Finite Difference Method for Computer Study of the Interfacial Heat Transfer Coefficient during Rapid Solidification of Spherical Samples on a Metallic Substrate

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Abstract:
In this paper a numerical model will be adopted to analyze the heat transfer process during rapid solidification of a spherical sample placed on a metallic substrate cooled by water. The interfacial heat transfer coefficient between the sample and the substrate will be evaluated by matching model calculations with the surface temperature history recorded by a digital camera during solidification of a sample melted in an Arc-image furnace.

Keywords: Rapid solidification, Interfacial heat transfer coefficient, Computer simulation.

Introduction

Interfacial heat transfer between a solidifying liquid and a colder substrate is a very interesting phenomenon in many technological processes involving rapid solidification. It affects the subsequent solidification process and controls the microstructure development of the solidified products. When the liquid cools down and solidifies, the interface (solidification front) changes from an initial liquid-solid contact to a solid-solid contact. It leads to a high variation in the rate of interfacial heat transfer. Therefore, a good understanding of these sample-on-substrate-melting and solidifying phenomena is necessary to obtain a good control of the solidification process. In this paper a two-dimensional (2-D) finite difference method will be developed and adopted to analyze the interfacial heat transfer process during solidification of a spherical sample on a metallic substrate. The interfacial heat transfer coefficient between the melt and the substrate will be evaluated by matching model calculations with the surface temperature history recorded by a digital camera during solidification.

Mathematical model

There are two major mechanisms controlling the rapid solidification - the external extraction of heat through convective and/or radiative heat transfer and the internal heat...
conduction combined with solidification. This radially symmetric heat-conduction problem, with a nonuniform temperature distribution inside the sample, and symmetric about the growth axis that coincides with direction substrate-top of the sample (solidification starts at some discrete points on the contact surface) becomes 2-D (multi-dimensional effects are believed to be small for cooling and solidification processes). The 2-D governing heat conduction equations for a solidifying sample on a water-cooled substrate (Fig. 1a), assuming constant conductivity, are given elsewhere [1].

\[
\frac{1}{\rho} \frac{\partial}{\partial r} \left( \rho \frac{\partial T_i}{\partial r} \right) + \frac{1}{\rho} \frac{\partial^2 T_i}{\partial \theta^2} = \frac{\partial T_i}{\partial t} \quad (\kappa_i = \frac{\rho c_i}{\lambda_i})
\]

where \( \lambda \) is the thermal conductivity, \( \rho \) is the density, \( c \) is the specific heat capacity, and the subscript \( i = 1,2,3 \) stands for the melt, solid and substrate, respectively.

As the initial condition, the sample is assumed to be at a uniform temperature \( T_0 \), the cooled substrate at uniform temperature \( T_{sub} \), and the surrounding air at ambient temperature \( T_a \) (300 K), where both \( T_{sub} \) and \( T_a \) are well below the melting temperature for the sample \( T_m \). Since there are no heat sources inside the sample, the sample surface temperature starts to decrease as a consequence of conduction, natural and forced convection and radiation. Thus the boundary condition at the sample surface will be defined as

\[
-\lambda_i \frac{\partial T_i}{\partial r} = h_c(T_i - T_a) + \varepsilon_i \sigma(T_i^4 - T_a^4),
\]

where subscript \( i = 1,2 \) stands for the sample and substrate, respectively, \( h_c \) is the convective heat transfer coefficient, \( \varepsilon \) is the surface emissivity, and \( \sigma \) is the Stefan-Bolzmann constant. After melting the sample does not change its shape and location except at the bottom of the sample where the initial point contact between sample and substrate will be replaced by a circle contact area \( (\pi a^2, \text{Fig. 1b}) \). The sample is much smaller than the substrate that will be permanently cooled by water during solidification. Therefore, the boundary condition for the substrate will be \( T_{sub} \) at all boundary points except on the interface between sample and substrate.

The solidification rate of a molten sample will be controlled by the interfacial heat transfer between the substrate and the sample. Because of the thermal contact resistance, the temperature drops across the interface substrate-sample governed by equation

\[
-\lambda_i \frac{\partial T_i}{\partial y} = h \cdot (\langle T_{sam} \rangle - \langle T_{sub} \rangle),
\]
where subscript \( i = 1,2,3 \) stands for the sample (liquid and/or solid) and substrate, respectively, \( y \) is the growth axis (Fig. 1c), \( h \) is the interfacial heat transfer coefficient, and \(<T_{sam}>\) and \(<T_{sub}>\) are the average temperature of the sample and substrate at the interface, respectively.

The entire solidification process can be divided into three stages: (i) liquid cooling, (ii) liquid solidification, and finally (iii) solid cooling. The numerical method that will be used will simulate the solidification process as a whole. Solidification begins on the bottom sample surface when the sample surface temperature reaches the interface undercooling \( \Delta T = T_m - T_i \). During this process the interface starts to change from the initial liquid-solid to a solid-solid contact, where the liquid-solid interface position, which follows the axisymmetric geometry depicted in Fig. 1c, will be defined by local equilibrium conditions

\[
\rho_2 v L = \lambda_2 \frac{\partial T_2}{\partial y}_{\text{interface}} - \lambda_1 \frac{\partial T_1}{\partial y}_{\text{interface}} \quad \text{and} \quad T_1 = T_2
\]

where \( v = -\frac{dy}{dt} \) is the moving velocity of the liquid-solid interface, and \( L \) is the latent heat of solidification. During recalescence (the effect of releasing the latent heat into the undercooled liquid) the interface will reach a temperature below but close to \( T_m \). After that the interface liquid-solid will be treated as a zero thickness domain kept at \( T_m \). The point at which the interface reaches the top of the sample is flagged as an end of the solidification process, and after that the sample will be cooled only up to room temperature.

The finite difference method will be used to solve iteratively the governing equations, which are formulated using the heat conduction model (Eq. (1)), where the formulation and numerical solution procedure are described elsewhere in detail [2].

**Experiment**

To get rapidly quenched samples of high temperature materials, many techniques (for example [3-5]) have been developed. However, these methods have been limited by factors in maximum temperatures, atmospheres, containers and/or cooling rates. An Arc-Image furnace can heat any samples rapidly to a very high temperature in a clean condition, without containers or crucibles, in any oxidizing, inert or reducing atmospheres.

![Recorded time-dependent sample surface brightness profile](image)

**Fig. 2** Recorded time-dependent sample surface brightness profile.
A detailed description of this method has been given elsewhere [6]. Pellets of the mixed powders with the eutectic composition were placed on a copper plate cooled by water and melted in air in an arc-image lamp furnace by the radiation of a 10 kW Xenon Lamp. The arc-melted specimens were quenched by rapidly moving the copper plate from the focal point. The cooling rate using such a method was estimated to be higher than $10^3$ Ks$^{-1}$ [7].

In order to measure the temperature distribution as a function of the solidification time, a sample on a stationary substrate was recorded with a video camera through a heat-ray cut filter (sensitivity of CCD was constant; the camera was a Fuji Fine Pix f601). The recorded movie was divided into bmp pictures. The brightness recorded on film was used to obtain the time-dependent spot temperature of the sample as follows: the pictures were converted to gray scale; the brightness in the sample surface (as integer numbers, "0" as perfect black (coldest), and "256" as perfect white (hottest)) was picked up and plotted in Fig. 2.

**Results and Discussions**

The alumina sample of a 4 mm radius was placed onto a cold Cu substrate with a diameter of 50 mm. For $T_o$ we will assume the liquid to be superheated uniformly by 175 K, $\Delta T = 75$ K, $T_{sub} = 308$ K, and $\pi a^2 = 10^{-6}$ m$^2$. The data used in this simulation were the same as in [1] and the convective heat transfer coefficient was $5 \times 10^3$ Wm$^{-2}$K$^{-1}$.

After superheating in an Arc-image furnace and moving away from the focal point the sample starts to cool down. At the beginning the rate of heat extraction dominates the rate of solidification and the surface temperature decreases further. At that time only the sample surface is at the temperature below the melting temperature, but the sample is still almost fully liquid. Relatively quickly the sample temperature at the interface reaches the interface undercooling $\Delta T$ and solidification starts. However, just after that the temperature gradients on both sides across the interface surface will be reduced. The gradient in the liquid part of the sample decreases as the process continues. The temperature jump between sample and substrate decreases further, and the interface cooling rate decreases significantly as the interface liquid-solid moves far away from the sample bottom. For the late stage of the interface the temperature jump approaches uniform value. When the solidification process ends the sample will be cooled down only.

![Fig. 3](image)

**Fig. 3** Matching of numerical predictions obtained for different values of the interfacial heat transfer coefficient (solid lines) with experimental data (open circles) for rapid solidification of an alumina sample on a copper substrate.
Fig. 3 shows the time-dependent average sample surface temperature during the whole solidification process as a function of the interfacial heat transfer coefficient. The brightness numbers were converted into temperatures using the equilibrium melting point of the melting materials (open circles). As one can see the experimental temperature history shows clearly a plateau indicating an isothermal solidification, where the plateau corresponds to the equilibrium melting temperature of alumina (2327 K). It also shows clearly the end of solidification.

Using the measured temperature data, the interfacial heat transfer coefficient can be estimated by solving the inverse heat transfer problem. The experimental temperature profile provides a direct comparison with the model prediction displayed in Fig. 3 and does not show good agreement with experimental data. It is assumed that when undercooling is achieved, the interface recalesces to the melting point immediately after nucleation. It can be seen that this condition is fulfilled approximately only for $h = 1.5 \times 10^3 \text{ Wm}^{-2} \text{K}^{-1}$. However, all three computed nucleation times differ very much compared to the experimental one. There is also no plateau indicating an isothermal solidification.

![Figure 3](image)

**Fig. 4** Matching of numerical prediction (solid line) with experimental data (open circles) for rapid solidification of an alumina sample on a copper substrate.

A good match was obtained between the calculations and the measured temperatures by using a single value of $h = 1.73 \times 10^3 \text{ Wm}^{-2} \text{K}^{-1}$ (Fig. 4). The computed surface temperature profile shows clearly a plateau indicating an isothermal solidification, where the plateau corresponds to the equilibrium melting temperature. The only problem could be no clear recalescence effect, which is probably a result of not a fine enough grid mesh.

Some observation can be derived from the obtained results. One needs to conduct a more extensive investigation, because the processing conditions such as the melt superheat, the surface roughness and the size of the contact area between sample and substrate have a great influence on the interfacial heat transfer coefficient value. Although there is a good match between experimental and theoretical results, future investigations should maybe include dividing the sample cooling process into two or three regions (time domains), each being represented by separate values of the interfacial heat transfer coefficient.
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References


Садржај: У овом раду ће бити развијен нумерички модел за анализу процеса преноса топлоте током брзог очвршћавања сферичног узора који се налази на металном супстрату који се хлади водом. Коефицијент преноса топлоте на међуповршини између узора и супстрата биће одређен на основу временске зависности температуре на површини узора снимљене дигиталном камером током процеса очвршћавања узора у Arc-image пећи.
Кључне речи. Брзо очвршћавање, међуповршински коефицијент преноса топлоте, компјутерска симулација.