Evolutionary Algorithm Based Approach to
Impedance Design

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Abstract: Bus bars of complex cross section were designed in order to achieve
desirable characteristics for use with high power switching devices. High
permeability material was introduced between the copper bus bars, to increase
non-linear rise of the bars' resistance with frequency. Micro genetic algorithm
(µGA) was used to search for the most suitable combination of parameters.
The main goal was to design bus bars with adequately adapted resistance and
inductance at 10 kHz and 50 Hz simultaneously. The frequency dependant
resistance and inductance were calculated utilizing 2D FEM software.

Keywords: Multiobjective optimisation, frequency dependant impedance,
FEM method.

1 Introduction

High power switching devices should be connected to power sources via bus
bars of low inductance, to minimise voltage over shots. As the fundamental
harmonic, 50 Hz, should produce minimal power loss, the bus bars' resis-
tance should be low at 50 Hz. Simultaneously, high resistance of the bus
bars is desirable at high frequencies, for instance at 10 kHz, to dump the
pulse rise and reduce ripple. To successfully meet most of the demands it
is necessary to design the bus bars with complex frequency behaviour. This
paper describes how the bus bars structure was designed utilizing the evolu-
tionary algorithm based concept and in particular micro genetic algorithm

Manuscript received February 3, 2002. A version of this paper was presented at the
fifth International Conference on Applied Electromagnetics, IIES 2001, October 8 -10,
2001, Niš, Serbia.

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(μGA), ([1], [2]), and how multiobjective optimisation offered a number of solutions.

Impedance of the bus bars with rectangular cross section depends on frequency. For the bus bars made of linear material, such as copper or aluminium, the frequency dependent behaviour of the resistance is well known: it rises proportionally to the square root of the frequency. Deviation from this canonical pattern was obtained with addition of ferromagnetic material, which introduced more possibilities into the observed system. In this paper, for the sake of simplicity, permeability of the material was considered constant, μr = 100. The solution was found with iron bars inserted between and attached to the copper bars. Although the shape of the copper conductors could have been considered, insertion of ferromagnetic material made it less important. Thus, μGA based procedure was performed to design the shape of the ferromagnetic parts only.

2 Problem Formulation

Starting from Maxwell’s equations

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \]
\[ \nabla \times \vec{H} \approx \vec{J}, \]
\[ \nabla \cdot \vec{D} = \rho, \]
\[ \nabla \cdot \vec{B} = 0, \text{ and} \]
\[ \nabla \times \vec{A} = \vec{B}, \quad \nabla \cdot \vec{A} = 0, \]

for the time harmonic excitations in linear materials, Helmholtz equation

\[ \nabla^2 \vec{A} - j\omega \mu \sigma \vec{A} + \mu \vec{J} = 0, \]

can be derived. This equation was solved for the 2D case, utilizing finite elements method (FEM) program. Two geometries were considered. The cross sections of the examined bus bars are illustrated in Figs. 1 and 2.

Per unit length resistance was calculated from the Joule’s power loss, PJ, like

\[ R = \frac{P_J}{I_m^2}. \]

Per unit length inductance was calculated from magnetic energy, Wm, stored in the region,

\[ L = \frac{2W_m}{I_m^2}. \]
Im stands for the current amplitude and was required to be as high as
$I_m = 600 \text{ A}$ at 50 Hz, and $I_m = 135 \text{ A}$ at 10 kHz. Dimensions and position
of the copper bus bars were defined in the beginning, while the dimensions
and positions of the iron bars were designed using $\mu$GA procedure.

3 Evolutionary Algorithm Based Search

Evolutionary algorithms are stochastic search techniques based on mech-
anism of natural selection and survival of the fittest. At each generation
individuals are created that offer better and better solution to the problem.
The objective of the procedure was to maximise the 10 kHz resistance of the
structure. Additionally, it was necessary to avoid ferromagnetic saturation
and to keep 50 Hz resistance and inductance low. Evolutionary algorithms
can provide a number of solutions or a family of pseudo optimal solutions.
Regarding practical problems, such as described in this paper, sometimes it
is useful to observe alternative solutions simultaneously.

As $\mu$GA is a maximizer by it’s nature, the objective function was formed
in a shape of

$$F = \frac{R_{50} - R_A R_B - R_{10k} L_{50} - L_C}{R_A R_B L_C}.$$  

The values $R_A$ and $L_C$ are minimal values found in the cycle, while $R_B$
is maximal value found in the cycle. The constants $R_A$, $R_B$ and $L_C$ are
calculated in each cycle independently.

Typically, 5 individuals formed the population. The initial population
of five was formed at random. Next populations were formed based on
selection, recombination and reproduction. Mutations were not introduced.

Due to the problem’s nature, there was no point in describing dimen-
sions of the cross section in fraction of millimetre. Rather, the penetration
depth was taken as a parameter. Equally, when calculating resistance and
inductance, number of significant digits was limited. These approximations
made use of micro genetic algorithm ($\mu$GA) possible. Unlike conventional
GA, the $\mu$GA has a small population size and uses start-restart procedure
to compensate for premature convergence.

In this work, decimal values were treated directly, thus avoiding binary
coding and decoding. The advantage was also in a possibility to use case
specific operator, such as combination operator - the arithmetic average of
the parents’ genes.
3.1 Selection

Tournament selection was performed. From population size of five individuals, two were taken at random. One of them, with better fitness, made parent1. The procedure was repeated for the parent2. Each couple of the parents gave one offspring. The procedure was repeated eight times in each cycle to form four new individuals. The fifth individual for the new generation was a replica from the previous generation.

3.2 Intermediate recombination

Intermediate recombination is a method applicable only to real variables (and not to binary variables). The values for the offspring are chosen between the values of the parents. Offspring were produced according to the rule

$$\text{offspring} = \text{parent1} + \alpha (\text{parent2} - \text{parent1}),$$

where $\alpha$ was between 0 and 1.

3.3 Elitism

Elitism was included: each time, genetic operators produced four new individuals, while the fifth individual was a copy of the best from the previous generation.

3.4 Restart and stopping criteria

Restart criteria were based on the difference between the individuals - genes; if it did not exceed some fraction of the best in population (typically 5 – 10%).

Stopping criteria were based on

- number of generations,
- satisfactory result of the search.

4 Examples

4.1 Example 1

The cross section of the bus bars, consisting of the two copper and two attached iron bars, is illustrated in Fig. 1. Only iron was considered for the optimisation by the $\mu$GA. Two variables were monitored: width, $a$, and
Fig. 1. The cross section of the two copper bus bars with two iron bus bars attached between them.

thickness, $b$, of the iron bars. Respective dimensions $a$ and $b$ may be observed from Fig. 1. Each dimension was limited according to the space available: $a$ (in mm) (60-180), $b$ (in mm) (0.5-2.5).

The $\mu$GA procedure was performed as explained. Tens of iterations were performed in each run. The limit was set to 100. The best results in each cycle were recorded. Clusters of the results for the 10kHz resistance, $R_{10kHz}$, are illustrated in Fig. 3. Some of the pseudo optimal solutions are compared in Fig. 4. More data are given in Table 1.

### 4.2 Example 2

The cross section of the bus bars, consisting of four copper and two perpendicularly attached iron bars, is illustrated in Fig. 2. Only iron was considered for the optimisation by the $\mu$GA. Three variables were monitored, width, $a$, thickness, $b$, of the iron bars, and the deviation, $e$, of the right edge from the middle point. Respective dimensions $a$, $b$ and $e$ may be observed from Fig. 2. Each dimension was limited according to the space available: $a$ (in mm) (18-100), $b$ (in mm) (0.5-4.5), $e$ (in mm) (9-$a$/2).

The $\mu$GA procedure was performed as explained. Tens of iterations were performed in each run. The limit was set to 100, because there was little improvement after approximately 60 iterations. Best results in each cycle were recorded. Clusters of the results for the 10kHz resistance, $R_{10kHz}$, 10kHz inductance, $L_{10kHz}$, 50Hz resistance, $R_{50Hz}$, and 50Hz inductance, $L_{50Hz}$, are illustrated in Fig. 5. Some of the pseudo optimal solutions are compared in Fig. 6. Corresponding data are given in Table 2.
5 Results

Tables 1 and 2 contain some of the practical results, ready for implementation. Final decision should be made based on additional criteria, such as minimum weight of the structure, mechanical demands, heating criteria. These criteria could have been implemented in the objective function, but it was found to be unnecessarily time consuming.

Table 1. Some quasi-optimal results for the bus bars from Fig. 1.

<table>
<thead>
<tr>
<th>No</th>
<th>$R_{10k}$ (mW/m)</th>
<th>$L_{10k}$ (nH/m)</th>
<th>$R_{60}$ (mW/m)</th>
<th>$L_{60}$ (nH/m)</th>
<th>$a$ (mm)</th>
<th>$b$ (mm)</th>
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<td>24.70</td>
<td>422.9</td>
<td>108.9</td>
<td>991.6</td>
<td>114.9</td>
<td>0.81</td>
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<td>2</td>
<td>24.55</td>
<td>425.6</td>
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<td>990.7</td>
<td>117.9</td>
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<tr>
<td>3</td>
<td>24.79</td>
<td>406.7</td>
<td>109.3</td>
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<td>114.4</td>
<td>0.85</td>
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<td>993.8</td>
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<td>0.76</td>
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<td>16.27</td>
<td>457.4</td>
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<td>1054.7</td>
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<td>1.24</td>
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<td>112.2</td>
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<td>116.3</td>
<td>1163.1</td>
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Fig. 2. The cross section of the four copper bus bars with two iron bus bars attached perpendicularly between them.
Table 2. Some quasi-optimal results for the bus bars from Fig. 2.

<table>
<thead>
<tr>
<th>No</th>
<th>$R_{10k}$ (mW/m)</th>
<th>$L_{10k}$ (nH/m)</th>
<th>$R_{50}$ (mW/m)</th>
<th>$L_{50}$ (nH/m)</th>
<th>$\alpha$ (mm)</th>
<th>$b$ (mm)</th>
<th>$c$ (mm)</th>
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<td>733.8</td>
<td>52.5</td>
<td>3.44</td>
<td>20.8</td>
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</tbody>
</table>

6 Discussion and Conclusion

![Graph](image)

Fig. 3. Clusters of the results (the best of tens of runs) for the 10kHz resistance, $R_{10k}$.

The bus bars structures were designed utilizing the evolutionary algorithm based concept. In particular, micro genetic algorithm, $\mu$GA, was used to model frequency dependant impedances in order to obtain the desired characteristics of the designed bus bars. The procedure is not limited to the particular structures and can be, with small modifications of the objective function, applied to some other geometry. The procedure in this work avoided binary coding, as was conveniently dictated by the nature of the problem. Multiobjective optimisation offered a number of solutions.
may be observed from Fig. 4, the designed 10kHz resistance for the Example 1 is approximately 35 times greater than $R_0$ the 10kHz resistance of the copper bars without iron, and with all other parameters of acceptable value. Similarly, for the Example 2, from Fig. 6 may be observed that the designed 10kHz resistance is approximately 10 times greater than $R_0$ the 10kHz resistance of the copper bars without iron, and with all other parameters of acceptable value. Obtained results were acceptable for practical applications since they achieved satisfactorily all given objectives.

![Graph](image)

Fig. 4. Some normalised (In respect to $R_0 = 0.7$ mW/m and $L_0 = 1$ mH/m) pseudo optimal solutions for the bus bars from Fig. 1, for the different width $a$ of the iron bars. More details are available in Table 1. $R_0$ stands for the 10kHz resistance of the copper bars without iron.

![Graphs](image)

Fig. 5. Clusters of the results (the best of tens of runs) for: (a) 50Hz inductance, $L_{50}$, (b) 10kHz inductance, $L_{10k}$. 
Fig. 5. Continue. (c) 50Hz resistance, \( R_{50} \), and (d) 10kHz resistance, \( R_{10k} \).

Fig. 6. Some normalised (in respect to \( R_0 = 0.982 \) mW/m, \( L_0 = 0.7 \) mH/m) pseudo optimal solutions for the bus bars from Fig. 2, for the different width \( a \) of the iron bars. For more details see Table 2. \( R_0 \) stands for the 10kHz resistance of the copper bars without iron.

Acknowledgement

This work was inspired by fruitful discussions with Dr. Nikola Čelenović and Mr. Pieder Jorg, Power Electronics Systems, ABB Corporate Research Center LTD, Baden-Dättwil, Switzerland. The authors are sincerely grateful for their continuous care and support.
References
