THE LUNAR ORBIT PARADOX

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Abstract: Newton's formula for gravity force gives greater force intensity for attraction of the Moon by the Sun than attraction by the Earth. However, central body in lunar (primary) orbit is the Earth. So appeared paradox which were ignored from competent specialist, because the most important problem, determination of lunar orbit, was immediately solved sufficiently by mathematical ingenuity - introducing the Sun as dominant body in the three body system by Delaunay, 1860. On this way the lunar orbit paradox were not canceled. Vujičić made a overview of principles of mechanics in year 1998, in critical consideration. As an example for application of corrected procedure he was obtained gravity law in some different form, which gave possibility to cancel paradox of lunar orbit.

The formula of Vujičić, with our small adaptation, contains two types of acceleration - related to inertial mass and related to gravity mass. So appears carried information on the origin of the Moon, and paradox cancels.

Keywords: Lunar orbit paradox, gravity law, gravity and inertial mass, concepts in physics

1. INTRODUCTION

The Earth's satellite Moon is the nearest celestial body, with very complex description from standpoint of celestial mechanics. The Earth / Moon mass ratio is equal 81.3, mean density ratio 1.647, enough that barycenter of this two body system lies inside of Earth and out of Earth's planetary nucleus. It produce many effects which can be important for geophysics, as ternal and tidal influence, and effect which are not negligible for celestial mechanics - Earth's barycentric motion along orbit around the Sun. Determination of lunar orbit around the Earth was additionally complex because the solar gravity force to the Moon calculated from Newton's gravity law formula gives 2.2 times greater value than Earth's gravity force to the Moon. So appeared a paradox that the Moon's orbital motion is around the Earth, and as secondary with the Earth around the Sun [1].
2. LUNAR ORBIT SOLUTION

The lunar orbit paradox was noted in Newton’s time. In the 18. century astronomers made attempts to solve this problem, but it was not satisfactory succesful. Clairaut (1742) introduced the furth order corrective term:

\[ F = -\kappa \cdot m_b \left( \frac{m^2_2}{r^2} - \alpha \frac{m^2_2}{r^2} \right). \]  

(1)

D’Alembert (1749) made the same using the third order term, [2]:

\[ F = -\kappa \cdot m_b \left( \frac{m^2_2}{r^2} - \beta \frac{m^2_2}{r^2} \right). \]  

(2)

Both solution can be mathematically satisfactory. But what is their physical sense? This type of solution today presents only a numerical fitting if additional term is not assumed as gravitational influence of the third mass. How the most might in gravity interaction with the Moon is the Sun, and solar gravity force is bigger than the one produced by Earth’s over two times, both formulas can not be adequate physically. Other words, these solutions are out of conceptual foundation of physics.

In the next century problem was pushed at margins of interest by succesful solving most important problem for astronomers – analitical determination of lunar orbit around the Earth, for needs of ephemeridal astronomy. Delaunay (1860) are simple considered the Earth – Moon system as double planet system in motion around the Sun [3], and solved orbital motion. (Fig.1)

Lunar motion in geocentric orthogonal coordinate system OXYZ was determined by equations [4]:

\[ \frac{d^2X}{dt^2} = -\kappa^2 \left( m_b + m_w \right) \frac{X}{r^3} + \frac{dR}{dX} \]

(3.1)

\[ \frac{d^2Y}{dt^2} = -\kappa^2 \left( m_b + m_w \right) \frac{Y}{r^3} + \frac{dR}{dY} \]

(3.2)
Here are: $R$ – perturbation function, $\kappa$ – gravity constant. Solar motion is along ellipse in XOY plane around baricenter (B) of the Earth – Moon system.

This solution were often quoted as argument that paradox of lunar orbit exist not. Solar force to the Moon converts into components with origin in lunar center, the first is parallel to direction of baricenter – Sun, and the second in direction to the Earth. But, this explanation is not correct as argument that lunar orbit paradox exists not. The Moon – Earth distance visible from the Sun is under angle of only $\leq 0.147$ degree, and additional force which the Sun gives to the Moon in direction towards Earth is insufficient to explain rotation around the Earth. Because the force component to the Sun stay twice bigger than the sum of solar force component of Moon to the Earth and Earth's attraction of the Moon, problem stay open.

3. GRAVITY LAW DERIVED BY NEWTON

Deduction of gravity law Isaac Newton started from Kepler laws, Galilean determination of gravity acceleration at Earth's surface, Piccard's determination of Earth's radius, and Huygens centripetal acceleration. All other were assumptions and principles introduced by himself. The first hypothesis was that at Earth surface centripetal acceleration $g$, determined by Galilei, must be equivalent to centrifugal acceleration caused by lunar rotation around the Earth – bigger (''central'') body in the Earth – Moon system. These hypothesis must be valid in the system Sun – Earth, general in solar system, too. Newton so obtained gravity law in well known form:

$$F = -\kappa \frac{m_1 \cdot m_2}{r^2},$$

with $\kappa$ – gravity constante. This form of law is usable for two massive body in relative quite too, because gravity constant value is known. Obvious in text books is not given what is contented in Newton's gravity constant. Here we quote result derived by Newton:

$$\kappa = \frac{4\pi^2 \cdot a^3}{m_1 \cdot T^2},$$

with $a$ – big semiaxis of planetary orbit, $T$ – period of planetary revolution. It presents the third Kepler's law – in astronomy known as Kepler’s constante, divided by mass of central body. It can be useful explanation with purpose to make Newton's procedure simplest to understand, now from standpoint of mathematical logic [5].

Kepler (1609) introduced hypothesis on the mass of central body as the cause of planetary orbital motion in Astronomia nova sive Physica coelestis. It is conceptually logical, because only this quantity is the same (or equal) in the interaction with each
planet. In the third Kepler’s law, the constant is expressed as product of the central body mass and new constant - gravity constant:

\[ v^2 r = \text{const} = \gamma \cdot m_1. \]  

(6)

Here we used mean planetary distance from the Sun, physical notion of mathematical term big semiaxis, and mean planet velocity along orbit. Now we easily insert centripetal acceleration which is equal to the ratio of squared velocity and distance. From previous formula dividing by \( r^2 \) obtain acceleration to the central body:

\[ \frac{v^2}{r} = \gamma \cdot m_2 \cdot r^2 = a_c. \]  

(7)

enough for description of planetary motion. Just connection of centripetal acceleration and planetary mass into gravity force made Newton, using his the second principle. So follows directly the gravity law:

\[ F = m_2 \cdot a_c = \gamma \cdot m_1 \cdot m_2 / r^2. \]  

(8)

The previous consideration gave a possibility for definition of the gravity constant in solar system as Kepler’s constant on mass unit of central body, i.e. via kinetic parameters of planetary body :

\[ \gamma = \frac{v^2 r}{m_1}. \]  

(9)

Also, we see that gravity constant connect Kepler’s kinetic concept with Newton’s dynamical concept in description of motion. Mass stay as a cause of motion, but kinetic parameters describe motion. (The other known explanation is Einstein’s space curvature, introduced 3 century later, into different mathematical concept.) In the pairs of body as Earth – Moon and other planet – satellite, the same numerical value were obtained for gravity constant, so that can be word on the universality of this constant. Validity of this constant for gravity attraction between two body in relative quite confirmed Cavendish (1798).

From inverse procedure, i.e. by application of Newton formula to binary star system (where mass ratio is not much bigger than 1, how it is in solar system) obtains more general formula for gravity constant:

\[ \gamma = r^2 \left( \frac{v_1^2}{m_1 r_1} + \frac{v_2^2}{m_2 r_2} \right). \]  

(10)

where are \( v_1, v_2 \) – baricentric velocities, \( r_1, r_2 \) – baricentric distances both bodies, and \( r = r_1 + r_2 \). It is important underline, because a general trend is colloquial present in many text books on the gravity constant as only proportionality parameter for dimensional equalization.
Vujčić made in Principles of Mechanics [6] an critical consideration and overview of foundation of mechanics, where gave few important critical opinion and suggestion very important from conceptual standpoint. As a result, by application of corrected procedure to two body motion appeared gravity law formula in some different form (see also [7],[8]):

$$\mathcal{F}_2 = \frac{m_1 m_2}{\rho^2} , \quad \chi = \frac{\hat{\rho}^2 + \rho \ddot{\rho} - \rho \dot{v}_{\text{or}}^2}{m_1 + m_2} , \quad (11)$$

where $\rho$ denote previously used $r$ and $\hat{\rho}, \ddot{\rho}$ - radial velocity and acceleration. Authors opinion is that this form of gravity law is more general than Newton's. Our opinion is the same, but after small intervention. Applied in given form equivalent of gravity constant (11) appears not constant, as it is (9). Physically, we underline two remarks:

Fig. 2. The Moon’s face visible from the Earth is the same, modified in period 19 years 11.3 day by Moon phases and lunar libration in latitude and longitude. This unique composite picture obtained Hubble space telescope. (Photo: NASA)
it is confirmed as reality that values of gravity force depend from distance not linear than squared (for two body as considered examples it is not controversial),

b/ constante in Newton formula really present a constant. (Into interval of time used for measurement, it is not discutable, too.)

This two fact previous formula give not. How harmonize new result with physical facts? The simplicity in natural science is often present, and here it appears as simultaneous multiplying and dividing with distance:

\[ F = \frac{m_1 m_2}{\gamma \rho^2}, \quad \gamma = \frac{\rho^2 + \rho \ddot{\rho} - v_0^2}{\rho}, \quad (12) \]

This form of Vujičić equation is fully adequate and really present generalization of Newton’s formula, and \( \gamma \) is equal to known gravity constant.

5. INERTIAL AND GRAVITY MASS

Before of application this corrected formula to the lunar orbit paradox we wish underline the importance of dual nature of the mass, which follow from kinetic description of motion. Galilean physics describe radial motion in gravity field, Kepler – Huygens – Newton’s physics describes circular / tangential motion around field source.

Einstein (1905) derived complete (kinetic) physical theory of gravity from Galilean starting position. Oetwes (1911) confirmed in ingenious experiment impossibility of differentiate in real measure eventually different numerical mass values for inertial and gravity mass. Eight decade later Hayashy derived complete physical theory of gravity starting from circular motion.

Einstein’s work on theory relativity presents just an example of building physical theory based on philosophical concept. In principles of general theory of relativity Einstein introduced the assumption on equivalence between each mass, gravity, inertial, electromagnetic, etc. But the equivalence is not the same as to be identical. Into same principle implicit is built a reserve, that in any way can exist situation in which these peculiar properties can be dominant, and can not be ignored.

6. THE PARADOX EXPLANATION

Vujičić’s formula for gravity force applied with purpose to calculate acceleration obtain form:

\[ a = \frac{F}{m_1 m_2 / (m_1 + m_2)} = \frac{\rho^2 + \rho \ddot{\rho} - v_0^2}{\rho^2}, \quad (13) \]

Strictly, this formula content two type of acceleration - \( a_g \) - which is related to gravity mass, \( a_i \) - related to inertial mass:

\[ a = a_g + a_i = \frac{\rho^2 + \rho \ddot{\rho} - v_0^2}{\rho^2}, \quad (14) \]
This formula we applied to explain and cancel paradox of lunar orbit. Orbital data - mean distance ($\rho$), period (T), eccentricity (e) and mean tangential velocity (v) for the Earth and the Moon used in calculation are:

$$\rho_{SZ} \approx 1.496 \cdot 10^{11} (m); \quad e = 0.01667$$

$$T_{SZ} = 365.2422 \cdot 86400 (s); \quad v_Z = 29700 (m/s)$$

$$\rho_{ZM} \approx 3.844 \cdot 10^8 (m); \quad e = 0.0549$$

$$T_{MZ} = 29.53 \cdot 86400 (s); \quad v_M = 1020 (m/s)$$

Both derivation of distance as mean value can be calculated via orbital eccentricity:

$$\rho = \frac{\rho \cdot e}{T}; \quad \rho = \frac{\rho}{T}.$$

Calculated values for the Sun - Moon / Earth system (index SM) and the Earth - Moon system (index EM) are:

$$(\rho_{SM}^2 / \rho) = 0.068 (\mu \cdot g); \quad \rho_{SM} = 4.084 (\mu g); \quad (v_{or}^2 / \rho)_{SM} = 601.052 (\mu g)$$

$$(\rho_{EM}^2 / \rho) = 0.290 (\mu \cdot g); \quad \rho_{EM} = 5.288 (\mu g); \quad (v_{or}^2 / \rho)_{EM} = 275.898 (\mu g).$$

Here acceleration is expressed in gravity acceleration $g = 9.81 (m/s^2)$ at Earth’s surface as unit, giving for acceleration:

$$a_{SM} = 4.152 (\mu g) - 601.052 (\mu g)$$

$$a_{EM} = 5.578 (\mu g) - 275.595 (\mu g).$$

Evidently, summary value is practically the same which gave Newton’s formula, but here we have possibility to separate acceleration connected with gravity mass (the first term) and with inertial mass.

Solar acceleration to the inertial mass of the Moon (the second term) is greater than Earth’s, 2.18 times but Earth’s acceleration to the gravity mass of the Moon (the first term) is 1.34 times greater than solar acceleration. The relation between inertial mass acceleration and gravity mass acceleration is equal 1.622 what is very close (99.97%) to Fibonacci golden ratio number, with meaning of stable harmonized ratio.

This can be explained as generic origin of the Earth and the Moon, strictly – the same primary mass from which were built both body, with the same kinetic properties according to the Sun as central body. This can be read just as the same what implicitly assumed Delaunay solving problem of lunar orbit!

If it is correct idea, paradox of lunar orbit is canceled. We have additional argument to support this opinion – the same side of the lunar surface (Fig. 2) is permanent visible from the Earth. What is realy meaning of this fact? Answer gave P. Savić and R. Kašanin in monograph “The Behaviour of the Materials under high Pressures”, I - IV, 1962 - 1965. Here we quote last paragraph No 22 in the fourth part of monograph [9]:
"The Moon: By studying the ionization of various elements (as we have done in part III), we reached the conclusion that the ionization due to pressure can be brought about at the earliest moment during transition from phase 2 to phase 3 (for instance, in case of aluminum). If thus, a certain celestial body has not the phase 3 in its interior, because of its small mass, then it certainly does not have a magnetic moment, no matter of which material it is made.

This is the case, for instance, with our Moon, since its mass is small for such a process; this was shown in parts II and III. This is why the Moon has neither a magnetic moment nor a rotation of its own."

Finally, we can add the sense of this comment, in context of our consideration: The Moon is generic originated with the Earth. Also, the same conclusion derived N. Tesla (1919) from analysis of lunar mechanical motion [10].

7. NIELSEN’S INTERPRETATION OF GRAVITY

Previous exposed present only one from few different form of gravity conceptual interpretation. For correct presentation mathematical forms must be in the same conceptual frame, what many people did not. Analogy between electricity and gravity was subject of many authors, but mostly not in the correct way. Maxwell’s equations are the crown of classical physics. It is not well known that exist analog equations form for gravity, too [11], [12], by Nielsen. For fully understand result exposed here, Nielsen’s paper is crucial appendix which must be present in the consciousness.

Nielsen introduced in fully correct way rotational gravity field as analog form with electricity and magnetic field, starting from special theory of relativity and invariance of electric charge. Static and dynamic components of electric interaction (here important – with very different amplitude, much stronger static) as conceptual correct notation, following formal mathematical analogy Newton’s and Coulomb’s formula obtained from measurement, must posses full analog for gravity, too. Electromagnetic induction as consequence of relativistic Thomas rotation, generate the same effect for gravity, what Nielsen shown. It is bright final completing of classical physics, but in the time (year 1972) in which it is not in main stream of physics, and so in fact - ignored!

Nielsen used Lorentz equations for position, time, velocity and force, and presumption that gravity mass is Lorentz invariant, too. Newton’s formula obtain form

\[ F_g = \frac{m_1 \cdot m_2}{\lambda_0 \cdot 4\pi \cdot r^2} \frac{r}{|r|} \]  

in which gravity constant is changed into form equivalent to electric constant

\[ \lambda_0 = -\frac{1}{4\pi \cdot \gamma} \]  

in Coulomb’s formula

\[ F_e = \frac{q_1 \cdot q_2}{\epsilon_0 \cdot 4\pi \cdot r^2} \frac{r}{|r|}. \]

Electric charges of the same sign show repulsiveness, charges of the different sign attractiveness. Gravity interaction is only attractiveness. Nielsen searched full analogy, so that mathematic isomorphism must be physical content, too. Formal analogy with
electricity is possible in physical sense if introduce two different mass, too. So appeared except of positive mass in static, negative mass in motion, what follows from Newton’s formula. Conceptual, this is condition sine qua non for analogy. And, this condition really equalized both law in the frame of classical physics.

Maxwell’s, for electricity:

\[ \vec{v} \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0}, \quad \vec{v} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \vec{v} \cdot \vec{B} = 0, \quad \vec{v} \times \vec{B} = \mu_0 \cdot j_e + \mu_0 \cdot \varepsilon_0 \cdot \frac{\partial \vec{E}}{\partial t} \]  

(19)

Nielsen’s, for gravity:

\[ \vec{v} \cdot \vec{G} = \frac{\rho_g}{\lambda_0}, \quad \vec{v} \times \vec{G} = -\frac{\partial \vec{N}}{\partial t}, \quad \vec{v} \cdot \vec{N} = 0, \quad \vec{v} \times \vec{N} = K_0 \cdot J_g + K_0 \cdot \lambda_0 \cdot \frac{\partial \vec{G}}{\partial t} \]  

(20)

Here are \( \varepsilon_0, \mu_0 \) - dielectric constant and magnetic permeability of vacuum, \( \lambda_0, K_0 \) - gravity constant and eddy permeability for mass in vacuum, \( \rho_g, \rho_e \) - density of mass and charge, \( j_e, J_g \) - density of charge and mass current. It is obvious that nature of acceleration given by formula presented here

\[ a = a_g + a_\rho = \frac{\rho^2 + \rho \dot{\rho}}{\rho} - \frac{a^2}{\rho} \]  

(21)

is in congruence with Nielsen’s conclusions, because tangential component of velocity (3rd term in formula) produce acceleration in direction normal to motion (for negative mass). If radial acceleration component (2nd term in formula) determines (static) radial acceleration for positive mass, radial velocity (1st term in formula) determines acceleration normal to radial acceleration, what can be responsible for baricentric motion, and evolution of circular orbit to eliptic.

8. REMARKS ON THE BARICENTRIC MOTION

Solar system are described in different paradigm, depending of accuracy in measurement. In all presentations planets motion is described as “around the Sun”, Moon motion “around the Earth”. Baricenter of solar system describes curve like pulsating (Arhimedes) spiral [13], and baricenter can be distant from center of the Sun up to 2.3 solar radius. (Fig.3.)

Objection of some criticist was that the Moon and the Earth motions must be described as motion around of his baricenter, and around the Sun. It is Delaunay interpretation, mostly correct mathematically. Baricenter of lunar motion is always into Earth. Conceptual correct is just motion around the Earth.

Each opinion with pretension to explanation must be presented in conceptual frame so that it can exist in time longer than time in which is reported. Mathematic is fundment of physics, but it is not physics. Needs measurement, concept, experiment, modeling, etc, what leads to development by permanent expansion of physics into other scientific area, also in philosophy.
9. CONCLUSIONS

The law of gravity interaction between two body was derived by Newton, 1687, primary from Kepler laws for planetary motion and few axioms which established dynamics. It has been applied on the lunar motion around the Earth and Earth's motion around the Sun. Newton's formula gives greater force for attraction of the Moon by the Sun than by the Earth. However, central body in lunar (primary) orbit is the Earth, not the Sun. Theoretical foundation of physics stay at formal logic and philosophical concepts. («Physics is an attempt of conceptual construction of the real world and its legal structure.» [14])

So appeared paradox which were ignored from competent specialist, because the most important problem, determination of lunar orbit, was immediately solved sufficiently by mathematical ingenuity - introducing the Sun as dominant body in the three body system (Delaunay, 1860). On this way the lunar orbit paradox were situated in the corner, not canceled. Vujićić (1998) in critical consideration made an overview of principles of mechanics. As an example of application corrected procedure was obtained gravity law in some different form, which gave possibility to cancel paradox of lunar orbit. With our small intervention presented as follows in text, the result of Vujićić present a generalization of classic gravity law. This formula content two type acceleration, one related to inertial mass, the second related to gravity mass. This appendix related to gravity mass carry information on generic origin of the Earth and the Moon, i.e. information that these two body present finally formation from the same initial mass condensed in process of planet birth in solar system genesis.
With small intervention by author, which we made here, little different formula related to Newton's formula for gravity law, which derived Vujičić in strictly defined circumstances, really present more general form of gravity law in classic physics. This formula content two type acceleration, one related to inertial mass, the second related to gravity mass. This appendix related to gravity mass carry information on generic origin of the Earth and the Moon, i.e. information that these two body present finally formation from the same initial mass condensation in process of planet birth in solar system genesis. Nielsen’s analogue to gravity of Maxwell equations for electricity is conceptual fully congruent with our conclusions.

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PARADOKS MESEC&Eacute;VE PUTANJE

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Ključne reči: paradoks mesečeve putanje, zakon gravitacije, gravitaciona masa, inerciona masa, koncepti u fizici.

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