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Parallel Inverter Analysis Using Mathematical Software

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Abstract: This paper examines the transient processes at turning on of the parallel current source inverter and parallel resonant inverters, using the program MatLab. The obtained results allow the transient process to be assessed and its design to be done to obtain favorable turn on transient process.

Keywords: Current source inverters, resonant inverters, MatLab, research, induction heating.

1 Introduction

Current-source inverters and resonance inverters are widely used in industry and household conditions. Their advantages as power supplies for the implementation of a variety of electrical technologies (inductive melting, hardening, etc.) are well-known [1, 2, 3]. The specialized literature contains analysis of both transition [4, 5, 6] and steady modes of operation [4, 5, 7].

Resonance inverters without reverse diodes can operate in three modes: continuous-current mode (current-source inverter), borderline mode and discontinuous-current mode [1, 4, 7]. On account of this, many authors have proposed a unified approach to the analysis of resonance inverters for electric technology applications. [5, 7].

Theoretical analysis of resonance inverters may be performed using different methods according to the equivalent schematics which are in effect between two consequent switches of the power elements. The most popular methods of analysis are as follows:[4, 5]:

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a) by classic or operator-based solving of the integral-differential equations constructed using the moment values method;

b) using harmonic analysis method;

c) using the switching functions;

d) solving the difference equations;

e) using the method of the first harmonic (the fundamental component);

f) using the geometric method;

g) by mathematical modelling of electromagnetic processes in the inverters based on the integral-differential equations using computer technology.

The first four methods are precise but lengthy and arduous and the last two of them are mainly being used for studying of transition processes, although being applicable to steady modes too. The method of the first harmonic and the geometric method in their most widely used variations are illustrative and simple but provide less accurate results. The method of mathematical modelling is mainly applied to studying of transition processes.

In addition to the aforementioned methods, widely used are purely computer methods of analysis using computers. Their use is very promising in the study of transition processes as well as steady processes in invertors by means of continuous integration of differential equations using the method of Runge-Kutta [6], which needs relatively uncomplicated preparation of the algorithm in case the computer has a set of suitable standard mathematical functions.

The present work proposes the creation of mathematical model describing the operation of a parallel resonance inverter, operating in forced (current-source inverter) or natural switching mode of elements in transient or steady modes of operation. The system of differential equations describing the operation of parallel resonance inverter is solved using specialized mathematical software - MatLab [8], without the help of the visualization package Simulink.

2 Model of Parallel Resonance Inverter

The schematic of a full-bridge parallel resonance inverter is shown on Fig. 1. Semiconductor elements are represented by one-way conductivity switches. $S_1 \div S_4$.

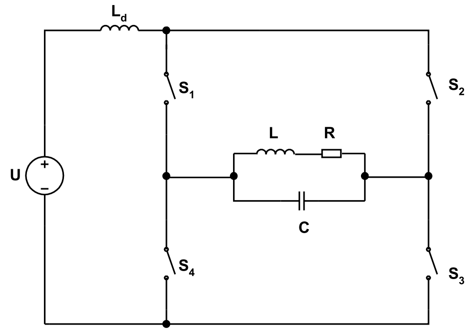

Fig. 1. Full-bridge parallel resonance inverter with parallel compensated load

2.1 Brief mathematical description of the model

The operation of the parallel resonance inverter is described by the following three systems of differential equations:

$$\dot{x}_i = \sum_{j=1}^{3} a^1_{ij} x_j + b_i \;, \tag{1}$$

with initial conditions $x_i(0) = x_i^1$, for the odd half-periods of circuit operation $i = 1,2,3$.

The coefficients in this system are:

$$\{a^1_{ij}\} = \begin{pmatrix} 0 & a_{12} & a_{13} \\ a_{21} & 0 & 0 \\ a_{31} & 0 & a_{33} \end{pmatrix} \quad \text{and} \quad \{b_i\} = \begin{pmatrix} 0 & b_2 & 0 \end{pmatrix}^T .$$

$$\dot{x}_i = \sum_{j=1}^{3} a^2_{ij} x_j + b_i \;, \tag{2}$$

with initial conditions $x_i(0) = x_i^1$, for the even half-periods of inverter full-bridge operation $i = 1,2,3$.

The following coefficients are used in this system:

$$\{a^2_{ij}\} = \begin{pmatrix} 0 & -a_{12} & a_{13} \\ -a_{21} & 0 & 0 \\ a_{31} & 0 & a_{33} \end{pmatrix} \quad \text{and} \quad \{b_i\} = \begin{pmatrix} 0 & b_2 & 0 \end{pmatrix}^T .$$

$$\dot{x}_i = a^3_{1j} x_1 + a^3_{3j} x_3 \;, \tag{3}$$

with initial conditions $x_i(0) = x_i^3$, for the second part of every half-period with the circuit operating in natural element switching mode when current is not being drawn from the power source $i = 1,3$.

In this system

$$\{a_{ij}^3\} = \begin{pmatrix} 0 & a_{13} \\ a_{31} & a_{33} \end{pmatrix}.$$

We calculate the coefficients:

$$a_{12} = 1/C,\ a_{13} = -1/C,\ a_{21} = -1/L,\ a_{31} = 1/L,\ a_{33} = -R/L \text{ and } b_2 = U/L_d.$$

We replace

$$x_1 = u_C,\ x_2 = i_d \text{ and } x_3 = i.$$

2.2 The model works with arbitrary number of steps

Step one: We solve system (1) with zero initial conditions $x_i(0) = 0$ keeping record if the current through L_d i.e. $x_2(t)$ turns to zero before the end of the half-period $T/2$. If $x_2(\bar{t}) = 0$ in moment $\bar{t}$ - occurring before the end of the half-period, then we proceed to solve system (3) and for the initial values of x_1 and x_3 we take the results from system (1) in moment $\bar{t}$. After the end of the half-period we unconditionally proceed to solve system (2) i.e. step two. If $x_2(t)$ does not turn to zero before the end of the half-period, then after the end of the half-period we proceed from system (1) again to solve system (2) i.e. step two.

Step two: We solve system (2) using the following initial conditions. If we have proceeded to solve system (2) after we have solved system (3) then for the initial conditions we take the values of x_1 and x_3 at the end of the previous half-period and also set an initial condition $x_2 = 0$. If we have proceeded to solve system (2) after solving system (1) then for the initial conditions we take the values of x_1, x_2 and x_3 at the end of the previous half-period. Then we proceed the same way as in step one keeping record if $x_2(\bar{t}) = 0$ in moment $\bar{t}$- occurring before the end of the half-period. Depending on this condition we proceed to solve system (1) or system (3) with the same initial conditions as in step one.

Step three: We return to step one but now we solve system (1) with initial values depending on whether we came from system (3) or system (2).

Remark: With certain values of the elements of the parallel resonance inverter the aforementioned algorithm may work by repeatedly connecting systems: (1)-(2)-(1)-(2)$\ldots$. This is the so-called continuous-current mode (current-source inverter) i_d (the variable x_2). With other initial data the same algorithm may work by repeatedly connecting systems: (1)-(3)-(2)-(3)-(1)-(3)-(2)-(3)-.... This is the so-called intermittent-current mode i_d or natural element switching mode (continuous-current mode). That's why we consider the implemented algorithm to be a general model of a parallel resonance inverter describing these two modes of operation. Depending on the initial data, the algorithm selects the necessary mode of execution.

2.3 Method used for solving the aforementioned systems of equations

Authors propose solving the systems of differential equations describing the transition and steady modes of parallel resonance inverter operation using the matrix exponent method.

The essence of the method is as follows: We represent the system as a matrix i.e..

$$\dot{x} = Ax + b \ , \tag{4}$$

with initial conditions $x(0) = x^0$.

In this system the matrix $A = \{a_{ij}\}$ is the matrix of coefficients and x, b and x^0 are column vectors of the functions to find, the free members and the initial conditions i.e.:

$$x = \begin{pmatrix} x_1(t) & x_2(t) & x_3(t) \end{pmatrix}^T, \quad b = \begin{pmatrix} 0 & b_2 & 0 \end{pmatrix}^T, \quad x^0 = \begin{pmatrix} x_1^0 & x_2^0 & x_3^0 \end{pmatrix}.$$

The general solution of the system is [9]:

$$x = e^{At}x^0 + A^{-1}(e^{At} - E)b \ . \tag{5}$$

Remark: Some authors [5, 6] resort on solving the aforementioned systems using the method of Runge-Kutta, but considering the potential of modern computing technology we believe that the matrix exponent method to be more suitable. This is mainly due to the fact that if we can calculate exactly the matrix exponent then (5) gives the exact solution of (4). The computer technology available to most users nowadays, combined with mathematical software (MatLab in particular) provide the means for computing this exponent with very small inaccuracy.

On the basis of the presented mathematical description of the model a program has been developed using MatLab5.3, which models the transition and steady modes of operation of a full-bridge parallel inverter.

The results of the simulation are presented as diagrams describing the changes over time of the load voltage, the input inductivity current and the load current.

3 Specialized Program for the Study of Parallel Resonance Inverter

A special program has been developed which allows input of initial data and output of the corresponding results as diagrams. Fig. 2 shows the program control panel which contains input boxes for initial data (input inductivity L_d, load parameters R and L, capacitor C in parallel, control frequency $1/T$, supply voltage U_d, duration of the simulation in milliseconds t). It also contains buttons which when pressed display the graphical results of the analysis. The results received by solving the system of equations can be used to calculate all other currents and voltages in the inverter power circuit. Fig. 3, 4, and 5 show the results of a simulation of a parallel current-source inverter (resonance inverter with forced element switching) and Fig. 6, 7, and 8 - the results of a simulation of a parallel resonance inverter (resonance inverter with natural element switching).

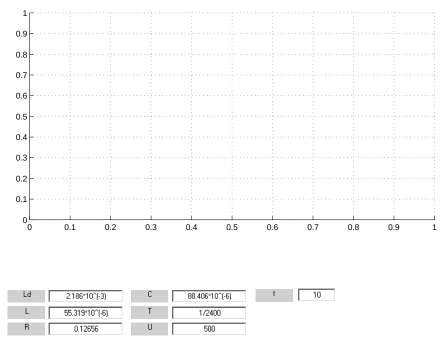

Fig. 2. Program control panel.

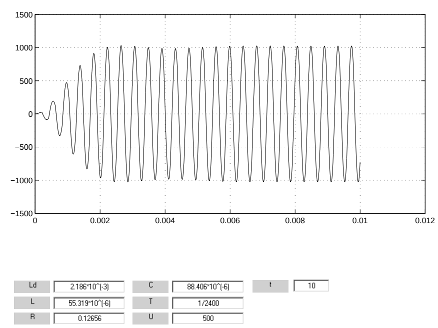

Fig. 3. Output voltage of current-source inverter.

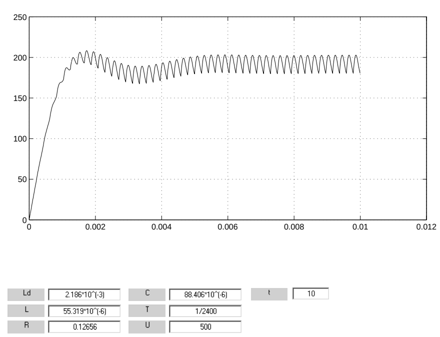

Fig. 4. Input current of parallel current-source inverter.

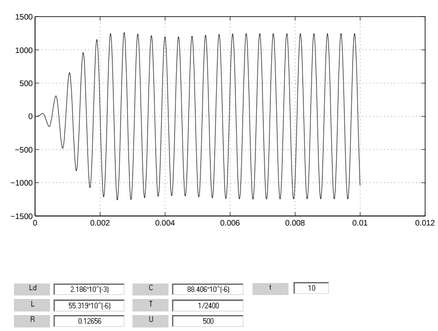

Fig. 5. Current through the load of parallel current-source inverter.

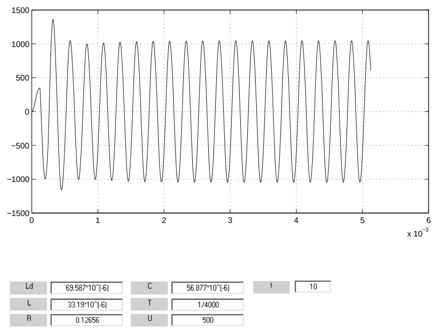

Fig. 6. Output voltage of parallel resonance inverter.

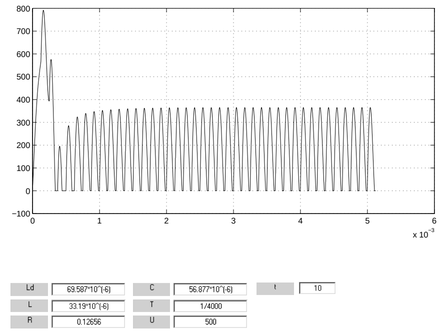

Fig. 7. Input current of parallel resonance inverter.

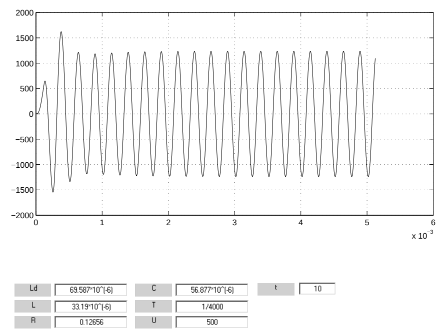

Fig. 8. Current through the load of parallel resonance inverter.

4 Comparison of Results and Accuracy Assessment

In order to check the accuracy of the received results they were compared with results received from simulation of a parallel current-source inverter using the computer simulator ”PSPICE” and also with results received from analysis of the steady mode of operation. The initial data used for the design of the parallel current-source inverter is: output active load power $P = 100kW$; load power factor $\cos\varphi_T = 0.15$; effective value of load voltage $U_T = 750V$; output frequency $f = 2400Hz$.

Table 1 is used to compare the results estimated using the method for design of a parallel current-source inverter with the results computed by the simulator and the MatLab application.

Table 1 shows that the results received from the program created by the authors are similar to the results received from the computer simulator and the steady mode analysis (the difference is less than 2%).

Initial data used for the design of the parallel current-source inverter: output

Table 1.

results	R_T, Ω	L_T, μ H	C, μF	L_d, mH	I_d,A
estimated	0.12656	55.319	88.406	2.187	200
PSPICE	0.12656	55.319	88.406	2.187	195.77
MATLAB	0.12656	55.319	88.406	2.187	197.4
results	UR_T,V	U_{Dm},V	t_q, μ s	f,Hz	
estimated	750	1060.66	48.874	2400	
PSPICE	751.7	1064.1	49	2400	
MATLAB	748.5	1058.38	48.5	2400	

active load power $P = 100kW$; load power factor $\cos\varphi_T = 0.15$; effective value of load voltage $U_T = 750V$; output frequency $f = 4000Hz$.

Table 2 is used to compare the results estimated using the method for design of a parallel current-source inverter with the results computed by the simulator and the MatLab application.

Table 2.

results	R_T, Ω	L_T, μH	C, μF	L_d,mH	I_d,A
estimated	0.12656	33.1919	56.877	69.578	215.256
PSPICE	0.12656	55.319	88.406	2.187	217.8
MATLAB	0.12656	55.319	88.406	2.187	216.43
results	I_{max},A	UR_T,V	U_{Dm},V	t_q, μs	f,Hz
estimated	396.82	750	1060.66	48.874	4000
PSPICE	377.117	751.7	1064.1	49	4000
MATLAB	379.2	750.5	1061.207	48.5	4000

Table 2 shows a very good match of the results of the three methods for parallel resonance inverter analysis (the difference is less than 5%.

5 Conclusion

The proposed program allows the evaluation of the transition process occurring on start-up of the parallel resonance inverter and enables designing an inverter with the most favorable start-up transition process which is essential with this type of converting devices. The steady modes of operation can also be examined after the transition process is over. The usage of specialized mathematical software [10, 11] enables the integration of the proposed program in a complete environment for research of autonomous resonance inverters which may include education, choosing of power circuit schematic, estimation of circuit elements' values, examination of transition and steady modes of operation using the computer simulator PSPICE and the mathematical software MATLAB, design enhancement based on certain criteria, evaluation of design accuracy and preparation of project documentation.

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