Determination of a Synchronous Generator Characteristics via Finite Element Analysis

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**Abstract:** In the paper a determination of characteristics of a small salient pole synchronous generator (SG) is presented. Machine characteristics are determined via Finite Element Analysis (FEA) and for that purpose is used the software package FEMM Version 3.3. After performing their calculation and analysis, one can conclude that most of the characteristics presented in this paper can be obtained only by using the Finite Element Method (FEM).

**Keywords:** Salient-pole synchronous generator, Characteristics, Magnetic field, Finite element method.

1 **Introduction**

When using the traditional methods for synchronous generator performance analysis the magnetic field is known only approximately. The evaluation of the machine characteristics is based on a rough idea of the field distribution in the magnetic core. These methods usually give satisfactory results for steady-state operation of the machine, but for transient operation the results are unreliable. It is one of the reasons why a lot of research work has recently been done associated with the numerical field analysis. The Finite Element Method (FEM) has been widely proved to be efficient when dealing with complicated geometries. The distribution of the magnetic field is solved by the system of Maxwell’s equations. Because of the nonlinearity of the core materials the system of equations must be solved iteratively. Equations should be solved simultaneously in the same iteration loop, if a fast convergence and a short solution time are wanted.

The proposed methodology is applied on experimental synchronous generator set, with rated data: \( S_n = 3 \text{ kVA} \), \( U_n = 400 \text{ V} \), \( f_n = 50 \text{ Hz} \), \( I_{dn} = 4.17 \text{ A} \), \( n_n = 1500 \text{ rpm} \), \( I_{fn} = 5.5 \text{ A} \).

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2 Mathematical Formulation

The mathematical formulation for magnetic field problems description is based on the system of relevant Maxwell’s equations, by which the magnetic field is described in closed and bounded system [1]. In our case the problem is 2D and here one can suppose that the axial length of the machine is infinite and the machine geometry is invariant along the $z$-axis. The synchronous generator is treated as a quasi-static magnetic system.

The field intensity $H$ and flux density $B$ must obey:

\[ \nabla \times H = J, \quad (1) \]
\[ \nabla \cdot B = 0. \quad (2) \]

The widely known subject to a constitutive relationship between $B$ and $H$ for each material is:

\[ B = \mu H. \quad (3) \]

In the case of electric machines the stator and rotor core are made from saturating iron and therefore the permeability $\mu$ is a function of $B$. The Finite Element Analysis (FEA) uses the magnetic vector potential approach about finding a field that satisfies Eqs. (1)-(3). The relationship between the flux density and the vector potential is

\[ B = \nabla \times A. \quad (4) \]

and from (4) we can express the distribution of the magnetic field by the following nonlinear partial differential equation:

\[ \nabla \times \left( \frac{1}{\mu} \nabla \times A \right) = J. \quad (5) \]

Assuming Coulomb gauge $\nabla \cdot A = 0$, for linear materials Eq. (5) reduces to a form:

\[ -\frac{1}{\mu} \nabla^2 A = J. \quad (6) \]

If there are no currents in the domain under consideration, the right side term in Eq. (6) turns to zero. In the general 3D case, $A$ is vector with three components, but in the analyzed case, is considered to be the planar 2D; consequently, two components of $A$ are becoming zero, leaving just the component in the “out of page” direction [2].

The use of vector potential formulation is an advantage that all the conditions to be satisfied have been combined into a single equation. We can notice that if $A$ is found, then $B$ and $H$ can be deduced by differentiating $A$. 
The procedure for numerical computation of magnetic field problems, by using the finite element method is divided into three main steps:

- **Pre-processing** – the derivation of the FE model of the electric machine under consideration, defining material properties, boundary conditions and mesh generation;
- **Processing** – solving the problem by the relevant Maxwell’s equations and obtaining the field distribution in the analyzed domain of the electric machine, at arbitrary chosen excitations and loading conditions;
- **Post-processing** – calculation and presentation of characteristics, as well as parameters, of the analyzed electric machine.

### 3 Results and Conclusions

Here are presented some of the synchronous generator characteristics determined in the step of post-processing when using FEM [3]. The results are obtained using the software package FEMM Version 3.3. We can prove that the most of these characteristics can be obtained only by using FEM.

#### 3.1 Open circuit characteristic

The open circuit characteristic of the synchronous generator is determined for values of the field current $0 < I_f < 1.3I_{fn}$. For each value of the field current the flux linkage $\psi$ is calculated from the finite element solution. Then the phase voltage $E$ is calculated by

$$E = 4.44 N k_w f \psi,$$

where: $N$ is the number of turns of the phase winding, $k_w$ is a winding coefficient and $f$ is the frequency. The open circuit characteristic is presented in Fig. 1.

![Fig. 1 – Open circuit characteristic.](image-url)
3.2 Characteristics of the flux in the air gap

The characteristics of the flux in the air gap of the generator represent a dependence of the flux in the middle of the air gap against the rotor position for different values of the armature current $I_a$. In Fig. 2 are presented the characteristics of the flux in the air gap for rated value of the field current $I_f$. Also, in Fig. 3 are presented characteristics of the flux in the air gap for different values of the armature current $I_a$ but for value of the field current $I_f = 0$.

![Fig. 2](characteristics_of_flux_air_gap_fI.png)

**Fig. 2** – Characteristics of the flux in the air gap for $I_f = I_fn$.

![Fig. 3](distribution_of_flux_density_air_gap.png)

**Fig. 3** – Characteristics of the flux in the air gap for $I_f = 0$.

3.3 Distribution of the flux density in the air gap

In the post-processing step when using the FEMM package we can determine the distribution of flux density in an arbitrary selected region in the machine under consideration.
In Fig. 4 is presented the distribution of the flux density in the middle line of the air gap over one pole pitch. The flux density is produced only by the field current $I_f$ and we can notice the influence of the stator slots and teeth on the shape and values of the flux density.

**Fig. 4 - Distribution of the flux density along the air gap.**

3.4 Characteristics of the magnetic coenergy in the air gap

The characteristics of the magnetic coenergy in the air gap of the analyzed synchronous generator represent a dependence of the coenergy in the air gap from the position of the rotor. Because the synchronous machine is a nonlinear system the magnetic coenergy is going to be calculated by

$$W_c = \int \int_{V} B'(H')dH'dV.$$  \hspace{1cm} (8)

**Fig. 5 – Characteristics of the magnetic coenergy in the air gap.**

In Fig. 5 are presented the characteristics of the magnetic coenergy $W_c$ in the air gap for different values of the armature current $I_a$ and rated value of the field current $I_f$. 

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3.5 Characteristic of electromagnetic torque

The angle-torque characteristic is the dependence of the electromagnetic torque developed in the air-gap, against the rotor position. The electromagnetic torque here is calculated by the Maxwell’s Stress Method. The calculation of this characteristic is less accurate than the other ones so we should use as fine a mesh as possible in order to obtain more accurate results. The determined characteristic of electromagnetic torque for rated field current and rated armature current, $I_f$ and $I_a$, of the synchronous generator is presented in Fig. 6.

![Fig. 6 – Characteristic of electromagnetic torque.](image)

4 Conclusion

As it was presented, the most efficient analysis is performed by using the Finite Element Method (FEM) for determination of synchronous generator characteristics. We can conclude that the most of the characteristics described in this paper can be obtained only by using FEM. The results always are obtained more easily and quickly compared to any other method. Also, an advantage is that FEM is excellent method when dealing with complicated geometries. Therefore, today FEM is considered to be the most powerful and elegant method for determination of a synchronous generator characteristics.

6 References