ABSTRACT: In order to reduce the exchange-rate risk, banks in emerging markets are typically denominating their loans in foreign currencies. However, in the event of a substantial depreciation of the local currency, the payment ability of a foreign-currency borrower may be reduced significantly, exposing the lender to additional default risk. This paper analyses how the exchange-rate risk of foreign-currency loans spills over into default risk. We show that in an economy where foreign-currency loans are a dominant source of financing economic activity, depreciation of the local currency establishes a negative feedback mechanism that leads to higher default probabilities, reduced credit supply, and reduced growth. This finding has some important implications that may be of special interest for regulators and market participants in emerging economies.

KEY WORDS: foreign currency loans, exchange-rate risk, default risk, banking regulation, integrated risk analysis

JEL CLASSIFICATION: E51, E58, G21, G28, G32
1. INTRODUCTION

The growing globalisation of capital markets made economies more reliant on foreign finance. Companies are increasing their international presence in order to improve their investment opportunities and diversify their assets. Similarly, banks and other financial institutions are borrowing and investing beyond national borders.

Today, on average, 40 to 45 percent of bank deposits in Emerging Europe, Latin America and the Middle East are denominated in or indexed to a foreign currency (International Monetary Fund, 2004). Patterns of such direct or indirect currency substitution, commonly known as “dollarisation”, are highly uneven: in some countries (e.g., Uruguay, Lebanon and Croatia) foreign-currency deposits greatly exceed domestic currency deposits, while in others (e.g., Brazil) their share is zero because banking legislation does not permit the holding of foreign-currency deposits.\(^1\)

In the event of domestic currency devaluation, the right-hand sides of banks’ balance sheets would be greatly inflated if the foreign-currency liabilities were dominant. In order to reduce the exchange-rate risk of their balance sheets, banks in many emerging markets hold a significant fraction of their assets in loans that are pegged to a „hard currency”. Thus, most local foreign-currency deposits are offset by domestic foreign-currency loans, not by assets held abroad. As a result, the banking sectors’ net foreign asset positions are typically positive but close to balance. From the point of view of the market risk department, currency risk is fully hedged in this way, since the exchange-rate exposure of assets approximately covers the exchange-rate exposure of liabilities. In addition, as Galai and Wiener (2009) show, foreign-currency borrowing may be cheaper when the exchange rate is positively correlated with the return on assets. In this way, dollarisation of placements also helps to reduce the costs of their financing.

Another aspect of dollarisation is that it allows the banking system to be the source of large foreign-currency liquidity needs in times of crisis. Large liquid

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\(^1\) Dollarisation of liabilities is typical for economies whose trade is dominated by imports. Within the European Union, there are monetary independent countries that have a trade deficit with the Eurozone, such as Hungary, where dollarisation seems natural. On the other hand, in export-oriented emerging markets foreign currency-denominated liabilities are scarce. This is the case with the Czech Republic, whose 2008 trade balance surplus with the Eurozone was around 3 billion Euros with an increasing tendency, leading to a very stable Czech Koruna.
foreign-currency assets can increase the resilience of dollarised banking systems both because they may be a source of emergency liquidity, and because these assets typically continue to perform in the event of moderate domestic shocks.

Exchange rate hedging through foreign currency-denomination of assets therefore seems straightforward and natural. However, such reasoning implicitly assumes that borrowers continue to service their debt obligations independently of the exchange rate volatility, i.e. that exchange rate fluctuations do not impact credit risk. In the case of small exchange rate moves such an assumption generally holds: for example, borrowers usually prefer not to default on their mortgage payments unless absolutely necessary. Nevertheless, in the event of a large exchange rate adjustment, this may not be true. Indeed, most individual borrowers have a stream of income that remains relatively stable in domestic currency. That implies that their obligations, expressed in terms of domestic currency, may significantly increase while their ability to pay may stay the same or even diminish. This is a mechanism by which exchange-rate risk gets transmitted into credit risk.

In this paper, we focus on the mechanisms and consequences of interaction between exchange rate and credit risk. In particular, we study how the likelihood of default of a foreign-currency borrower may substantially increase in the case of a significant depreciation of the local currency. We explore how the increased default risk feeds back, negatively, into the supply of credit in the economy. As we will show, this leads to a reduction of growth and initiates a new wave of procyclical defaults.

In financial literature, the issue of the currency structure of loans is usually analysed from the point of view of the borrower. A common approach used by academics and policy-makers is to focus on the mismatch between the currency composition of assets and liabilities held by corporations and sovereigns. A significant number of papers show that currency mismatch is an important factor in financial crises, particularly in developing economies. This paper rather concentrates on the implications that reduction in the payment ability of foreign-currency borrowers may have on the entire financial system.

The remainder of the paper is structured as follows: Section 2 develops a model of interaction between the exchange rate and default risk. Section 3 discusses

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the model equilibrium and explains how currency risk spills over into default risk, creating a negative feedback effect. Our results reinvigorate the notion that in emerging markets an appropriate risk measure should integrate exchange rate and credit risk. This has important implications for regulators and the financial industry, especially in small emerging economies. These implications are discussed in Section 4. Section 5 concludes. Proofs of the propositions are given in the Appendix.

2. THE MODEL

Ever since the pioneering work of Robert Merton (1974), there has been a substantial number of publications on various issues related to credit risk modelling (see Duffie and Singleton, 2003, or Capuano et al., 2009, for a review of credit risk models). Far less effort, however, has been dedicated to the interaction of credit and market risk and, in particular, to the interaction of credit and exchange rate risk. One of the first models of credit and market risk interaction can be found in Jarrow and Turnbull (2000). This paper studied the interaction of credit and interest rate risk. More recently, Alessandri and Drehmann (2007), among others, have focused on the issue of whether separate estimation of market and credit risk, which is currently a standard banking practice, under or overestimates the total risk. Breuer et al. (2008) compare regulatory capital based on integrated credit and exchange-rate modelling with the sum of the regulatory capital when credit and exchange-rate risk are treated separately. Using econometric methods of Pesaran et al. (2000) and Pesaran et al. (2006), they find that required regulatory capital for Swiss Franc-denominated loans in a stylized Austrian bank may be significantly underestimated if credit and currency risks are considered to be independent.

In this paper, the transfer of currency into credit risk is modelled within the framework of a financial market in a partial equilibrium setup with rational expectations. The economy lives for one period, between dates 0 and 1. The banking sector is represented by a single commercial bank, which we loosely refer to as the representative bank. The local interbank market will be inessential for our analysis. At time 0, the representative bank borrows the amount \( B \) in foreign currency (say, from the parent bank) at the exogenous foreign interest rate \( r \). The bank immediately places \( B \) locally, at the interest rate \( r + s_i \), where \( s_i \) is the spread
that accounts for the default risk premium\(^3\) associated with the borrower \(i = 1, 2, ..., n\). At time 0, the bank selects the set of spreads it charges to its clients, \(\{s_i\}_{i=1, 2, ..., n}\), in order to maximize the expected profit. At the end of the period the bank is liquidated and the proceeds are distributed to its shareholders, while the individual agents consume their final income.

There is a single foreign currency and both the bank and the agents can exchange it for domestic currency at a single spot exchange rate without a bid-ask spread. For simplicity, we also assume that there is no informational asymmetry among market participants, that the foreign exchange rate is exogenous, that there are no loans in domestic currency and that domestic currency is in fixed supply. Hence, a central bank that would target the domestic currency interest rate or intervene in the interbank market is unnecessary for the model, although the introduction of a central bank would be the logical next step in our analysis. In addition, we assume that there is no secondary market for loans, including securitization, in this economy. The latter assumption is quite realistic if we wish to apply the model to an emerging economy.

The model incorporates \(n\) distinct agents who act as rational borrowers. We can think of them as households or firms, each with its own compensated demand for money. At time 0, agent \(i\) borrows from the bank the amount \(b_i\), denominated in the foreign currency, at the interest rate \(r + s_i\). We assume that initially all the borrowers have sufficient payment ability \(A_{0,i}\) to qualify for a loan. The total foreign-currency amount lent by the bank to the private sector at time 0 is given by the sum of individual placements:

\[
B = \sum_{i=1}^{n} b_i \tag{1}
\]

Agent \(i\) pays the loan back in full only if her final payment ability, expressed in the domestic currency, exceeds her liability – that is, if \(A_{1,i} \geq X_1 b_i (1 + r + s_i)\), where \(X_1\) is the exchange rate at time 1. Otherwise, the agent defaults and repays only the amount \(A_{1,i}\). Following Breuer et al. (2008), we assume that the payment ability evolves according to

\[
\ln \left( \frac{A_{1,i}}{A_{0,i}} \right) = \ln \left( \frac{Y_1}{Y_0} \right) + \varepsilon_i \tag{2}
\]

\(^3\) The spread can also account for other premia and margins. We focus our analysis on the default-risk aspect of the overall premium.
where $Y_t$ is the value of GDP at time $t = 0, 1$, whereas

$$\epsilon_i \sim N\left(-\sigma^2_A / 2, \sigma^2_A\right)$$

for any $i$. The intuition behind the process given by (2) is that the payment ability will be higher, on average, during periods of economic growth and lower during recessions. The total foreign currency amount repaid at time 1 by agent $i$ is therefore

$$\min\left\{\frac{A_{t,i}}{X_t}, b_i (1 + r + s_i)\right\} = (S_{1,i} - \max\{S_{1,i} - K_i, 0\}), \quad (3)$$

where $S_{t,i} = A_{t,i} / X_t$ is the payment ability in foreign currency at $t = 0, 1$, while $K_i = b_i (1 + r + s_i)$, \quad (4)

is the foreign currency amount due at time 1. The payoff schematic is given in Figure 1.

**Figure 1.** Possible payments by agent $i$ at time 1.

As a standard assumption, the demand for loans, $m_i$, decreases with the lending rate offered by the representative bank. In addition, we assume that it is positively correlated with the expected GDP at time 1 expressed in terms of the foreign
currency. We assume that the borrowers at time 0 rationally anticipate GDP at time 1, and hence implicitly their expected income at time 1. The borrowers adjust their loan demand on the initial date in order to smooth their consumption over time. The money demand function manifests the standard Hicksian elements whereby it responds positively to expected foreign-currency income and negatively to interest rates. In particular,

$$\ln(m_i) = \alpha + \beta \mathbb{E}_{1} \left[ \ln \left( \frac{Y_1}{X_1} \right) \right] - \gamma (r + s_i).$$

(5)

for every $i$, with $\alpha, \beta$ and $\gamma$ positive (see, for example, Goodhart et al., 2005). An agent can invest the amount borrowed in production – directly, in case of a company, or indirectly as a stakeholder in case of a household. There are no bank deposits available to agents or other investment opportunities. Thus, all the borrowings from time 0 are ploughed into production and will be reflected onto GDP at time 1. Therefore, in condition (5), the foreign-currency income at time 1 is entirely captured by $\frac{Y_1}{X_1}$.

On the other hand, higher credit extension generates a positive real balance effect that raises consumption demand and, ultimately, the GDP. Hence, we can endogenise GDP by assuming that it is a positive function of the aggregate credit supply (expressed in domestic currency), available at the beginning of the period:

$$\ln Y_1 = \phi_0 + \phi_1 \ln (BX_0) + \eta,$$

(6)

where

$$\eta \sim N \left(0, \sigma^2 \gamma \right)$$

is the exogenous shock to GDP, uncorrelated to all $\varepsilon_i$. Since we assume that domestic currency is in fixed supply, there is no inflation in this economy, and hence real and nominal GDP growth will coincide. The absence of inflation is not quite realistic, especially in the context of an emerging economy. However, introducing a variable supply of domestic currency would not only complicate the model, but also make the effects of interaction of exchange rate and default risk more difficult to isolate.

The equilibrium in the economy is characterised by a set of interest rate spreads $\{s_i\}_{i=1, 2, ..., n}$ charged by the bank. The representative bank maximizes its expected profit $\Pi$, which is equal to the expected value of its assets (given by the right-hand
side of (3) summed across all the agents) net of the bank’s end-of-period liabilities (i.e., the amount that the bank has borrowed abroad, $B$, plus the associated interest):

$$\max_{\{s_i\}} \mathbb{E}(\Pi) = \mathbb{E}\left[ \sum_{i=1}^{n} \left( s_{i,t} - \max \{s_{i,t} - K_i, 0\} \right) \right] - B(1 + r). \tag{7}$$

The summand in the expectation on the right hand side of (7) has the same form as a payoff of a portfolio consisting of a long position in an asset (in this case, the foreign-currency amount that the borrower is able to pay at the end of period) and a short position in a call option on this asset. Such payoff structure is common for any risky debt (see Merton, 1974).

The equilibrium is established in the following way. The bank maximises the expected future profit by choosing the optimal interest rates (i.e. spreads). When the optimal spreads are substituted into (5), we obtain the optimal individual loan demands. The corresponding compensated credit demand levels will give the borrowers the optimal expected utility levels in equilibrium. Finally, the equilibrium total demand and supply of foreign-currency loans have to be equal at any given moment:

$$M = B \tag{8}$$

where $M = \sum_{i=1}^{n} m_i$ is the total demand for loans at the beginning of the period. Condition (8) determines the amount of foreign-currency loans initially supplied by the bank, $B$, given by condition (1).

4. THE SPILLOVER EFFECT

**Proposition 1.** Agent $i$ defaults at time 1 with probability $\text{PD}_i = \mathbb{N}\left(-d_{z,i}\right)$.

Here, $\text{PD}_i = \mathbb{P}\left(S_{z,i} < K_i\right)$ is the probability of default for agent $i$, $\mathbb{N}(\cdot)$ is the cumulative density of a standard normal distribution, while

$$d_{z,i} = \frac{\ln(S_{z,i} / K_i) + \mathbb{E}[\ln(Y_t / Y_0)] - \ln(X_t / X_o) - \sigma^2 / 2}{\sqrt{\sigma^2 + \sigma^2_t}}. \tag{9}$$
Proposition 1 hence implies that, *ceteris paribus*, an increase in the exchange rate would lead to a decrease in \( d_{z,i} \), which would in turn lead to an increase in default probabilities in the economy. The sensitivity of default probability with respect to changes in the exchange rate may be substantial. Figure 2 illustrates a generic case where the foreign currency is allowed to appreciate or depreciate by 20 percent, while the loan amount and interest rate spreads are fixed. The figure shows the probability of default, \( PD_i \), relative to the baseline case, in which there are no changes in the exchange rate or GDP. The corresponding probability of default is labelled by \( PD_i^0 \).

**Figure 2.** Behaviour of the probability of default as a function of exchange rate.

Proposition 1 implies two important results, summarized in Propositions 2 and 3.

**Proposition 2.** A decrease in the expected growth rate of GDP leads to an increase in default probabilities.

**Proposition 3.** If \( \phi_i \beta < 1 \), an increase in the interest rate spread charged to borrower \( i \) leads to a higher default probability \( PD_i \).
A *combination* of the effects described in Propositions 2 and 3 may lead to a substantial fragility of the financial system. In fact, we will show that for such a combination to arise it is sufficient to have a depreciation of local currency.

**Proposition 4.** The maximand in (7) can be written as

\[
\sum_{i=1}^{n} S_{0,i} e^{\mathbb{E}[\ln(Y_i/Y_b)|-\ln(X_i/X_b) + \sigma_f^2 + \sigma_y^2}/2} \left(1 - N(d_{1,i}) \right) + K_i N(d_{2,i}) - B(1 + r),
\]

(10)

where

\[
d_{1,i} = d_{2,i} + \sqrt{\sigma_A^2 + \sigma_y^2}
\]

(11)

and \(d_{2,i}\) is given by (9).

Using Proposition 4, the term in the bracket in (10) has the following interpretation.

The expression

\[
S_{0,i} e^{\mathbb{E}[\ln(Y_i/Y_b)|-\ln(X_i/X_b) + \sigma_f^2 + \sigma_y^2}/2} \left(1 - N(d_{1,i}) \right).
\]

can be interpreted as the foreign-currency equivalent of the expected payment ability of agent \(i\), in the case that he defaults by the end of the period. Similarly, using Proposition 1, the second term in the bracket in (10) can be written as

\[
K_i N(d_{2,i}) = K_i \left[1 - N(-d_{2,i}) \right] = K_i \left(1 - PD_i \right).
\]

This is the face value of the loan if fully recovered, multiplied by the survival probability (cf. Figure 1).
Proposition 5. The first-order condition for the optimization problem in (6) is given by

\[\sum_{j=1}^{n} \left[ S_{0,j} e^{x_i - \ln(x_i/x_0) + \sigma_A^2 + \sigma_X^2/2} \left( 1 - N(d_{i,j}) \right) \right] - \sum_{j=1}^{n} \left[ S_{0,j} e^{x_i - \ln(x_i/x_0) + \sigma_A^2 + \sigma_Y^2/2} \frac{f(d_{i,j})}{\sqrt{\sigma_A^2 + \sigma_Y^2}} - K_j \frac{f(d_{2,j})}{\sqrt{\sigma_Y^2 + \hat{\sigma}}} \right] - \frac{1 - \phi_1 \beta}{\gamma} e^{\gamma t} b_i N(d_{2,i}) \sum_{j=1}^{n} e^{-\gamma t_j} - \frac{1}{\phi_1} B(1 + r) = 0,\]

for all \( i \), where \( f(\cdot) \) is the probability density function of a standard normal random variable.

Equation (12) determines the set of optimal spreads \( \{s_i\}_{i=1}^{n} \) that the bank charges to the borrowers in equilibrium, provided that its left-hand side has a real zero. Given that the equation (12) does not allow for a closed-form solution for optimal spreads, the system of equations has to be solved numerically.

To analyze the equilibrium, we fix the model parameters in the following way. The annualized interbank rate is set to 5%, i.e. \( r = 0.05 \). The annual volatilities are \( \sigma_A = 0.2 \) for the payment ability and \( \sigma_Y = 0.04 \) for the GDP growth rate. The money demand function has the parameters \( \alpha = 0.1, \beta = 0.5 \) and \( \gamma = 0.8 \), while the parameters in the condition (6) that determines the rate of endogenous growth are set to \( \phi_0 = 0 \), and \( \phi_1 = 0.1 \). We focus on an individual borrower whose initial payment ability (when the loan was granted) was 10 percent above the borrowed amount, i.e. \( S_{0,i} = 1.1 b_i \), and investigate the effect of appreciation or depreciation of the foreign currency with respect to the domestic one. The equilibrium is obtained by simultaneous numerical solving of (4), (5), (6), (8) and (12) using the Brent algorithm.\(^4\)

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\(^4\) The Brent (1973) algorithm uses the combination of bisection, secant, and inverse quadratic interpolation methods.
Figure 3. Behaviour of the equilibrium total supply of credit as a function of exchange rate.

Figure 3 illustrates the behaviour of the equilibrium total supply of credit as a function of the exchange rate. Relative to the case when there is no change in the exchange rate, $X_1 = X_0$, the total supply of credit increases when foreign currency depreciates, and vice-versa. The intuition is rather simple: a higher exchange rate implies higher optimal spread, and hence lower equilibrium demand for credit. Thus, with a higher exchange rate the local currency becomes cheap and consequently the foreign-currency loan becomes more difficult to afford.

A decrease in credit supply in the economy feeds backs negatively into the GDP through condition (6). This leads to reduction of growth. Figure 4 displays the principal outcome of the feedback effect: if the foreign currency appreciation is significant, the expected growth rate can become negative for the particular choice of parameters. If in addition the economy is in a downturn, which will be the case for an interval of realisations of the variable $\eta$ in condition (6), the effect is even stronger.
Thus, a spillover of the exchange-rate risk into default risk works in the following way. First, the exchange-rate risk increases the default risk of a foreign currency borrower directly, through the optionality embedded in the payoff structure of a foreign-currency risky debt, as given by (3). This is a first-order effect that would exist even if the GDP were assumed to be exogenous, as in Breuer et al. (2008). Next, higher default risk implies higher interest rates, combined with a lower aggregate credit supply. This credit contraction leads to a negative effect on future growth, condition (6), which in turn reduces the credit demand today through a rational expectation of borrowers. As a result, the equilibrium supply of credit will be further reduced, which creates a circle of negative feedback, see Figure 5. The economy ends up in an equilibrium in which the growth may slow down or decrease substantially.
Figure 5. Schematic representation of the spillover effect.

We can illustrate the importance of the spillover effect using actual data. Consider, for example, the case of Serbia, where around 65 percent of loans to retail and the corporate sector were indexed to a foreign currency in 2008 (National Bank of Serbia, 2009). The Serbian Dinar (RSD), depreciated from 77.09 RSD/EUR on September 1, 2008, to 95.46 RSD/EUR on March 31, 2009. This corresponds to 24% appreciation of the Euro with respect to the Dinar, or, using notation of the model, \( \frac{X_i}{X_0} = 1.24 \). As the baseline scenario, we will assume the absence of any interaction of currency and credit risk, fixed supply of Euro-denominated credits, and the GDP growth rate equal to zero. The latter assumption would correspond to a stagnation, which might have seemed plausible in September 2008, when the true repercussions of the global economic crisis were still unforeseen. In comparison, when the interaction of exchange rate and credit risk is included, the equilibrium outcome becomes quite different. If parameter values are set to \( r = 0.05, \sigma_A = 0.2, \sigma_Y = 0.04, \alpha = 0, \beta = 0.68, \gamma = 0.8, \varphi_0 = 0, \varphi_1 = 0.1, \) and \( S_{0,i} = 1.1b_i \), the numerical solution of the system of equations given by (4), (5), (6), (8) and (12) shows that the equilibrium supply of credit decreases by about 18%, while the annualized GDP growth rate drops from 0 to about –4%. Hence, the impact of the spillover effect on economic activity may be substantial.

5. PRACTICAL AND REGULATORY IMPLICATIONS

The implications of our findings described in Section 3 raise important issues for financial institutions as well as their regulators. These issues are of particular significance for financial systems in Emerging Europe, where foreign banks are dominant market players and where a significant fraction of loans is denominated in or indexed to a foreign currency. In this section, we discuss possible strategies that commercial and central banks might adopt in order to mitigate the spillover effect.
First, the banking sector needs to consider the alternatives to foreign-currency indexing. For example, one way for the banks to be hedged against the combined exchange rate and credit risk is to denominate their loans in domestic currency, but to tie the interest payments to the consumer price index (CPI). This solution would be a natural substitute for foreign-currency indexing in import-oriented countries (unless, of course, the exchange rate is not in a fixed regime). In such economies, inflation is typically positively correlated with the exchange rate. We see at least two reasons why commercial banks in emerging markets rarely resort to CPI indexing in practice. Firstly, exchange rates are established on a daily basis and always involve some market mechanisms. The official CPI data, on the other hand, are published by government-sponsored statistical offices once every month, and usually with a lag. Secondly, CPI is often calculated using a methodology that is arguably not the most appropriate one to represent the prevailing payment abilities of the borrowers.

An alternative for banks is to maintain the foreign-currency indexing of their credits, but to use combined hedging mechanisms. This, for instance, may include hedging with exchange rate and credit derivatives, achieved through combined positions in currency forwards or swaps and credit default swaps.

On the other hand, a possible regulatory response to prevent the spillover of market into credit risk is to manipulate the obligatory reserve requirements. In order to cushion the effect, the central bank has to provide an incentive for commercial banks to either increase their supply of foreign currency-denominated credit or decrease the interest rates they charge for such placements. If the exchange rate in question is in a floating or managed floating regime, one way to provide such an incentive is to reduce the required reserves for the foreign currency placements whenever the exchange rate increases significantly. This effectively decreases the regulatory costs for banks, which leaves them the opportunity to reduce the spreads in order to be more competitive while remaining profitable. Lower interest rates on foreign-currency loans should then lead to a higher demand for credit. In the framework of our model, the reduction of reserve requirements results in equilibrium where the credit supply and GDP growth rate are higher, probabilities of default are lower, while the bank retains optimum level of the expected profit.

Unlike market risk, the hedging of credit risk with derivatives is more complex. For instance, in Optimum whatse2008 during the financial crisis, AIG could not recover any of the short positions on credit default swaps they held as a credit risk hedge.

Strictly speaking, in our model regulatory costs are not explicitly taken into account. However, the reduction of obligatory reserve requirements for foreign currency-denominated placements will have the same effect as if the bank’s cost of capital r is reduced.
The Global Financial Stability Report (International Monetary Fund, 2009) shows that from the end of 2007 credit growth declined continuously in Europe, where many countries are heavily reliant on cross-border funding which became scarce during the crisis. On the other hand, in Latin America and Asia (excluding China) bank credit growth stabilized in the second half of 2009, suggesting that policy actions have been successful in halting the downward spiral in financial conditions.

An alternative solution for central banks is related to an improvement in the way in which regulatory capital is determined. Regulatory standards, such as those proposed by the Basel Committee, are based on the traditional approach, where the implicit assumption is that market and credit risks are independent. This view parallels the customary organization of bank departments, which are often impermeable to each other. Consequently, procedures for regulatory capital calculations, such as those based on Pillar 1 of the Basel II Accord, treat credit and market risk separately. As a result, the overall regulatory capital is simply the sum of regulatory capitals for credit, market and operational risks. The conventional wisdom is that such regulatory standards are always conservative, since the measure of total portfolio risk should be smaller than – or at most equal to – the sum of individual risk measures. This belief is a natural consequence of the fact that the current regulatory framework is based on the Value-at-Risk (VaR) logic. In spite of being established as a de-facto industry as well as regulatory standard, VaR is often criticized for not being a coherent risk measure. For example, for two sources of risk, X and Y, there might be situations in which \( \text{VaR}(X + Y) > \text{VaR}(X) + \text{VaR}(Y) \). In practice, however, banks’ assets are simultaneously sensitive to both market and credit risk factors. Hence, any change in value of the total portfolio that is separated into a pure-market and a pure-credit risk change may lead to an underestimation of the total risk. This is the case whenever market and credit risk factors exhibit a positive non-linear co-dependence or whenever they are driven by other, common, risk factors in a non-linear fashion. The negative feedback described in Section 3 clearly reinforces the necessity for a careful treatment of market risk in the banking book.

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7 Some forms of market risk in the banking book, such as exchange rate and interest rate risk, do carry additional capital charges under Pillar 2 of Basel II.
8 Furthermore, VaR completely ignores statistical properties of losses beyond the specified quantile of the profit-loss distribution, i.e. the tail risk.
9 Examples of such non-subadditivity of VaR can be found in Artzner et al. (1997, 1999), Acerbi and Tasche (2002) and Breuer et al. (2008).
6. CONCLUSION

This paper contributes to the ongoing challenge of the traditional banking approach of dividing market and credit risk. When portfolio positions depend simultaneously on market and credit risk factors, the nature of the problem changes. Foreign currency loans are a good example where the conventional additivity of risk measures is violated in a potentially dangerous way. In fact, this paper shows that the probability of default of a foreign-currency borrower may increase significantly when the local currency depreciates. In this case, the peril is in fact twofold. First, the non-linear nature of payoffs in a risky debt leads to a positive non-linear co-dependence between the two risk factors. Second, in an economy where GDP is endogenous, the increased default risk reduces the total supply of credit. This creates a negative feedback and leads to a higher default risk through reduced payment ability in the future, that is, a spillover of the exchange-rate risk into default risk. In sum, when the domestic currency depreciates, the rational expectations equilibrium is characterized by a higher default probability, reduced credit supply and reduced growth, compared to the equilibrium resulting from a stable exchange rate.

Our results reinforce the notion that an appropriate risk measure must integrate exchange-rate risk with credit risk. There are mechanisms that market participants and regulators can adopt in order to reduce the negative effects of the spillover of one type of risk into another. In addition, development of a fully integrated approach to risk assessment and mitigation will certainly be of great help to financial institutions in future crises.

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APPENDIX

PROOF OF PROPOSITION 1

Probability of default for agent $i$ can be written as the probability that her payment ability at time 1 is lower than the face value of debt (both in local currency):

$$PD_i = \mathbb{P} \left( A_{1,i} < X_1 K_i \right)$$
$$= \mathbb{P} \left( \ln A_{1,i} - \ln A_{0,i} < \ln X_1 - \ln X_0 + \ln X_0 - \ln A_{0,i} + \ln K_i \right)$$
$$= \mathbb{P} \left( \ln \left( \frac{A_{1,i}}{A_{0,i}} \right) < \ln \left( \frac{X_1}{X_0} \right) - \ln \left( \frac{S_{0,i}}{K_i} \right) \right)$$

Using (2) and then (6) we obtain

$$PD_i = \mathbb{P} \left( \ln \left( \frac{Y_1}{Y_0} \right) + \varepsilon_i < \ln \left( \frac{X_1}{X_0} \right) - \ln \left( \frac{S_{0,i}}{K_i} \right) \right)$$
$$= \mathbb{P} \left( \mathbb{E} \left[ \ln \left( \frac{Y_1}{Y_0} \right) \right] + \eta \varepsilon_i < \ln \left( \frac{X_1}{X_0} \right) - \ln \left( \frac{S_{0,i}}{K_i} \right) \right)$$
$$= \mathbb{P} \left( \eta \varepsilon_i < \ln \left( \frac{X_1}{X_0} \right) - \ln \left( \frac{S_{0,i}}{K_i} \right) - \mathbb{E} \left[ \ln \left( \frac{Y_1}{Y_0} \right) \right] \right)$$

The random variable $\eta \varepsilon_i$ is distributed normally, with an expected value of $-\sigma^2_A/2$ and a variance of $\sigma^2_A + \sigma^2_i$. Hence,

$$PD_i = \mathbb{P} \left( z_i < -\frac{\ln \left( \frac{S_{0,i}}{K_i} \right) + \mathbb{E} \left[ \ln \left( \frac{Y_1}{Y_0} \right) \right] - \ln \left( \frac{X_1}{X_0} \right) - \frac{\sigma^2_A}{2}}{\sqrt{\sigma^2_A + \sigma^2_i}} \right)$$

where $z_i$ is a standard normal random variable. This finally gives

$$PD_i = \mathbb{N} \left( -d_{2,i} \right)$$

where $d_{2,i}$ is defined by (9). ♦
**PROOF OF PROPOSITION 2**

Let

\[ g = \mathbb{E} \left[ \ln \left( \frac{Y_t}{Y_0} \right) \right] \]

be the expected growth rate of GDP. Using Proposition 1 and formula (9), we find

\[
\frac{\partial PD_i}{\partial g} = \frac{\partial \mathcal{N}(-d_{2,i})}{\partial g} = -f(-d_{2,i}) \frac{\partial d_{2,i}}{\partial g} = -f(-d_{2,i}) \frac{\partial d_{2,i}}{\partial \sigma} \frac{\sigma}{\sqrt{\sigma_A^2 + \sigma_i^2}} < 0, \tag{A1}
\]

where \( f(\cdot) \) is the probability density function of a standard normal random variable. In other words, all default probabilities increase when GDP growth rate decreases, and vice-versa. ♦

**PROOF OF PROPOSITION 3**

Let

\[ g = \mathbb{E} \left[ \ln \left( \frac{Y_t}{Y_0} \right) \right] \]

be the expected growth rate of GDP. Using (6), we can write it as

\[ g = \phi_0 + \phi \ln(BX_0) - \ln Y_0. \tag{A2} \]

Next, note that in equilibrium

\[
B = \sum_{i=1}^{n} m_i = \exp(\alpha + \beta g - \beta \ln(X_1 / X_0) - \gamma r) \sum_{i=1}^{n} e^{-\gamma \lambda_i}, \tag{A3}
\]
where the equalities follow from (8) and (5). Combining (A2) and (A3), and then solving explicitly for the equilibrium growth rate, we obtain

\[
g = \frac{1}{1-\phi_1 \beta} \left\{ \phi_0 + \phi_1 \ln X_0 - \ln Y_0 + \phi_1 \left[ \alpha - \beta \ln (X_1 / X_0) - \gamma r + \ln \left( \sum_{i=1}^{n} e^{-\gamma s_i} \right) \right] \right\}. \quad (A4)
\]

This implies

\[
\frac{\partial g}{\partial s_i} = -\frac{1}{1-\phi_1 \beta} \sum_{j=1}^{n} e^{-\gamma s_j}. \quad (A5)
\]

Therefore, (A1) and (A5) give

\[
\frac{\partial \text{PD}_i}{\partial s_i} = \frac{\partial \text{PD}_i}{\partial s_i} \frac{\partial s_i}{\partial g} = \frac{f(-d_{2,i})}{\sqrt{\sigma^2 + \sigma_Y^2}} \left(1-\phi_1 \beta\right) \sum_{j=1}^{n} e^{-\gamma s_j} \gamma e^{-\gamma s_i}.
\]

If \( \phi_1 \beta < 1 \), the last expression will be positive. In that case, the higher the spread, the higher the default probability. ♦

**PROOF OF PROPOSITION 4**

First, note that the expected profit in (7) can be written as

\[
\mathbb{E}(\Pi) = \mathbb{E} \left[ \sum_{i=1}^{n} \left( S_{i,i} - \max \{ S_{i,i} - K_i, 0 \} \right) \right] - B(1+r)
\]

\[
= \sum_{i=1}^{n} \left( \mathbb{E} [S_{i,i} | S_{i,i} < K_i] + \mathbb{E} [K_i | S_{i,i} > K_i] \right) - B(1+r).
\]

On the other hand,
\[ S_{1,i} = \frac{A_{1,i}}{X_1} \]
\[ = \exp \left( \ln A_{1,i} - \ln X_1 \right) \]
\[ = S_{0,i} \exp \left( \ln \left( \frac{A_{1,i}}{A_{0,i}} \right) - \ln \left( \frac{X_1}{X_0} \right) \right) \]

Using (2) and (7) we can write the above expression as

\[ S_{1,i} = S_{0,i} \exp \left( \mathbb{E} \left[ \ln \left( Y_1 / Y_0 \right) \right] - \ln \left( X_1 / X_0 \right) \right) + \eta + \varepsilon_i \]

Then, using the distributional assumptions for \( \eta \) and \( \varepsilon_i \), we can calculate the first conditional expectation

\[ \mathbb{E} \left[ S_{1,i} \mid S_{1,i} < K_i \right] = S_{0,i} \exp \left( \mathbb{E} \left[ \ln \left( Y_1 / Y_0 \right) \right] - \ln \left( X_1 / X_0 \right) \right) \mathbb{E} \left[ e^{\eta+\varepsilon_i} \mid \ln \left( S_{1,i} / K_i \right) < 0 \right] ; \]

\[ \mathbb{E} \left[ e^{\eta+\varepsilon_i} \mid \ln \left( S_{1,i} / K_i \right) < 0 \right] = \mathbb{E} \left[ e^{z_i \sqrt{\sigma_\eta^2 + \sigma_\varepsilon^2} + \frac{\sigma_\varepsilon}{2}} \left| \frac{\ln \left( S_{0,i} / K_i \right) + \mathbb{E} \left[ \ln \left( Y_1 / Y_0 \right) \right] - \ln \left( X_1 / X_0 \right) - \sigma_A^2 / 2}{\sqrt{\sigma_A^2 + \sigma_\varepsilon^2}} \right] \right] \]

\[ = e^{\frac{\sigma_A^2}{2}} \int_{-\infty}^{\ln \left( S_{0,i} / K_i \right) + \mathbb{E} \left[ \ln \left( Y_1 / Y_0 \right) \right] - \ln \left( X_1 / X_0 \right) - \frac{\sigma_A^2}{2}} e^{z_i \sqrt{\sigma_\eta^2 + \sigma_\varepsilon^2} \left( \frac{\sigma_\varepsilon}{\sqrt{\sigma_A^2 + \sigma_\varepsilon^2}} \right)} f(z_i) \, dz_i. \]

After completing the squares, we obtain

\[ \mathbb{E} \left[ e^{\eta+\varepsilon_i} \mid \ln \left( S_{1,i} / K_i \right) < 0 \right] = e^{\frac{\sigma_A^2}{2} + \frac{\sigma_\varepsilon^2}{2}} \mathcal{N} \left( -d_{1,i} \right), \]

where \( d_{1,i} \) is defined by (11). That is,

\[ \mathbb{E} \left[ S_{1,i} \mid S_{1,i} < K_i \right] = S_{0,i} e^{\mathbb{E} \left[ \ln \left( Y_1 / Y_0 \right) \right] - \ln \left( X_1 / X_0 \right) + \frac{\sigma_A^2}{2} + \frac{\sigma_\varepsilon^2}{2}} \mathcal{N} \left( -d_{1,i} \right). \]

Similarly, the second conditional expectation can be written as

\[ \mathbb{E} \left[ K_i \mid S_{1,i} > K_i \right] = K_i \mathbb{P} \left( S_{1,i} > K_i \right) \]

\[ = K_i \left( 1 - PD_i \right) \]

\[ = K_i \mathcal{N} \left( d_{2,i} \right). \]

Putting it all together, we obtain condition (10).
PROOF OF PROPOSITION 5

The first-order condition is obtained by setting the partial derivative of (10) with respect to every $s_i$ to zero. This is equivalent to

\[
\sum_{j=1}^{n} \left[ S_{0,j} e^{\gamma j} \left( 1 - N(d_{1,j}) \right) \frac{\partial g}{\partial s_i} - S_{0,j} e^{\gamma j} \left( 1 - N(d_{1,j}) \right) \frac{\partial f(d_{1,j})}{\partial s_i} \right] \\
+ \sum_{j=1}^{n} \left[ \frac{\partial K_j}{\partial s_i} N(d_{2,j}) + K_j f(d_{2,j}) \frac{\partial d_{2,j}}{\partial s_i} \right] \\
-(1+r) \frac{\partial B}{\partial g} \frac{\partial g}{\partial s_i} = 0. \tag{A6}
\]

We can write

\[
\frac{\partial B}{\partial g} = \frac{1}{\phi_1} e^{(\phi_0 - \phi \ln x_0 + \phi y) \gamma},
\]

where we used (A2). Since $\gamma > 0$, (A5) implies that $\partial g / \partial s_i$ will never be equal to zero. A straightforward derivation from (A6) gives

\[
\sum_{j=1}^{n} \frac{\gamma e^{-\gamma_{s_j}}}{1-\phi_1} \sum_{j=1}^{n} e^{-\gamma_{s_j}} \left[ S_{0,j} e^{\gamma j} \left( 1 - N(d_{1,j}) \right) \right] \\
+ \sum_{j=1}^{n} \frac{1}{1-\phi_1} \frac{\gamma e^{-\gamma_{s_j}}}{\phi_1} \left[ S_{0,j} e^{\gamma j} \left( 1 - N(d_{1,j}) \right) \right] \\
+ b_i N(d_{2,i}) + \frac{1}{1-\phi_1} \frac{\gamma e^{-\gamma_{s_j}}}{\phi_1} B(1+r) = 0,
\]

for all $i$, which is equivalent to (12). ♦