AN INTUITIONISTIC LOGIC
WITH PROBABILISTIC OPERATORS

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Abstract. A probabilistic extension of intuitionistic logic is introduced. The corresponding completeness and decidability theorems are proven.

1. Introduction

In this paper we combine probabilistic operators with intuitionistic logic. There are two possible approaches to do that. We may treat probabilistic operators intuitionistically or we may assume that they behave classically. The former approach was analyzed in [3, 4, 5, 6], while we consider here the later one which is more in spirit of [13, 14, 15]. At the syntax level we add probabilistic operators to the propositional intuitionistic language which enables making formulas such as $P_{\geq s}\alpha$. The intended meaning of the formula is "the probability of truthfulness of $\alpha$ is greater than or equal to $s$". In our logic nesting of probabilistic operators, i.e., higher order probabilities, will not be allowed. Thus, on the first level we have intuitionistic propositional calculus, and on the second level we start with the formulas of the form $P_{\geq s}\alpha$ as atoms (where $\alpha$ is an intuitionistic propositional formula) and apply to them classical conjunction and negation, i.e., on the second level the rules of classical logic hold. Syntactically, this corresponds to the approach in [13, 14, 15] except that on the first level we have intuitionistic logic, so e.g., we have $\neg$, $\land$, $\lor$, and $\rightarrow$ as independent propositional connectives. Our choice in combining intuitionistic and probabilistic logics makes it possible to give a simple and natural interpretation of probabilistic formulas, quite in line with Boole’s original ideas, based on the ‘size’ of the set of possible worlds in which a proposition is true.
In axiomatization of our logic we follow the ideas from [13, 14, 15], but in this paper we give a new inference rule which allows to determine ranges of probabilities syntactically.

2. Syntax

Let $S$ be a recursive subset of $[0,1]$ which contains all rational numbers from $[0,1]$. The language of the logic consists of a denumerable set $\phi = \{ p, q, r, \ldots \}$ of propositional letters, connectives $\neg, \wedge, \vee, \rightarrow$ and two lists of unary probabilistic operators $(P_{\geq s})_{s \in S}$, and $(P_{\leq s})_{s \in S}$.

The set $\text{For}_I$ of intuitionistic propositional formulas is the smallest set $X$ containing $\phi$ and closed under the formation rules: if $\alpha$ and $\beta$ belong to $X$, then $\neg \alpha$, $\alpha \wedge \beta$, $\alpha \vee \beta$, and $\alpha \rightarrow \beta$ are in $X$. Elements of $\text{For}_I$ will be denoted by $\alpha, \beta, \ldots$

The set $\text{For}_P$ of probabilistic propositional formulas is the smallest set $Y$ containing all formulas of the form $P_{\geq s} \alpha$ and $P_{\leq s} \alpha$ for $\alpha \in \text{For}_I$, $s \in S$, and closed under the formation rules: if $A$ and $B$ belong to $Y$, then $\neg A$, and $A \wedge B$ are in $Y$. Probabilistic literals are formulas of the form $P_{\geq s} \alpha$, $P_{\leq s} \alpha$, or $P_{\leq s} \alpha$. Formulas from $\text{For}_P$ will be denoted by $A, B, \ldots$. We use $A \lor B$, $A \rightarrow B$, $P_{\geq s} \alpha$, $P_{\geq s} \alpha$ and $P_{\leq s} \alpha$ to denote the formulas $\neg (\neg A \land \neg B)$, $\neg A \lor B$, $\neg P_{\leq s} \alpha$, $\neg P_{\leq s} \alpha$, and $P_{\geq s} \alpha \land P_{\leq s} \alpha$, respectively.

Let $\text{For}_I \cup \text{For}_P$ be denoted by $\text{For}$. We use $\phi, \psi, \ldots$ to denote formulas from $\text{For}$. For $\alpha \in \text{For}_I$, and $A \in \text{For}_P$, we abbreviate both $\neg (\alpha \rightarrow \alpha)$ and $\neg (A \rightarrow A)$ by $\bot$.

3. Semantics

We propose a possible-world approach to give semantics to formulas from the set $\text{For}$. According to the structure of $\text{For}$, there are two levels in the definition of models. At the first level there are the notions of intuitionistic Kripke models and the forcing relation ($\models$) [11, 12, 18]. We suppose that the reader is familiar with them. At the second level probabilistic models and the satisfiability relation are defined.

Let $M = \langle W, \leq, v \rangle$ be an intuitionistic Kripke model. We use $\lfloor \alpha \rfloor_M$ to denote $\{ w \in W : w \vDash \alpha \}$ for every $\alpha \in \text{For}_I$. Note that the family $H_I = \{ \lfloor \alpha \rfloor_M : \alpha \in \text{For}_I \}$ is a Heyting algebra which may not be closed under complementation.

**Definition 3.1.** A probabilistic model is a structure $\langle W, \leq, v, H, \mu \rangle$ where:

- $\langle W, \leq, v \rangle$ is an intuitionistic Kripke model,
- $H$ is the smallest algebra on $W$ ($H$ contains $W$, and it is closed under complementation and finite union) containing $H_I = \{ \lfloor \alpha \rfloor_M : \alpha \in \text{For}_I \}$ and the family $\{ W \setminus \lfloor \alpha \rfloor_M : \alpha \in \text{For}_I \}$, and
- $\mu : H \rightarrow S$ is a finitely additive probability ($\mu(W) = 1$, $\mu(G_1 \cup G_2) = \mu(G_1) + \mu(G_2)$ for all disjoint $G_1$ and $G_2 \in H$).

**Definition 3.2.** The satisfiability relation $\models$ is defined by the following conditions for every probabilistic model $M = \langle W, \leq, v, H, \mu \rangle$:

- if $\alpha \in \text{For}_I$, $M \models \alpha$ iff ($\forall w \in W w \vDash \alpha$),
- $M \models P_{\geq s} \alpha$ iff $\mu([\alpha]_M) \geq s$,
\[ M \models P_{\leq s} \alpha \text{ iff } \mu([\alpha]_M) \leq s, \]
\[ \text{if } A \in \text{For}_F, \ M \models \neg A \text{ iff } M \models A \text{ does not hold, and} \]
\[ \text{if } A, B \in \text{For}_F, \ M \models A \land B \text{ iff } M \models A, \text{ and } M \models B. \]

A formula \( \varphi \in \text{For} \) is satisfiable if there is a probabilistic model \( M \) such that \( M \models \varphi \); \( \varphi \) is valid if for every probabilistic model \( M \), \( M \models \varphi \); a set of formulas is satisfiable if there is a probabilistic model \( M \) such that for every formula \( \varphi \) from the set, \( M \models \varphi \).

4. A sound and complete axiomatization

We shall prove that the set of all valid formulas can be characterized by the following sound and complete set of axiom schemata:

1. all \( \text{For}_F \)-instances of intuitionistic propositional tautologies
2. all \( \text{For}_F \)-instances of classical propositional tautologies
3. \( P_{\geq 0} \alpha \)
4. \( P_{\geq 1-r} \neg \alpha \rightarrow \neg P_{\geq s} \alpha, \) for \( s > r \)
5. \( P_{\geq r} \alpha \rightarrow P_{\geq s} \alpha, \) for \( r > s \)
6. \( P_{\geq s} \alpha \rightarrow P_{\geq s} \alpha, \)
7. \( P_{\geq 1}(\alpha \leftrightarrow \beta) \rightarrow (P_{\geq s} \alpha \rightarrow P_{\geq s} \beta) \)
8. \( (P_{\geq s} \alpha \land P_{\geq r} \beta \land P_{\geq 1}(\neg (\alpha \land \beta))) \rightarrow P_{\min(1,s+r)}(\alpha \lor \beta) \)

and inference rules:

1. From \( \varphi \) and \( \varphi \rightarrow \psi \) infer \( \psi \).
2. If \( \alpha \in \text{For}_F \), from \( \alpha \) infer \( P_{\geq 1} \alpha \).
3. From \( B \rightarrow \neg P_{\geq s} \alpha \), for every \( s \in S \), infer \( B \rightarrow \bot \).

The axioms and rules are similar to the ones given in [13, 14, 15], except for the adjustment required by Axiom 1, i.e., the fact that \( \text{For}_F \)-formulas obey the intuitionistical laws. Rule 3 is a new one. The axioms 5 and 6 are equivalent to

\[ (5') P_{\leq s} \alpha \rightarrow P_{\leq s} \alpha, \ r > s \]
\[ (6') P_{\leq s} \alpha \rightarrow P_{\leq s} \alpha \]

respectively. Note that by substituting \( \neg \alpha \) for \( \alpha \) in Axiom 3, and using the axioms 4 and 6 the formula \( P_{\leq 1} \alpha \) is obtained which means that every formula is satisfied by a set of worlds of the measure at most 1. Finally, note that the monotonicity of the measure can be expressed by the formulas \( P_{\geq r} \alpha \rightarrow P_{\geq s} \alpha \), for \( r > s \), and \( P_{\leq s} \alpha \rightarrow P_{\leq s} \alpha \), for \( r \leq s \). These formulas are easy consequences of the axioms 5, 6, 5' and 6'.

A formula \( \varphi \in \text{For} \) is deducible from a set \( T \) of formulas \( (T \vdash \varphi) \) if there is an at most countable sequence of formulas \( \varphi_0, \varphi_1, \ldots, \varphi \), such that every formula in the sequence is an axiom or a formula from the set \( T \), or it is derived from the preceding formulas by an application of an inference rule. If \( \emptyset \vdash \varphi \), we say that \( \varphi \) is a theorem of the deductive system, also denoted by \( \vdash \varphi \).

A set \( T \) of formulas is consistent if neither \( T \vdash \neg(\alpha \rightarrow \alpha) \) nor \( T \vdash \neg(A \rightarrow A) \) for arbitrary \( \alpha \in \text{For}_F, A \in \text{For}_F \). Otherwise, \( T \) is inconsistent. A set \( T \) of formulas is maximal consistent if the following conditions are satisfied:

1. \( T \) is consistent,
• for every $\alpha \in \text{For}_T$, if $T \vdash \alpha$, then $\alpha \in T$, $P_3 \alpha \in T$, and
• for every $A \in \text{For}_P$, either $A \in \text{For}_P$ or $\neg A \in \text{For}_P$.

5. Soundness and completeness

Soundness of the system follows from the soundness of propositional intuitionistic and classical logics, as well as from the properties of probabilistic measures, so the proof is straightforward.

Theorem 5.1 (Deduction theorem). If $T$ is a set of formulas and $T \cup \{\varphi\} \vdash \psi$, then $T \vdash \varphi \rightarrow \psi$, where either $\varphi, \psi \in \text{For}_T$ or $\varphi, \psi \in \text{For}_P$.

Proof. We use the transfinite induction on the length of the proof of $\psi$ from $T \cup \{\varphi\}$. We consider the case where $\psi = C \rightarrow \bot$ is obtained from $T \cup \{\varphi\}$ by an application of the inference rule3, and $\varphi \in \text{For}_P$. Then:

1. $T, \varphi \vdash C \rightarrow P_2, \delta$, for every $s \in S$
2. $T, \varphi \vdash (C \rightarrow P_2, \delta)$, for every $s \in S$, by the induction hypothesis
3. $T \vdash (\varphi \land C) \rightarrow P_2, \delta$, for every $s \in S$
4. $T \vdash (\varphi \land C) \rightarrow \bot$, from (3) by Rule3
5. $T \vdash \varphi \rightarrow \psi$.

The other cases follow by standard arguments. $\square$

Theorem 5.2. Every consistent set $T$ can be extended to a maximal consistent set.

Proof. Let $T$ be a consistent set $T$ of formulas, $\text{ipconseq}(T) = \{\alpha \in \text{For}_T : T \vdash \alpha\}$ be the set of all intuitionistic propositional consequences of $T$, and $\text{ipconseq}(T)$ be a consistent disjunctive closure of $\text{ipconseq}(T)$, i.e. if $\beta \lor \gamma \in \text{ipconseq}(T)$, then $\beta \in \overline{\text{ipconseq}(T)}$ or $\gamma \in \overline{\text{ipconseq}(T)}$. Note that $\text{ipconseq}(T)$ is consistent because it is the set of consequence of a consistent set, and that in that case $\text{ipconseq}(T)$ always exists. Let $A_0, A_1, \ldots$ be an enumeration of all formulas from $\text{For}_P$. Let $\alpha_0, \alpha_1, \ldots$ be an enumeration of all formulas from $\text{For}_T$. We define a sequence of sets $T_i, i = 0, 1, 2, \ldots$, and a set $T^*$ such that:

1. $T_0 = T \cup \overline{\text{ipconseq}(T)} \cup \{P_3 \alpha : \alpha \in \overline{\text{ipconseq}(T)}\}$
2. for every $i \geq 0$, if $T_{2i} \cup \{A_i\}$ is consistent, then $T_{2i+1} = T_{2i} \cup \{A_i\}$, otherwise, $T_{2i+1} = T_{2i} \cup \{\neg A_i\}$,
3. for every $i \geq 0$, $T_{2i+2} = T_{2i+1} \cup \{P_{r} \alpha_i\}$, for some $r \in S$, so that $T_{2i+2}$ is consistent.
4. $T^* = \bigcup T_i$.

$T_0$ is consistent because it is a set of consequences of a consistent set. Suppose that $T_{2i+1}$ is obtained by the step2 of the above construction and that neither $T_{2i} \cup \{A_i\}$, nor $T_{2i} \cup \{\neg A_i\}$ are consistent. It follows by the deduction theorem that $T_{2i} \vdash A_i \land \neg A_i$, which is a contradiction. Consider the step3 of the construction, and suppose that for every $r \in S$, $T_{2i+1} \cup \{P_{r} \alpha_i\}$ is not consistent. Let $T_{2i+1} = T_0 \cup T_{2i+1}^r$, where $T_{2i+1}^r$ denotes the set of all formulas from $\text{For}_P$ that are added to $T_0$ in the previous steps of the construction. It means that:

1. $T_0, T_{2i+1}^r, P_{s} \alpha_i \vdash \bot$, for every $s \in S$, by the hypothesis
(2) $T_0 \vdash (\bigwedge_{B \in T_{2i+1}^+} B) \rightarrow \neg P_{s} \alpha_i$, for every $s \in S$, by Deduction theorem

(3) $T_0 \vdash (\bigwedge_{B \in T_{2i+1}^+} B) \rightarrow \bot$, by Rule3

(4) $T_{2i+1} \vdash \bot$,

which contradicts consistency of $T_{2i+1}$.

Finally, we have to prove that $T^*$ is maximal consistent. We do it by showing that $T^*$ is a deductively closed set that contains all formulas from $\text{For}_T$ nor all formulas from $\text{For}_P$.

Since $T$ is a consistent set, there is an $\alpha \in \text{For}_T$ such that $T \not\vdash \alpha$, $\alpha \not\in T_0$, and $\alpha \not\in T^*$. For a formula $A \in \text{For}_P$ the set $T^*$ does not contain both $A = A_i$ and $\neg A = A_j$, because $T_{\text{max}}(2i+2j+1)$ is consistent.

Next, if $T_i \vdash \varphi$ for some $i$ and $\varphi \in \text{For}_T$, it must be $\varphi \in T^*$, because if $\varphi \in \text{For}_T$, it follows from the construction of $T_0$, and if $\varphi = A_j \in \text{For}_P$, it follows from consistency of $T_{\text{max}}(2i+2j+1)$. Also, note that if $\neg P_{s \alpha} \in T^*$, then it follows from classical Axiom $A \rightarrow (B \rightarrow A)$ that for every $B \in \text{For}_P$, $B \rightarrow P_{s \alpha} \in T^*$.

If a formula $\alpha \in \text{For}_I$ and $T^* \vdash \alpha$, then by the construction of $T_0$, $\alpha \in T^*$ and $P_{\geq 1} \alpha \in T^*$.

Let $A \in \text{For}_P$. It can be proved by the induction on the length of the inference that if $T^* \vdash A$, then $A \in T^*$.

Suppose that the sequence $\varphi_1, \varphi_2, \ldots, A$ forms the proof of $A$ from $T^*$. If the sequence is finite, there must be a set $T_i$ such that $T_i \vdash A$, and $A \in T^*$. Thus, suppose that the sequence is countably infinite. We can show that for every $i$, if $\varphi_i$ is obtained by an application of an inference rule, and all the premises belong to $T^*$, then it must be $\varphi_i \in T^*$. If the rule is a finitary one, then there must be a set $T_j$ which contains all the premises and $T_j \vdash \varphi_i$. Reasoning as above, we conclude $\varphi_i \in T^*$. Otherwise, let $\varphi_i = B \rightarrow \bot$ be obtained from the set of premises $\{\varphi_k^i = B \rightarrow \neg P_{s_k \gamma} : s_k \in S\}$ by Rule3. By the induction hypothesis, $\varphi_k^i \in T^*$ for every $k$. By the step 3 of the construction, there are some $l$ and $s_l \in S$ such that $P_{s_l \gamma} \in T_l$. Reasoning as above, we conclude that $B \rightarrow P_{s_l \gamma} \in T^*$. Thus, there must be some $j$ such that $T_j \vdash B \rightarrow \neg P_{s_l \gamma}$, $T_j \vdash B \rightarrow P_{s_l \gamma}$, and $T_j \vdash B \rightarrow \bot$, which means that $B \rightarrow \bot \in T^*$.

Hence, from $T^* \vdash \varphi$, we have $\varphi \in T^*$, and $T^*$ is consistent. Finally, according to the above definition of a maximal set, it is provided by the construction of the set $T^*$ that $T^*$ is maximal.

Being a maximal consistent set, $T^*$ has all the expected properties, and additionally the following ones.

**Theorem 5.3.** Let $T^*$ be as above. Then the following holds:

1. There is exactly one $s \in S$ such that $P_{s \alpha} \in T^*$.
2. If $P_{\geq s \alpha} \in T^*$, there is some $r \in S$ such that $r \geq s$ and $P_{r \alpha} \in T^*$.
3. If $P_{\leq s \alpha} \in T^*$, there is some $r \in S$ such that $r \leq s$ and $P_{r \alpha} \in T^*$.

**Proof.** (1) It is easy to see that $\vdash P_{s \alpha} \rightarrow \neg P_{r \alpha}$, for $r \neq s$. Thus, if $P_{s \alpha} \in T^*$, then for every $r \neq s$, $P_{r \alpha} \notin T^*$. Suppose that for every $s \in S$, $\neg P_{s \alpha} \in T^*$. It follows that $T^* \vdash \neg P_{s \alpha}$ for every $s \in S$, and by Rule3, $T^* \vdash \bot$. 


which contradicts consistency of $T^*$. Thus, for every $\alpha \in \text{For}_\land$, there is exactly one $s \in S$ such that $P_{z_s}\alpha \in T^*$.

(2) If $P_{\geq s}\alpha \in T^*$, we have that $\neg P_{< s}\alpha \in T^*$. Also, there is some $r \in S$ such that $P_{= r}\alpha \in T^*$. It means that $P_{\geq r}\alpha \in T^*$, and $P_{\leq r}\alpha \in T^*$. If $r < s$, then by Axiom 5 from $P_{< s}\alpha \in T^*$ it follows that $P_{> s}\alpha \in T^*$, a contradiction. Thus, it must be $r \geq s$.

(3) Analogously to (2). \hfill \square

Theorem 5.4 (Extended completeness theorem). Every consistent set of formulas has a model.

Proof. Let $T$ be a consistent set of formulas. According to Theorem 5.2 there is a maximal consistent set $T^*$ which contains $T$. Let $w_0 = \text{fix}\text{conseq}(T)$, and $W$ be the set of all consistent, deductively closed extensions of $w_0$ having the property that for every $\alpha, \beta \in \text{For}_\land$, $w \vdash \alpha \lor \beta$ implies $w \vdash \alpha$ or $w \vdash \beta$, $w \in W$. Let for every $w \in W$, $\nu(w) = \{\alpha \in \phi : \alpha \in w\}$. Then, Axiom 1 guarantees that $(W, \subseteq, \nu)$ is an intuitionistic Kripke model.

Let $H_T = \{[\alpha]_M \mid [\alpha]_M \in \text{For}_\land\}$, and for every $\alpha \in \text{For}_\land$, $\mu_H([\alpha]_M) = s$ iff $P_{z_s}\alpha \in T^*$. The probabilistic part of our axiomatic system guarantees that $\mu_H$ is a finitely additive probability on the lattice $H_T$. Let $H$ be the smallest algebra on $W$ containing $H_T$ and the family $\{W \setminus [\alpha]_M : \alpha \in \text{For}_\land\}$. Using [2, Theorem 3.10] we can find a finitely additive probability $\mu$ on $H$ which is an extension of $\mu_H$. It follows that $M = (W, \subseteq, \nu, H, \mu)$ is a probabilistic model.

Finally, $M$ has the property that for every $\varphi \in \text{For}_\land$, $M \models \varphi$ iff $\varphi \in T^*$. Let $\varphi = P_{\geq s}\alpha$. If $P_{\geq s}\alpha \in T^*$, then, by Theorem 5.3, there is some $r \geq s$ such that $P_{= r}\alpha \in T^*$, i.e., such that $\mu([\alpha]_M) = r \geq s$. Thus, $M \models P_{\geq s}\alpha$. On the other hand, suppose that $M \models P_{\geq s}\alpha$, i.e., that $\mu([\alpha]_M) = r \geq s$, so $P_{\geq r}\alpha \in T^*$. It means that $P_{\geq r}\alpha \in T^*$, and by the theorem $\vdash P_{\geq r}\alpha \rightarrow P_{\geq s}\alpha$, for $r > s$ it follows that $P_{\geq s}\alpha \in T^*$. The case $\varphi = P_{= s}\alpha$ follows analogously, while the other cases are routine (see [9, 16, 18] for the intuitionistic part).

Thus, $T^*$ and $T$ are both satisfiable. \hfill \square

6. Decidability

Note that a formula $\alpha \in \text{For}_\land$ is intuitionistically satisfiable iff it is forced in the root of a tree-like model which is decidable [9, 12, 18]. It follows that satisfiability problem of $\text{For}_\land$-formulas in our probabilistic logic is decidable. To prove decidability of our logic we have to show that satisfiability problem for probabilistic formulas is decidable.

Let $A \in \text{For}_\land$ and $\text{Sub}_\land(A) = \{\alpha \in \text{For}_\land : \alpha \text{ is a subformula of } A\}$. Let $|A|$ and $|\text{Sub}_\land(A)|$ denote the length of $A$, and the number of formulas in $\text{Sub}_\land(A)$, respectively. Obviously, $|\text{Sub}_\land(A)| \leq |A|$.

Theorem 6.1. The satisfiability problem for probabilistic formulas is decidable.

Proof. Let $A$ be a probabilistic formula. Using [16, Theorem 5.3.4], we can prove that $A$ is satisfiable iff it is satisfiable in a finite probabilistic model containing at most $2^{|A|}$ worlds.
Let $\text{DNF}(A)$ be the formula $\bigvee_i \land_j \pm P_{\rho_{i,j}} \alpha_i \land_j$ which is equivalent to $A$, where $\rho \in \{\geq, \leq\}$, and $\pm P_{\rho_{i,j}} \alpha_i \land_j$'s are probabilistic literals. For every $A \in \mathbb{F}_P$, there is at least one $\text{DNF}(A)$, because propositional connectives behave classically at the probabilistic level. $A$ is satisfiable iff at least one disjunct $D$ from $\text{DNF}(A)$ is satisfiable. Since $D$ is a conjunction of probabilistic literals, without loss of generality we can assume that $A$ is of the same form.

We can check satisfiability of $A$ in the following way. For every $l, 1 \leq l \leq 2^{|A|}$, there is only finitely many intuitionistic models with different valuations with respect to the set of propositional letters that occur in $A$. For every such intuitionistic model $M_l = \langle W, \leq, v \rangle$ we can find the algebra $H$ generated by the set \{ $[\alpha]_{M_l} : \alpha \in \text{Sub}_l(A)$ \}. Thanks to [2, Theorem 3.3.4], we can suppose that every world from $W$ belongs to $H$ as well, and consider the following linear system:

\[
\sum_{w \in W} \mu(w) = 1 \\
\mu(w) \geq 0, \text{ for } w \in W \\
\sum_{w \in [\alpha]_{M_l}} \mu(w)\rho_r, \text{ for every } P_{\rho_r} \alpha \text{ which appears in } A \\
\sum_{w \in [\alpha]_{M_l}} \mu(w)\rho'_r, \text{ for every } -P_{\rho_r} \alpha \text{ which appears in } A, \text{ where } \leq' \text{ denotes } >, \text{ and } \geq' \text{ denotes } < .
\]

Obviously, if the above system is solvable, $M = \langle W, \leq, v, H, \mu \rangle \models A$.

There is a finite number of models and linear systems we have to check. Since linear programming problem is decidable, the same holds for the considered satisfiability problem. \qed

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