One Approach to the Integration of Inertial and Visual Navigation Systems

Dedicated to Professor Milić Stojić on the occasion of his 65th birthday

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Abstract: The algorithm of simultaneous estimation of motion parameters and scene structure using the integrated navigation system consisting from inertial sensors (three rate gyros and three accelerometers) and TV camera has been presented. All mentioned sensors are rigidly fixed to the body of a moving object. It is assumed that the inertial sensors are characterized by constant biases. The recognizable landmarks existing in the scene on known locations in the reference coordinate frame are assumed also. It is enabled by parallel processing of information in two independent navigation systems that they may correct each other, in order to estimate moving object's linear and angular position relative to the landmark as well as it's linear and angular velocities in an optimal fashion.

Keywords: Inertial navigation system, visual navigation system, dynamic vision, TV camera.

1 Introduction

The integration of different navigation systems is a highly widespread method of increasing the overall system reliability as well as of improving the resultant accuracy in navigation parameter estimations. The most usual case is that an inertial navigation system (INS) is aided by some others like: satellite global positioning systems (GPS, GLONASS), systems based on radar altimeters and Doppler radars, or visual navigation systems (VNS) based on TV or IR cameras. The basic idea consists in extending the set of measurements outside INS itself and providing the means for the optimal estimation of navigation parameters (vectors of linear position and linear velocity of a moving object relative to the reference coordinate system).
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frame) as well as for the estimation of error parameters characterizing the inaccuracies of inertial sensors inside INS.

Speaking about the integration of INS and VNS it is possible to consider different levels of coupling, according to the particular application. There are some surveillance and guidance systems of military purpose where the INS is responsible for the guidance of the moving object until it reaches the area where it is possible to recognize the target by TV camera, and after that, the guidance is transferred to the VNS. In such cases, two independent systems act inside one complex guidance system, but practically without exchanges of information. The existence of recognizable visual landmarks inside the field of view of TV camera may be used in different ways as the source of corrective information for an INS. General principles of integration of these two systems in order to improve the overall navigation system accuracy are considered in [1], while their equalized treatment for the purposes of system initialization and mutual corrections was the subject of interest in articles [2, 3]. However, in all those approaches, INS was implicitly considered as “master” system, continuously operating with high sampling frequency, while initially and/or in work, it is calibrated/corrected by the information originating from VNS (supposing their higher level of accuracy). The similar level of coupling can be found in examples given in [4]-[6] where the VNS based on dynamic vision algorithm has been used as the primary system while the required information regarding the linear velocity and angular attitude are provided by INS and considered as the accurate ones. This later category of applications is generally oriented toward the field of robotics and automatic motion control of land vehicles.

This paper makes an approach toward the consideration of the highest level of interactions between INS and VNS (tightly coupled systems). VNS will be provisionally considered as the primary one, having in minds that the suitable applications in robotics and automatic control of cars on roads would primarily require the estimation of the scene structure (distance of the moving object from the landmark/obstacle and angular orientation of the object carrying the TV camera). VNS algorithm includes two mechanisms in parallel:

1. autonomous estimations based on prior knowledge of landmark characteristics, calculated using the general principles of projective geometry [7];
2. estimations obtained by the processing of the sequence of frames, using the additional information originating in INS (dynamic vision algorithm).

The main idea in usage of dynamic vision is to incorporate the required INS data after the process of optimal estimation of linear and angular velocities obtained by INS aided by autonomous VNS. The accuracy of these data is improved by the estimation of inaccuracies of inertial sensors.
The second part of the paper is related to the basic mathematical models of INS and VNS algorithms. The scenarios of motion of an object as well as the characteristics of ground landmarks suitable for the verification are specified. Suggested algorithm of INS/VNS integration suitable for the simultaneous estimation of scene structure and motion parameters is explained in the third part of the paper. The fourth part consists of typical results obtained throughout the simulations of this algorithm.

2 MODELS OF NAVIGATION ALGORITHMS

2.1 Inertial navigation algorithm

Relevant coordinate frames are specified on Fig. 1. Reference coordinate frame (ICF) used for representation of a moving object’s position is denoted as $O_1x_1y_1z_1$ and will be considered as the stationary one (inertial - index $I$). The moving coordinate frame $O_Cx_Cy_Cz_C$ is fixed to the body of a moving object. TV camera is rigidly fixed to the body of a moving object and for the sake of clarity it is supposed that the central line of its field of view coincides with the object’s longitudinal axis.

![Fig. 1. Reference (ICF) and camera fixed (CCF) coordinate frames.](image)

Kinematic model of translational motion of an object in ICF is represented as

\[
\begin{align*}
\vec{v}_I &= T_{I/C}\vec{a}_C + \vec{g}_I, \\
\vec{x}_I(0) &= \vec{x}_{I0},
\end{align*}
\]

where the notations are as follows: $\vec{x}_I = [x_I \ y_I \ z_I]^T$ - vector of linear position of a moving object; $\vec{v}_I = [v_{xI} \ v_{yI} \ v_{zI}]^T$ - vector of linear velocity of a moving object; $\vec{a}_C = [a_{xC} \ a_{yC} \ a_{zC}]^T$ - vector of linear acceleration of a moving object (measured onboard); $\vec{g}_I = [0 \ 0 \ g]^T$ - gravitational acceleration; $\vec{x}_{I0}, \vec{v}_{I0}$ - initial conditions.

Angular orientation of a moving object (TV camera) relative to ICF is specified via transform matrix connecting CCF to ICF, denoted as $T_{I/C}$. Introducing four
parameter operator \( \vec{b} = [b_1, b_2, b_3, b_4] \), the transform matrix could be expressed as

\[
T_{IC} = \begin{bmatrix}
  b_1^2 + b_2^2 - b_3^2 - b_4^2 & 2(b_1b_2 - b_3b_4) & 2(b_1b_3 + b_4b_2) \\
 2(b_1b_2 + b_3b_4) & b_1^2 - b_2^2 + b_3^2 - b_4^2 & 2(b_2b_3 - b_4b_1) \\
2(b_1b_3 - b_4b_2) & 2(b_2b_3 + b_4b_1) & b_4^2 - b_1^2 + b_2^2 - b_3^2
\end{bmatrix}
\] (2)

Vector \( \vec{b} \) has the geometric interpretation:

\[
\vec{b} = [E \sin \frac{\varepsilon}{2} \quad \cos \frac{\varepsilon}{2}]^T
\] (3)

Vector \( \vec{E} \) has the meaning of the ort of a principal axis (it is a direction in space about which the object should be rotated in order to coincide with ICF), while the angle \( \varepsilon \) represents the amount of this rotation. Angular orientation dynamics is defined by the following set of differential equations:

\[
\dot{b} = \frac{1}{2} \begin{bmatrix}
  0 & r & -q & p \\
-\dot{r} & 0 & p & q \\
q & -p & 0 & \dot{r} \\
-q & -q & -p & 0
\end{bmatrix}
\] (4)

The elements of matrix \( \Omega \) are the components of object’s angular velocity vector relative to body fixed CCF - \( \vec{\omega} = [p \quad q \quad r]^T \).

Linear accelerometers, rigidly fixed to the body of a moving object, are measuring

\[
\vec{a}_C^* = \vec{a}_C + \vec{a}_b + \vec{a}_n
\] (5)

where \( \vec{a}_C \) is physically existing acceleration, \( \vec{a}_b \) is a constant accelerometer bias, while \( \vec{a}_n \) represents the measurement noise. Similarly, set of rate gyros rigidly fixed to the body of a moving object measures

\[
\vec{\omega}_C^* = \vec{\omega}_C + \vec{\omega}_b + \vec{\omega}_n
\] (6)

Due to assumption that the biases of inertial sensors are constant, the following relationships are valid:

\[
[\dot{\vec{b}}_b \quad \dot{\vec{\omega}}_b] = [0 \quad 0]
\] (7)

Complete state vector of the inertial navigation system is of dimension 16, encompassing:

\[
\vec{X} = [\vec{x}_f^T \quad \vec{v}_f^T \quad \vec{b}_b^T \quad \vec{a}_b^T \quad \vec{\omega}_b^T]^T
\] (8)

The elements of state vector are the linear position and velocity of a moving object relative to ICF, its angular orientation relative to ICF, as well as the constant biases of linear accelerometers and rate gyros.
Excluding the uncertainty regarding the initial position and velocity, inaccuracy of an INS is a result of the fact that during numerical integration of differential equations (1) the measured acceleration (5) acts as an operand instead of physically existing one, while in integration of (4), the measured angular velocity (6) is included.

2.2 Autonomous Visual Navigation Algorithm

Autonomous estimation of the position of a moving object relative to the stationary object (landmark) existing inside the field of view of TV camera, time rate of change of this position, and angular orientation of a camera relative to ICF, assumes the existing of a priori knowledge of landmark’s shape and dimensions. Assuming additionally that landmark’s position in ICF is known also, the calculation of relative position of a moving object in respect to the landmark at the same time enables the calculation of its absolute position in ICF. The requirement that the landmark’s shape and dimensions should be known in advance is the fact limiting the area of possible applications. However, this field is wide enough, encompassing the number of possible robotic applications as well as the applications where the land vehicles are to be automatically guided in some indoor situations or on the highways, etc. In order to simplify the following calculations, it is assumed without the loss of generality that the central point of the landmark is located at the origin of ICF. Moreover, it is assumed that the landmark’s central point lies at the cross section of two strip-like areas located in the horizontal plane of ICF (Fig. 2.) In aerial navigation applications, this case could be related to the cross section of two roads (at known angle).

Dynamics of changes of a landmark central point position relative to CCF is given with

\[ \dot{x}_c = \omega \times x_c + \nu_c \]  

(9)
Position and velocity of a moving object relative to ICF are defined with

\[
\vec{x}_I = -T_{I/C} \vec{x}_C, \quad \vec{v}_I = -T_{I/C} \vec{v}_C
\] (10)

Points from three-dimensional space are projected onto the focal pane of an optical system of TV camera at the distance \( f \) from the camera’s sensitive element. Inside this plane, the coordinate frame fixed to the picture (PCF) is specified. After the appropriate image processing the coordinates of images of space points are specified in PCF as

\[
\vec{y} = [x_L \ y_L]^T = \left[ \begin{array}{c} \frac{y}{x_C} \\ \frac{z_c}{x_c} \end{array} \right]^T
\] (11)

Every point inside the picture is defined by its \( m \)-vector

\[
\vec{m} = [f \ x_L \ y_L]^T
\] (12)

while the picture lines specified by \( n_1 x_L + n_2 y_L + n_3 f = 0 \) are determined by the appropriate \( n \)-vector

\[
\vec{n} = [n_1 \ n_2 \ n_3]^T
\] (13)

Under the assumption that the relevant part of landmark could be distinguished as the rectangle ABCD, relative angular attitude of the camera can be calculated using the \( m \)-vectors of vanishing points \( P \) and \( Q \) (shown on Fig. 2) as

\[
T_{I/C} = [\vec{m}_P \ \vec{m}_Q \ \vec{m}_P \times \vec{m}_Q]^T
\] (14)

The image of landmark central point can be obtained at the cross-section of rectangle diagonals as

\[
\vec{m}_O = \pm \frac{[\vec{n}_{AC} \times \vec{BD}]}{\|\vec{n}_{AC} \times \vec{n}_{BD}\|}
\] (15)

The distance from camera to the landmark center is calculated as

\[
|\vec{R}| = \frac{\vec{m}_A(\vec{m}_P \times \vec{m}_Q)|O_IA|}{\|\vec{m}_{O_I}(\vec{m}_P \times \vec{m}_Q)\vec{m}_A - \vec{m}_A(\vec{m}_P \times \vec{m}_Q)\vec{m}_{O_I}\|}
\] (16)

and, finally, moving object’s position vector relative to ICF is found as

\[
\vec{x}_I = -|\vec{R}|T_{I/C}\vec{m}_{O_I}
\] (17)

A priori knowledge of the shape (rectangle) and dimensions (distance between points \( O_I \) and \( A \), in this particular case) enabled the reconstruction of scene structure (camera’s linear position in ICF (17) and its angular orientation (14)).
2.3 Visual navigation algorithm based on dynamic vision

Basic relationships characterizing visual navigation (state model (9-10) and model of measurements (11)) are valid in this case also. Linear velocity vector $\mathbf{v}_C$ and angular velocity vector $\mathbf{w}_C$ as well as the angular orientation of camera determined by $\mathbf{T}_{C/I} = \mathbf{T}^T_{I/C}$, are considered as a priori known variables (measured by independent sensors). These variables, appearing in (10-11) as parameters, are originating from an INS.

By processing of sequence of images obtained from TV camera it is possible to determine the inter-frame shift of characteristic landmark point between two consecutive frames. The new position of reference point in CCF is denoted as $[x_c + \Delta x_C \quad y_c + \Delta y_C \quad z_c + \Delta z_C]^T$ while the increment of position vector $\Delta x_C$ is a result of moving object’s motion between two frames and its change in angular orientation, given as

$$\Delta x_C = \Delta T_{C/I} \mathbf{T}^T_{C/I} \Delta x_C - (T_{C/I} + \Delta T_{C/I}) \Delta x_I \quad (18)$$

Shift of the image of reference point in PCF is determined by

$$\Delta x_L = f \frac{\Delta y_C}{x_c + \Delta x_C} - x_L \frac{\Delta x_C}{x_c + \Delta x_C}$$
$$\Delta y_L = f \frac{\Delta x_C}{x_c + \Delta x_C} - y_L \frac{\Delta x_C}{x_c + \Delta x_C} \quad (19)$$

Having in minds that $\Delta x_C$ can be expressed as $\Delta x_C = x_c [f \quad x_L \quad y_L]^T$, the knowledge of $\Delta x_I$, $T_{C/I}$ and $\Delta T_{C/I}$ from INS enables that components of $\Delta x_C$ from (18) could be expressed as functions of $x_c$, $x_L$ and $y_L$. By their replacement in (19) it is possible to calculate $x_C$, and after that, based on (11), the remaining two components $(x_C, y_C)$. Finally, it is possible to calculate the moving object’s position in ICF as: $\bar{x}_I = -T_{I/C} \bar{x}_C$.

2.4 Scenario of application

The simple straight line trajectory of motion is assumed. The starting position is specified at $\bar{x}_{I0} = [-1000 \quad -100 \quad -100]^T$ and the constant velocity of motion along x-axis of ICF is assumed as $\bar{v}_{I0} = [10 \quad 0 \quad 0]^T$. CCF coincides with ICF. Linear accelerometers are characterized by constant bias of 10mg (= 0.1m/s$^2$), while the rate gyros are of low accuracy (constant bias of 36°/h = 0.174 mrad/s). Measurement noises of accelerometers and rate gyros are assumed as white, Gaussian, with zero mean values and standard deviations of: $\sigma_{acc} = 10mg$ i $\sigma_{rg} = 1 mrad/s$, respectively. For the optical system of TV camera it is assumed that focal
distance is known \( f = 1 \). The process of determination of characteristic points in PCF is characterized by the measurement noise which is white, Gaussian, with zero mean value, and standard deviation of one pixel. Assuming that the overall field of view is \( 25.6^0 \) and that digitized image has 512 pixels along both directions, the angular equivalent of this error is equal to \( 0.05^0 \) (= 0.87 mrad). Central point of the reference object is located at the origin of ICF. Strip-like areas are of 20m width and characterized by recognizable contrast relative to the surrounding background. Reference distance \((OA)\) is equal to 14.1m in this case. The sampling frequencies are 100Hz in INS and 10 Hz in VNS.

3 Integrated Navigation Algorithm

3.1 General structure

The structure of intended integrated navigation algorithm is shown on Fig. 3.

![Fig. 3. Structure of an integrated navigation algorithm.](image)

The estimations of moving object’s position in ICF are continuously present as the output of INS. They are corrected by averaging with the position estimates originating from two separate parts of VNS algorithm, at the moments when the later ones are available. The estimate of a linear velocity is obtained in INS based on corrections from autonomous VNS and transferred toward VNS based on dynamic vision. The same is valid for the attitude information also. The scene structure as well as the motion parameters is simultaneously estimated based on the full available set of measurements: linear accelerations, angular rates, landmark points’ positions in every frame, and their shifts between two consecutive frames.

3.2 Corrections of INS

System model in state space obtained by grouping of equations (1), (4) and (7) is obviously nonlinear and it is needed to apply its linearization in the neighborhood of the estimated state vector in order to implement the optimal recursive state
estimator. The linearized model has the form

$$\dot{\hat{X}} = \frac{d}{dt}(\hat{X} - \hat{\tilde{X}}) = F\hat{X} - G\tilde{u}$$

(20)

where the matrices $F$ and $G$ are obtained by partial differentiation of nonlinear vector function $\tilde{X} = \tilde{h}_i(\hat{X}, \tilde{u})$ relative to state variables (8) and stochastic inputs specified in (5,6) as the components of measured signals at the outputs of inertial sensors.

The differences between INS and autonomous VNS outputs are supplied as system measurements. As the state model is of relatively high order (sixteen) it is beneficial to make its decoupling onto two subsystems. Rotational motion model is going to be considered separately (encompassing state variables $\hat{b}$ and $\tilde{b}_b$). After the replacement of expressions for measured angular rates (6) into the equation (4) and its partial differentiating, the appropriate matrices specified by general form in (20) would have the following meanings

$$F_{7\times 7}^1 = \begin{bmatrix} \Omega_{4\times 4} & \hat{B}_{4\times 3} \\ 0_{3\times 4} & 0_{3\times 3} \end{bmatrix}, \quad G_{7\times 3}^1 = \begin{bmatrix} \hat{B}_{4\times 3} \end{bmatrix}, \quad \hat{B}_{4\times 3} = \begin{bmatrix} \hat{b}_4 & -\hat{b}_3 & \hat{b}_2 \\ \hat{b}_3 & \hat{b}_4 & -\hat{b}_1 \\ -\hat{b}_2 & \hat{b}_1 & \hat{b}_4 \\ -\hat{b}_1 & -\hat{b}_2 & -\hat{b}_3 \end{bmatrix}.$$  \hspace{1cm} (21)

Vector $\tilde{b}$ calculated in VNS corresponds to matrix transform $T = T_{I/C}^{\text{VNS}}$ in the following way

$$\tilde{b} = \begin{bmatrix} t_{32} - t_{23} \\ t_{13} - t_{31} \\ t_{21} - t_{12} \end{bmatrix} 4b_4 \quad 0.5\sqrt{1 + \text{Trace}(T)}^T$$

(22)

Statistical parameters of the state model noise are assumed based on the assumptions regarding the rate gyro's measurement noise. Parameters of the system measurement noise are specified by the uncertainty in calculation of locations of vanishing points $P$ and $Q$.

In order to increase the accuracy of angular orientation calculation in autonomous VNS (matrix $T_{I/C}^{\text{VNS}}$) two important processing steps have been applied:

1. $m$-vectors of vanishing points are calculated by collecting all edge points of strip-like areas and by finding the best fitting line through them, rather than by using of rectangle vertices only for this purpose. This way the effective resolution of TV picture is improved.

2. in spite of this procedure, transform matrix obtained in (14) does not satisfy orthogonality condition in general case (the condition $\tilde{m}_P\tilde{m}_Q = 0$ is not fulfilled exactly). From this reason, an iterative procedure of minimizing
the scalar product is used, giving as a result: \( \bar{m}_P \bar{m}_Q \leq \epsilon \) (the inner product is made to be arbitrarily small). Vertical direction is obtained as the outer product of these estimates: \( \hat{m}_P \times \hat{m}_Q \).

The second part of correction algorithm consists in optimal estimation of the remaining part of state vector from equation (8) - \( [\bar{x}_T^T \bar{v}_T^T \bar{a}_b^T]^T \). Transform matrix \( \hat{T}_{IC} \) from the first phase of correction is used in the subsystem state model (1).

### 3.3 Dynamic Vision Algorithm Based on Corrected INS

After INS calculations of angular attitude and linear velocity have been corrected, part of VNS based on dynamic vision uses these data and by processing of two consecutive frames calculates the new estimate of moving object’s position - \( \bar{x}_{DV} \).

### 3.4 Resultant estimates

In 2.4 it is realistically assumed that the sampling frequency of INS is greater than inside VNS. According to this fact, INS continuously estimates scene structure while the corrections are made periodically, after calculations made by two parts of VNS algorithm. Motion parameters (linear and angular velocity) are calculated by INS only, but they are improved based on VNS calculations used in order to determine inertial sensors’ biases.

### 4 Illustrative Results

Some of the most illustrative results obtained through the simulations of suggested integration algorithm are given here. Figure 4. illustrates unlimitedly increasing position errors obtained during 50s of work of uncorrected INS. It is obvious that due to relatively high values of accelerometer biases, position estimates based on uncorrected INS could be valid just in a short time interval.

![Fig. 4. Position errors in the case of uncorrected INS.](image-url)
The appropriate position errors obtained by autonomous VNS are shown on Fig. 5. These errors are of oscillatory nature, but generally decreasing while the moving object approaches the landmark.

The benefits of corrections made in INS based on calculations inside autonomous

![Fig. 5. Position errors in the case of autonomous VNS.](image)

![Fig. 6. Position errors obtained with Dynamic Vision algorithm: (a) uncorrected INS; (b) corrected INS.](image)
VNS are obvious from the Fig. 6 where the position errors are illustrated for two cases of VNS algorithm based on dynamic vision. Diagrams (a) are obtained with uncorrected INS while diagrams (b) are results obtained when corrected INS data have been used. While the first ones are showing very low accuracy in the case of \( x \) coordinate, the later ones illustrate that the accuracy is now approximately the same as in autonomous VNS. Finally, Figure 7. is an illustration of the estimate of the most critical (\( x \)) position obtained by averaging of results all three navigation algorithms as it was specified on Fig. 3. It is obvious that the error is limited and that by approaching the landmark it becomes of order of 1m.

5 Conclusion

The integration of inertial and visual navigation algorithms have been considered here, following the main idea that by extending the number of available measurements and using the fact that the accuracy of different sensors is affected by different physical sources, the overall navigation system accuracy can be improved by proper combination of these data. The main step in this integration consists in corrections of INS calculations based on the scene structure estimates provided by autonomous VNS. As a result of this, INS internal errors are estimated and its short term good accuracy is enabling the acceptable results between the moments when VNS supplies its data. However, the accuracy of autonomous VNS is highly affected by image processing effects (image noise, finite resolution) and in order to obtain the useful results two internal optimization procedures in calculation of camera angular orientation have been suggested. The next step making the coupling of INS and VNS more strong is done by including the second part of VNS algorithm based on dynamic vision principle. Calculation of position of just one characteristic landmark point is less noise sensitive than in the case of autonomous VNS where four characteristic points are of interest. On the other side, dynamic vision
algorithm is critically dependent on the accuracy of data supplied by INS. If the data regarding angular orientation and linear velocity are provided from corrected INS, the accuracy of position estimates obtained by dynamic vision algorithm is becoming comparable to other two. This way the redundancy is increased and the resultant position estimates are obtained as a result of averaging of three separate ones.

Although the analyzed moving object trajectory and type and dimensions of a landmark are of the aerial application type, it seems that the indoor applications are the most promising field where this approach could be applied. Using the maps of buildings, extra added reference objects fixed on walls and floor as well as wall/floor edges and corners existing in such an environment, it is possible to overcome the problem of usual non-existence of GPS signals and to allow the usage of such type of navigation system integration.

References


