configuration and the Coordinate System.](image)

The modeled differential equations (equation of continuity, equations of momentum, energy equation) governing the axi-symmetric unsteady FHD boundary layer flow in component form are given as [34]:

\[ \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 , \]
\[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} - \frac{v^2}{r} + w \frac{\partial u}{\partial z} \right) + \frac{\partial p}{\partial r} = \frac{\partial}{\partial r} \left( \mu(z,T) \frac{\partial u}{\partial r} \right) + \frac{\partial}{\partial z} \left( \mu(z,T) \frac{\partial u}{\partial z} \right) + \mu_0 |\vec{M}| \frac{\partial}{\partial r} |\vec{H}| - \frac{u}{k_0} v, \]

(3)

\[ \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{w}{r} + w \frac{\partial v}{\partial z} \right) = \frac{\partial}{\partial r} \left( \mu(z,T) \frac{\partial v}{\partial r} \right) + \frac{\partial}{\partial z} \left( \mu(z,T) \frac{\partial v}{\partial z} \right) + \mu_0 |\vec{M}| \frac{\partial}{\partial \phi} |\vec{H}| - \frac{u}{k_0} v, \]

(4)

\[ \rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) + \frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( \mu(z,T) \frac{\partial w}{\partial r} \right) + \frac{\partial}{\partial z} \left( \mu(z,T) \frac{\partial w}{\partial z} \right) + \mu_0 |\vec{M}| \frac{\partial}{\partial z} |\vec{H}| - \frac{u}{k_0} w, \]

(5)

\[ \rho C_p \left( \frac{\partial \varphi}{\partial t} + u \frac{\partial \varphi}{\partial r} + w \frac{\partial \varphi}{\partial z} \right) = k \left( \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} + \frac{\partial^2 \varphi}{\partial z^2} \right) - \frac{\partial q_r}{\partial z}, \]

(6)

where \( \rho \) is the fluid density, \( \mu_0 \) is magnetic permeability in free space, \( k_0 \) is Darcy permeability parameter, \( k \) is the thermal conductivity of heat, \( C_p \) is the specific heat with constant pressure and \( q_r \) is radiative heat flux.

The boundary conditions for the flow are [35]:

\[ \begin{cases} 
  u = 0, \ v = r\Omega, \ w = 0, \ T = T_w & \text{at } z = 0, \\
  u \to 0, \ v \to 0, \ T \to T_\infty & \text{as } z \to \infty, \\
  w \to \text{some finite negative value} & \text{as } z \to \infty.
\end{cases} \]

(7)

where \( \Omega \) is the angular velocity of the disk and \( z \to \infty \) is not exactly infinity but a large distance beyond which the boundary layer vanishes.

### 3. Modeling and Solution of the Problem

The flow of magnetic nanofluid is affected by the magnetic field due to the magnetic dipole \( m \), whose magnetic scalar potential is given by,

\[ \psi_m = \frac{m}{2\pi r} \cos \phi, \]

and the corresponding magnetic field \( H \), considering negligible variation along \( z \)-axis, has the components,

\[ H_r = -\frac{\partial \psi_m}{\partial r} = \frac{1}{2\pi} \frac{m \cos \phi}{r^2}, \quad H_\phi = -\frac{1}{r} \frac{\partial \psi_m}{\partial \phi} = \frac{1}{2\pi} \frac{m \sin \phi}{r^2}, \quad H_z = 0. \]

Hence, the resultant applied magnetic field is given by,

\[ H = \sqrt{H_r^2 + H_\phi^2 + H_z^2} = \frac{m}{2\pi r^2}. \]

(8)

Assuming that the applied magnetic field \( H \) is sufficiently strong to saturate the magnetic nanofluid and the variation of magnetization \( M \) with temperature can be approximated by a linear equation of state,

\[ M = K (T_c - T), \]

(9)

where \( T_c \) is curie temperature and \( K \) is pyromagnetic coefficient.

Using Rosseland approximation, the radiative flux \( q_r \) is modeled as,

\[ q_r = -\frac{\sigma T^4}{3k^* \frac{\partial T}{\partial z}}, \]

(10)

where \( \sigma \) is Stefan-Boltzmann constant and \( k^* \) is the mean absorption coefficient. Assuming that the differences in temperature within the flow are such that \( T^4 \) can be expressed as a linear combination of temperature; expanding \( T^4 \) in Taylor’s series about \( T_\infty \) and neglecting the higher order terms, we get
\[ T^4 \approx 4T_{\infty}^3 T - 3T_{\infty}^4. \] (11)

The revolving flow of magnetic nanofluid due to the rotating disk with constant angular velocity \( \Omega \) is in equilibrium under the influence of centrifugal force which is balanced by the radial pressure gradient. So, the boundary layer approximation for equation (3) is:
\[ \frac{1}{\mu} \frac{\partial p}{\partial r} = r \Omega^2. \] (12)

Introducing the following similarity transformations to non-dimensionalized the governing equations:
\[ \{ u = r \Omega U'(\eta), \quad v = r \Omega V(\eta), \quad w = \frac{\nu}{\delta} W(\eta), \quad T - T_{\infty} = \Delta T \theta(\eta), \] (13)
where \( \eta = \frac{r}{\delta}, \Delta T = T_w - T_{\infty} \) and \( \delta(t) \) is a scalar factor responsive for unsteady flow.

Using equation (13) in equation of continuity (2), we get
\[ W = -2RU. \] (14)

Using equations (8) – (14) in the set of governing equations (3) – (6), we get a system of non-linear coupled ordinary differential equations as:
\[
\begin{align*}
(1 - \varepsilon_1 \eta)U'''' - \varepsilon_1 U'' - (1 - \varepsilon_1 \eta)(1 + \varepsilon \theta)^{-1} \varepsilon \theta' U'' + \\
(1 + \varepsilon \theta) \left[ \delta \frac{d}{dt} \eta U''' + R(V^2 - U'^2) + 2RUU'' - R - \frac{2B}{R} - R \beta U' \right] &= 0, \\
(1 - \varepsilon \eta)V'' - \varepsilon_1 V' - (1 - \varepsilon_1 \eta)(1 + \varepsilon \theta)^{-1} \varepsilon \theta' V' + \\
(1 + \varepsilon \theta) \left[ \delta \frac{d}{dt} \eta V'' + 2R(UV' - VU') - R \beta V \right] &= 0, \\
(3Q_r + 4) \theta'' + 3 \Pr Q_r \left( \frac{\delta}{\nu} \frac{d}{dt} \eta + 2RU \right) \theta' &= 0,
\end{align*}
\] (15) – (17)
where \( \varepsilon = \alpha \Delta T, \varepsilon_1 = \alpha \delta(t) \) are viscosity variation parameter and modified viscosity variation parameter respectively, unsteady rotation parameter \( R = \Omega \delta^2 / \nu \), FHD interaction parameter \( B = m \mu_0 K(T_c - T) / 2 \pi \mu^2 \), permeability parameter \( \beta = \mu / k \theta_0 \), Prandtl number \( \Pr = \mu C_p / k \) and radiation parameter \( Q_r = \kappa k^2 / 4 \alpha T_{\infty}^2 \).

For unsteady flow problem, the term \( (\delta / \nu) (d \delta / dt) \) should not be dropped from the equations (15) – (17). Considering the usual scaling factor for various unsteady boundary layer flows [35],
\[ \delta = 2 \sqrt{\nu t} + L, \] (18)
where \( L \) represents the length scale of steady flow.

Introducing (18) in equations (15) – (17) respectively, we have the following dimensionless nonlinear ordinary differential equations:
\[
\begin{align*}
(1 - \varepsilon_1 \eta)U'''' - \varepsilon_1 U'' - (1 - \varepsilon_1 \eta)(1 + \varepsilon \theta)^{-1} \varepsilon \theta' U'' + \\
(1 + \varepsilon \theta) \left[ 2 \eta U''' + R(V^2 - U'^2) + 2RUU'' - R - \frac{2B}{R} - R \beta U' \right] &= 0, \\
(1 - \varepsilon_1 \eta)V'' - \varepsilon_1 V' - (1 - \varepsilon_1 \eta)(1 + \varepsilon \theta)^{-1} \varepsilon \theta' V' + \\
(1 + \varepsilon \theta) \left[ 2 \eta V'' + 2R(UV' - VU') - R \beta V \right] &= 0, \\
(3Q_r + 4) \theta'' + 6 \Pr Q_r (\eta + RU) \theta' &= 0.
\end{align*}
\] (19) – (20)
(21)

Also, the boundary conditions (7) reduces to
\[
\begin{align*}
U'(0) = 0, \quad V(0) = 1, \quad W(0) = 0, \quad \theta(0) = 1 \\
U'(\infty) \rightarrow 0, \quad V(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0 \\
W(\infty) \text{tends to some negative value.}
\end{align*}
\] (22)
A fourth order Runge Kutta method is employed to obtain the numerical solution followed by shooting technique with a systematic guessing of missing boundary conditions \( U''(0), V'(0) \) and \( \theta'(0) \) as employed by Ram et al.\[36\].

With the above scheme, the solution of the coupled differential equations (19) – (21) along with the boundary conditions (22) provides us the variations in velocity and temperature profiles besides the skin friction coefficients and the rate of heat transfer at the surface of the plate. The Newtonian formulae are used to calculate the radial stress \( \tau_r \) and tangential shear stress \( \tau_\phi \) as:

\[
\begin{cases}
\tau_r = \left( \mu(z, T) \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) \right)_{z=0} = \mu_\infty (1 - b) Re^{1/2} \Omega U''(0), \\
\tau_\phi = \left( \mu(z, T) \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial \theta} \right) \right)_{z=0} = \mu_\infty (1 - b) Re^{1/2} \Omega V'(0),
\end{cases}
\]

(23)

where \( Re \) is the local rotational Reynolds number. Therefore, the radial and tangential skin frictions are, respectively, given by

\[
(1 - b)^{-1} Re^{1/2} \tilde{C}_{fr} = U''(0), \quad (1 - b)^{-1} Re^{1/2} \tilde{C}_{f\phi} = V'(0);
\]

(24)

where \( \tilde{C}_{fr} \) and \( \tilde{C}_{f\phi} \) are the coefficients of radial and tangential skin frictions, respectively. Also, the rate of heat transfer from the surface of the plate to the magnetic nanofluid is calculated by using Fourier law given as:

\[
q = -\left( k \frac{\partial T}{\partial z} \right)_{z=0} = -k \Delta T \sqrt{\frac{\Omega}{\nu_\infty}} \theta'(0)
\]

(25)

Therefore, the Nusselt number (Nu) is given by, \( Re^{1/2} Nu = -\theta'(0) \).

4. Results and Discussion

The system of non-linear coupled ordinary differential equations (19)-(21) governing the fluid flow along with boundary conditions (22) have been solved using the numerical procedure in MATLAB as given in Appendix-1. This enables us to carry out the influence of rotation, viscosity variation and thermal radiation on flow behaviour and enhancement of heat transfer of a Hydrocarbon based magnetic nanofluid. Also, the thermal behaviour of C1-20B has been compared with Taiho W-40 and FC-72 taking FHD interaction parameter \( B=1.0 \), permeability parameter \( \beta = 1.0 \), viscosity variation parameters \( \epsilon \) & \( \epsilon_1 \) ranging from 0.0 to 1.0, unsteady rotation parameter ranging from 0.5 to 2.0 and radiation parameter ranging from 0.5 to 5.0.
Figure 2: Effect of Viscosity Variation on Radial Velocity Profile.

Figure 3: Effect of Viscosity Variation on Tangential Velocity Profile.
The effect of increase in viscosity variation parameter $\varepsilon$ & $\varepsilon_1$ for the set of values $R=1.0, B=1.0,$ $\beta = 1.0, \Pr=128 \& Qr=1.0$ on the dimensionless fluid flow profiles (velocity and temperature profiles) is depicted in figures (2) – (5). An increasing value of $\varepsilon$ implies the dependency of viscosity on temperature (heated surface) while viscosity depends on depth with the increase in the value of $\varepsilon_1$. From the figures, we may see how the velocity distribution is affected by variation in viscosity dependency on temperature and geophysical position in comparison to the uniform viscosity i.e. when $\varepsilon = \varepsilon_1 = 0$. The numerical results show an increase in dimensionless radial and axial velocity profiles for increasing values of $\varepsilon$ and $\varepsilon_1$. Here, $\eta = 0.21$ is a critical point as for $0 \leq \eta \leq 0.21$, the tangential velocity decreases with increasing values of $\varepsilon$ & $\varepsilon_1$, however, the trend is reverse for $0.21 < \eta \leq 0.96$ i.e. as we move away from the centre disk.
Figures (6) – (8) describe the influence of rotation of the plate on flow profiles. From these figures, it has been noted that on increasing the rotation of the disk there is a significant change in velocity profiles (radial, tangential & axial). An increase in rotation of the disk causes decrease in velocity distribution profiles and attains the steady state more rapidly. However, for slower rotation of the plate, the radial and axial profiles show large influence while the influence becomes smaller when rotation increases.
We examined the effects of different values of radiation parameter $Q_r$ on the velocity and temperature profiles with a set of values $\varepsilon = \varepsilon_1 = 0.1$; $R = 1.0$; $B = 1.0$ and $\beta = 1.0$ for Hydrocarbon based magnetic nanofluid (C1-20B). It is observed that the thermal radiation does not affect significantly on the velocity of the fluid and it is only the temperature distribution [Figure (9)] which is affected the most. The increasing radiation parameter $Q_r$, decreases the temperature profile due to the faster dissipation of the heat. Also, it is observed that the temperature is maximum at the surface of the disk and asymptotically decreases to its steady state.

Figures (10) shows a comparative study of the modeled problem for three type of MNF: Hydrocarbon based magnetic nanofluid (C1-20B), Water based magnetic nanofluid (Taiho W-40) and Fluorocarbon based magnetic nanofluid (FC-72) with a specified set of values of various physical parameters, such as $\varepsilon = \varepsilon_1 = 0.1$; $R = 1.0$; $B = 1.0$ and $Q_r = 1.0$. As expected, the numerical results show that on increasing the prandtl number, the temperature profile decreases and reaches to its boundary condition more fastly due to the fact that the thermal diffusion of the fluid decreases with increase in the Prandtl number. It means that Hydrocarbon based magnetic nanofluid (C1-20B) dissipates heat faster than water based magnetic nanofluid (Taiho W-40) and much faster.
than Fluorocarbon based magnetic nanofluid (FC-72). Precisely, we can conclude that the maximum cooling of the rotating disk is achieved in C1-20B.

Table 2. Skin Friction Coefficients and Rate of Heat Transfer for B=1.0; β=1.0.

<table>
<thead>
<tr>
<th>ε</th>
<th>ε₁</th>
<th>R</th>
<th>Qr</th>
<th>Pr</th>
<th>(U''(0))</th>
<th>(V'(0))</th>
<th>(-\theta'(0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
<td>1.0</td>
<td>128</td>
<td>1.186871532109</td>
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<tr>
<td>0.2</td>
<td>0.2</td>
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<td>1.0</td>
<td>128</td>
<td>1.360915832109</td>
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<tr>
<td>0.4</td>
<td>0.4</td>
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<td>1.0</td>
<td>128</td>
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<tr>
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<td>1.0</td>
<td>128</td>
<td>1.830685321096</td>
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<td>1.0</td>
<td>128</td>
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The effects of the rheological parameters; viscosity variation parameters \(\varepsilon\) & \(\varepsilon_1\), unsteady rotation parameter \(R\), radiation parameter \(Qr\) and Prandtl number \(Pr\) on the shear stresses \(U''(0), V'(0)\) and the rate of heat transfer \(\theta'(0)\) have been illustrated in Table 2. We noticed that the effect of the increasing \(\varepsilon\) & \(\varepsilon_1\) and \(R\), increases the tangential skin friction. Also the rate of heat transfer increases with increasing value of \(Qr\) and \(Pr\). The negative values of \(\theta'(0)\) indicates the heat flow from the disk surface to the ambient fluid. This is in perfect agreement with the physical fact that the thermal boundary layer thickness decreases with increasing Prandtl number and radiation parameter. Table 2 reveals that heat transfer of Taiho W-40 is 95.8739% and C1-20B is 239.7458% faster than FC-72.

5. Validation and Comparison with the Existing Results

The present numerical model has been validated with the help of previous research findings. A comparison of rate of heat transfer of the reduced case of steady and ordinary (\(\delta = L, \beta = 0\)) viscous fluid flow for \(Pr = 10\) and 100 have been presented in Table 3 and are in good agreement with those of Gregg and Sparrow [37] and Maleque [33]. Also, a comparison of the present study with its reduced case is described in Figure 11. A remarkable difference in thermal boundary layer thickness is observed when we replace ordinary fluids with MNF.
Table 3. The Rate of Heat Transfer: Comparison and Validation.

<table>
<thead>
<tr>
<th>Pr</th>
<th>Present Study</th>
<th>Reduced Case of the Present Study</th>
<th>Maleque [33]</th>
<th>Gregg and Sparrow [37]</th>
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6. Concluding Remarks

The numerical investigations exhibit many interesting rheological features concerning the effects of thermal radiation and depth and temperature dependent viscosity on Hydrocarbon based magnetic nanofluid (C1-20B) flow over a rotating disk embedded in the porous medium. The limiting case of the numerical model has an excellent agreement with the previous works. Also, a comparative study has been done for three types of magnetic nanofluids: Hydrocarbon based magnetic nanofluid (C1-20B), Water based magnetic nanofluid (Taiho W-40) and Fluorocarbon based magnetic nanofluid (FC-72). The main findings of this work are:

i. Viscosity variation of the fluid and the rotations of the plate alters the velocity distribution however temperature profile is unaltered. The velocity profiles increases and approaches to its steady state when we move from uniform viscosity to variable viscosity (temperature and depth dependent) while the trends are reverse when we increase the rotation of the plate.

ii. The achievement of the steady state in temperature distribution is faster as we move from low thermal radiation to high thermal radiation and increase the rate of heat transfer from the disk surface to the fluid.

iii. Thermal boundary layer thickness of C1-20B is thinner than Taiho W-40 and much thinner than FC-72, meaning thereby that Hydrocarbon based magnetic nanofluid C1-20B is better coolant than Taiho W-40 and FC-72.

iv. Thus fast cooling of the plate can be achieved by implementing these effects.

In the present study, the hydro-carbon based magnetic nanofluids has been considered, which is directly applicable to control the heat losses/ or in keeping cool the instrument and avoiding the
damage caused due to the heat generation by the motion of its blades/shafts. This study is also applicable in thermal power generating systems, high speed rotating machinery and aerodynamic extrusion of plastic sheets. In the future study, we generalize this model for the CNTs nanofluids and non-Newtonian nanofluids with the effects of magnetic fields.

References


[37] J Gregg and E Sparrow, Heat transfer from a rotating disk to fluids of any Prandtl number, (1959),
Appendix-1

Numerical Coding for the Model

The code is written in the MATLAB environment using ODE45. To reduce the equations to first order equations, we set:

\[ y_1 = U; \quad y_2 = U'; \quad y_3 = U''; \]
\[ y_4 = V; \quad y_5 = V'; \]
\[ y_6 = \theta; \quad y_7 = \theta'. \]

The initial condition are given by,

\[ y_1(0) = 0; \quad y_2(0) = 0; \quad y_4(0) = 1; \quad y_6(0) = 1. \]

Viscosity variation parameter due to temperature and depth:

\[ \varepsilon = m = 0.1, 0.4, 0.8, 1.0; \quad \varepsilon_1 = n = 0.1, 0.4, 0.8, 1.0. \]

Unsteady rotation parameter: \[ R = 0.5, 1.0, 2.0. \]
Prandtl Number: \[ Pr = 44.3, 79.3, 128. \]
FHD interaction parameter: \[ B = 1.0. \]
Permeability parameter: \[ \beta = b = 1.0. \]
Radiation Parameter: \[ Qr = 0.5, 1.0, 5.0. \]

Model equations used in the problem:

\[ y_3' = (n * y_3 + (1 - n * t) * (m * y_7 * y_3/(1 + m * y_6) - (1 + m * y_6) * (2 * t * y_3 - R * (y_2^2 - y_4^2)) + 2 * R * y_4 * y_5 - R - b * y_2))/((1 - n * t)), \tag{26} \]
\[ y_3 = s_3; \]
\[ y_5' = (n * y_5 + (1 - n * t) * m * y_7 * y_5/(1 + m * y_6) - 2 * (1 + m * y_6) * (t * y_5 - R * (y_2 * y_4 - y_1 * y_5) - b * y_4))/((1 - n * t)), \tag{27} \]
\[ y_5 = s_5; \]
\[ y_7' = -3 * Qr * Pr * (2 * t * y_2 + 2 * R * y_1 * y_7)/(3 * Qr + 4), \tag{28} \]
\[ y_7 = s_7; \]

where \( s_3, s_5 \) & \( s_7 \) are constants.

The tool ODE45 reduces the boundary value problem (BVP) to initial value problem (IVP). Shooting technique is used to guess the values of \( s_3, s_5 \) & \( s_7 \) satisfying the boundary conditions: \( y_2(\infty) \to 0, y_4(\infty) \to 0, y_1(\infty) \to -c, \quad (c > 0) \) & \( y_6(\infty) \to 0 \) with the desired degree of accuracy, viz. \( 10^{-12} \).

The solution is then compared quantitatively and qualitatively through table [Table 2] and graphs [Figures (2-10)].