ABSTRACT: Considering a model of discrete demand with two consumers, this article shows that irrespective of the difference between the willingness to pay of consumers with high and low incomes, if interest rates are low, a durable goods monopolist has an advantage in discriminating prices over time. If the difference in willingness to pay is limited and interest rates high, the monopolist has an advantage in setting a price equal to the low-income consumer’s willingness to pay. Finally, in the case of great difference in willingness to pay and high interest rates, the monopolist has an advantage in setting a price equal to the high-income consumer’s willingness to pay, and not selling the durable good to the low-income consumer. The results show that the Coase conjecture can fail if the difference in willingness to pay is great, and interest rates are high.

KEY WORDS: Coase Conjecture, Discrete Demand, Durable Goods, Willingness to Pay

JEL CLASSIFICATION: D21, D42
1. INTRODUCTION

Coase (1972) shows that the price of durable goods can quickly reach marginal cost. From the hypotheses of continuous demand and the difference over time tending towards zero, i.e., time being a continuous variable, the author shows that the monopoly firm can set a price equal to marginal cost immediately in the current period.

The conclusions of Coase (1972) are based on the notion that if consumers foresee a reduction in the price of durable goods in the future, their willingness to pay in the current period diminishes, as they wait for a lower price in the future.


Despite some studies (Gul, 1987; Ausubel and Deneckere, 1989; Sobel, 1991; Waldman, 1996; Denicolò and Garella, 1999; Mason, 2000; Maçãs Nunes, 2006; Maçãs Nunes and Serrasqueiro, 2008), using discrete demand models with consumers with different willingness to pay, specific study of individual and
joint effects of differences in willingness to pay and interest rates on the Coase conjecture has been somewhat neglected. Therefore, this paper aims to fill that gap in the literature on the Coase conjecture.

The specific contribution of this paper is to show that: 1) in the context of discrete demand, 2) a finite number of consumers, and 3) time as a discrete variable, if the difference in willingness to pay of consumers with different income levels is high, and the interest rate is also high, the Coase conjecture fails since the monopoly firm has an advantage in selling only one unit of the durable goods, immediately in the current period, to the high-income consumer, and not reducing the price to satisfy the lesser willingness to pay of the low-income consumer.

The paper is divided as follows, after this introduction. Section 2 presents the model, including the assumptions, the optimization process, and finally, calibration of the model. Section 3 presents the conclusions of the paper.

2. THE MODEL

2.1 Assumptions

The assumptions of the model are now presented. Just as Gul (1987), Ausubel and Deneckere (1989), Sobel (1991), Waldman (1996), Denicolò and Garella (1999), Mason (2000), and Maçãs Nunes and Serrasqueiro (2008), we consider a discrete demand model, in which the utility is the difference between what consumers are willing to pay and what they effectively pay for the durable goods, i.e., \( UT_i = (V_i - P_i)\delta' \), being \( \delta \) is the discount factor given by:

\[
\delta = \frac{1}{1 + r},
\]

where \( r \) is the interest rate. In addition, we consider two periods, the current period (period 1) and the future period (period 2). Just as Maçãs Nunes (2006), we consider that demand is made up of two consumers with different willingness to pay, the high-income consumer (consumer \( i \)) who is willing to pay \( V_i \) for the durable goods, and the low-income consumer (consumer \( j \)) willing to pay \( V_j \) for the durable goods, with \( V_i > V_j \). In the same way as Bulow (1982), Gul (1987), Denicolò and Garella (1999), and Maçãs Nunes (2006), we consider that the monopolist’s marginal costs are constant and equal to zero. At
the end of the two periods, the durable goods become obsolete, and so whether consumers $i$ and $j$ buy the durable goods in the current period or postpone purchase until the future period, they cannot use it after the end of the future period. Finally, we admit that if consumer utility is the same in the current and future periods, they always opt to acquire the durable goods in the current period, i.e., if utility is identical in the current and future periods, consumers prefer to acquire the durable goods in the current period rather than put off purchase until the future period.

### 2.2. Optimization

The monopolist has three options:

A. Sell one unit of the durable goods, at price $V_i$, to the high-income consumer in the current period.

B. Sell two units of the durable goods, at price $V_j$, to the high-income and low-income consumers in the current period.

C. Discriminate prices over time, selling one unit of the durable goods, at a price between $V_i$ and $V_j$, to the high-income consumer in the current period, and one unit of the durable goods, at price $V_j$, to the low-income consumer in the future period.

If the monopoly opts to sell one unit of the durable goods in the current period, given that only the high-income consumer buys one, profit will be:

$$\pi_A = P_1 \times q_1 - 0,$$

and so we have:

$$\pi_A = V_i \times 1 = V_i. \quad (2)$$

If the monopoly opts to sell two units of the durable goods in the current period, given that both consumers (high and low-income) acquire it, profit will be:

$$\pi_B = 2V_j = 2V_j. \quad (3)$$
Finally, if the monopoly opts to discriminate prices over time, to determine the balance between consumers’ expectations and the process of maximizing the monopoly’s prices over time, we must resort to the process of backward induction.

We will define consumers’ utility as their surplus, i.e., the difference between what consumers are ready to pay and what they effectively pay. If consumers acquire the durable goods in the current period (period 1) they are able to use it in two periods, and so the total utility will be given by the sum of utility in the current period and utility in the future period. If consumers choose to acquire the durable goods only in the future period, the total utility they have from use of the durable goods will only be that referring to the future period.

Consumer $i$ (high-income) correctly anticipates the price the durable goods monopoly will set in the future period. Opting to discriminate prices over time, the monopoly firm sets in the future period a price equal to the willingness to pay of the low-income consumer, i.e., $P_2 = V_j$.

The decision of consumer $i$ will depend on utility in the current and future periods. If consumer $i$ acquires the durable goods in the current period, he will have a utility given by:

$$UT_{1+2} = (V_i - P_1) + (V_i - P_1)\delta .$$

(4)

If consumer $i$ chooses to acquire the durable goods only in the future period, his utility will be given by:

$$UT_2 = (V_i - P_2)\delta$$

(5)

Consumer $i$ will be willing to acquire the durable goods in the current period if his total utility from using the durable goods in the two periods is at least equal to the utility from using the durable goods only in the future period. Therefore, the consumer acquires the durable goods in the current period, if:

$$UT_{1+2} = UT_2 \iff (V_i - P_1) + (V_i - P_1)\delta = (V_i - P_2)\delta .$$

(6)
Consumer \( i \) knows the monopoly sets in the future period \( P_2 = V_j \), and so:

\[
(V_i - P_i) + (V_i - P_1)\delta = (V_i - V_j)\delta .
\]  

(7)

For consumer \( i \) to acquire the durable goods in the current period, the monopoly will have to set a price in that period equal to:

\[
P_1 = \frac{V_i + V_j\delta}{1 + \delta} .
\]  

(8)

As already mentioned, in the future period, the monopoly producing durable goods sets:

\[
P_2 = V_j .
\]  

(9)

**Lemma 1:** If a monopoly producing durable goods discriminates prices over time, the price of the durable goods has necessarily a decreasing tendency, because \( V_i > V_j\delta^2 \).

**Proof:**

Considering equations (8) and (9) corresponding to the prices of durable goods in the current and future periods respectively, the price of the durable goods tends to decrease, since \( P_1 > P_2\delta \), because:

\[
\frac{V_i + V_j\delta}{1 + \delta} > V_j\delta ,
\]  

(10)

given that:

\[
V_i > V_j\delta^2 .
\]  

(11)

As the willingness to pay of the consumer of high-income level \( (V_i) \) is greater than the willingness to pay of the consumer of low-income level \( (V_j) \), and the
discount factor \((\delta)\) is less than or equal to 1, then always found \(V_i > V_j \delta^2\), and the price always assume a decreasing tendency over time. It is therefore proven that if a monopoly producing durable goods discriminates prices over time, the price necessarily tends to decrease.

**Lemma 2:** The difference in willingness to pay between the high-income consumer and the low-income consumer, and interest rates, influence the optimal strategy of price-setting by a monopoly producing durable goods.

Proof:

Considering that the durable goods monopoly sells one unit in the current period and another in the future period, discriminating prices over time, the profit over time is given by:

\[
\pi_C = P_1 \times 1 - 0 + (1 \times q_2 - 0) \times \delta, \tag{12}
\]

and so we have:

\[
\pi_C = \frac{V_i + V_j \delta}{1 + \delta} + V_j \delta, \tag{13}
\]

simplifying the terms, we have finally:

\[
\pi_C = \frac{V_i + 2V_j \delta + V_j \delta^2}{1 + \delta}. \tag{14}
\]

Comparing \(\pi_A\), \(\pi_B\) and \(\pi_C\), we find the durable goods monopoly sells only one unit in the current period to the high-income consumer, setting \(P_1 = V_i\), if:

\[
\pi_A > \pi_B \quad \text{and} \quad \pi_A > \pi_C, \tag{15}
\]

and so, simplifying the terms we have:
The durable goods monopoly sells two units of the durable goods to high and low-income consumers in the current period, setting \( P_1 = V_j \), if:

\[
\pi_B > \pi_A \quad \text{and} \quad \pi_B > \pi_C ,
\]

and so, simplifying the terms we have:

\[
2V_j > V_i \quad \text{and} \quad V_j (2 - \delta^2) > V_i .
\]

Finally, the durable goods monopoly sells one unit of durable goods to the high-income consumer in the current period, and one unit to the low-income consumer in the future period, setting \( P_1 = \frac{V_j + V_j \delta}{1 + \delta} \) and \( P_2 = V_j \), if:

\[
\pi_C > \pi_A \quad \text{and} \quad \pi_C > \pi_B ,
\]

and so, simplifying the terms we have:

\[
V_j (2 + \delta) > V_i \quad \text{and} \quad V_i > V_j (2 - \delta^2 ) .
\]
We find that, depending on the willingness to pay of high-income consumers (consumer \( i \)) and low-income consumers (consumer \( j \)) and on the interest rate determining the value of the updating factor (\( \delta \)), the durable goods monopoly can choose to: 1) sell just one unit of the durable goods in the current period to the high-income consumer; 2) sell two units of the durable goods in the current period to high and low-income consumers; and 3) discriminate prices over time, selling one unit of the durable goods in the current period to the high-income consumer, and one unit of the durable goods in the future period to the low-income consumer. Based on these results, we can conclude that the difference in willingness to pay between the high-income consumer and the low-income consumer, and interest rates, influence the optimal pricing strategy of a durable goods monopoly.

2.3. Calibration

We now present various calibration exercises of the model so that its results can be more easily understood.

Initially, we will consider that \( V_i = 10 \) and \( V_j = 8 \). The monopoly’s marginal costs are 0. The interest rate is 0%. In these circumstances, if the monopoly sells one unit of the durable goods to the high-income consumer in the current period, it sets \( P_1 = 10 \), and has \( \Pi_A = 10 \times 1 = 10 \). If the monopoly sells two units of the durable goods in the current period, it sets \( P_1 = 8 \), and has a profit of \( \Pi_B = 8 \times 2 = 16 \). Finally, if the monopoly discriminates prices over time, it sets \( P_1 = 9 \) and \( P_2 = 8 \), and so its profit will be \( \Pi_C = 9 \times 1 + 8 \times 1 = 17 \). In these circumstances, the most advantageous option is to sell one unit of durable goods in the current period to the high-income consumer, and one unit in the future period to the low-income consumer, in this way discriminating prices over time.

We admit that \( V_i = 10 \) and \( V_j = 8 \). The monopoly’s marginal costs are 0. The interest rate is 25%. In these circumstances, if the monopoly opts to discriminate prices over time, it sets \( P_1 = 9.11 \) and \( P_2 = 8 \). In this case, profit will be \( \Pi_C = 9.11 \times 1 + 8 \times 1 \times 0.8 = 15.51 \). Therefore, the most advantageous option will be for the monopoly to set a price in the current period equal to what the low-income consumer is willing to pay, selling two units of the durable goods in the current period and having a profit of \( \Pi_B = 8 \times 2 = 16 \).
We see that with a limited difference between consumers’ willingness to pay and a high interest rate, the best solution for the monopoly is no longer to discriminate prices over time but rather to sell two units of durable goods at the present moment. This result is important and shows that with high interest rates and little difference between consumers’ willingness to pay, the price can drop right away in the current period, equal to what the low-income consumer is willing to pay.

Let us now consider that $V_i=10$ and $V_j=3.5$. The monopoly’s marginal costs are 0. The interest rate is 0%. If the monopoly sells a unit of durable goods to the high-income consumer, it sets $P_1=10$, and has a profit of $\Pi_A=10\times1=10$. If the monopoly chooses to sell two units of durable goods at the present time, one to the high-income consumer and another to the low-income consumer, it sets $P_1=3.5$, and has a profit of $\Pi_B=3.5\times2=7$. Opting to discriminate prices over time, selling one unit of durable goods to the high-income consumer and another to the low-income consumer, it sets $P_1=6.75$ and $P_2=3.5$, and profit will be $\Pi_C=6.75\times1+3.5\times1\times1=10.25$. We see the most advantageous option for the monopoly is to discriminate prices over time, selling one unit of durable goods to the high-income consumer, and another in the future period to the low-income consumer.

Finally, we consider that $V_i=10$ and $V_j=3.5$. The monopoly’s marginal costs are 0. The interest rate is 25%. In these circumstances, if the monopoly chooses to discriminate prices over time, it sets $P_1=7.11$ and $P_2=3.5$, with profit being $\Pi_C=7.11\times1+3.5\times1\times0.8=9.91$. We find that in this case the most advantageous option for the monopoly is to sell one unit of durable goods in the current period to the high-income consumer, profit being $\Pi_A=10\times1=10$. It is important to mention that the combination of a great difference in consumers’ willingness to pay and a high interest rate means the price may not drop to levels corresponding to low-income consumers’ willingness to pay.

3. CONCLUSION

Considering a discrete demand model and two consumers with different willingness to pay, this article shows that confirmation of the Coase conjecture
depends on the difference in willingness to pay between consumers with different levels of income and interest rates.

When interest rates are low, irrespective of the difference in the willingness to pay of consumers with different levels of income, the monopoly’s best strategy is to discriminate prices over time, selling one unit of durable goods to the high-income consumer in the current period and another unit to the low-income consumer in the future period. In these circumstances, we find a tendency for the price to decrease over time, this result being in agreement with the Coase conjecture.

If the difference in the willingness to pay of consumers with different levels of income is limited, and interest rates are high, the monopoly has an advantage in setting a low price in the current period, selling two units of durable goods in the current period to high and low-income consumers. This result corroborates the Coase conjecture since the monopoly sets a low price immediately in the current period, allowing it to sell its entire production.

Finally, if there is a great difference in the willingness to pay of consumers with different income levels, and interest rates are high, the monopoly has an advantage in selling only one unit of durable goods in the current period to the high-income consumer. In these circumstances we do not find a tendency for the price of durable goods to drop, since the price never reaches what the low-income consumer is willing to pay. Therefore, with a great difference in the willingness to pay of consumers of different income levels, and high interest rates, the Coase conjecture tends to fail.
REFERENCES


Received: November 15, 2014
Accepted: April 16, 2015