 Forced Dynamics Control Of An Actuator With Linear PMSM

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Abstract: The paper presents design and verification of Forced Dynamics Control of an actuator with linear permanent magnet synchronous motor. This control method is a relatively new one and offers an accurate realisation of a dynamic speed response, which can be selected for given application by the user. In addition to this, the angle between stator current vector and moving part flux vector is maintained mutually perpendicular as it is under conventional vector control. To achieve prescribed speed response derived control law requires estimation of an external force, which is obtained from the set of observers. The first observer works in pseudo-sliding mode and observes speed of moving part while the second one has filtering effect for elimination of the previous one chattering. The overall control system is verified by simulations and experimentally. Preliminary experiments confirmed that the moving part speed response follows the prescribed one fairly closely.

Keywords: Feedback linearization, Linear permanent magnet synchronous motor, Nonlinear system control, Observers.

1 Introduction

TWO major categories of linear motors with low and high acceleration have been developed. Low-acceleration linear motors are suitable mainly for ground-based transportation applications while high acceleration motors are designed to accelerate an object up to a very high speed. This paper develops a low acceleration speed control of linear permanent magnet synchronous motor (LPMSM).
Control system exploits principles of feedback linearization [1] and forces moving part speed to follow demanded speed with prescribed closed-loop dynamics.

Presented speed control system for LPMSM is based on Forced Dynamics Control (FDC) [2] while respecting also principles of vector control [3, 4]. FDC method enables to design such control algorithms, which have precisely defined the acceleration profile and therefore it can contribute to the travel comfort of the moving objects. Derived control algorithms respect mutual orthogonality of the force producing primary part current vector and moving part magnetic flux vector as it is in conventional vector controlled rotational drives with PMSM.

To achieve prescribed settling time during speed transients the information about external force is needed. The estimate of external force together with the estimate of translation speed is obtained from the set of two observers. The first observer is based on sliding-mode principles and to avoid chattering of the estimated variable this observer is completed with another one, which has filtering effect. Disturbances estimation gives FDC a certain degree of robustness not only with respect to external disturbances but also to plant parameter variations since the plant parameter variations are equivalent to load forces applied to the unperturbed plant model. The original contribution of this paper is a preliminary experimental verification of the control algorithms presented in [5] capable to control speed of moving part with prescribed closed loop dynamics.

The overall speed control system of LPMSM, which is shown in Fig. 1, has a nested structure comprising an outer master control loop and inner slave control loop. Outer control loop computes such demanded primary part currents, which realizes the closed loop prescribed dynamic behavior of the drive. This way selected operational modes with defined acceleration and speed profile can be prescribed. The inner slave control loop forces real three-phase currents of primary part to fol-
low their computed demands from master algorithm with negligible lag via control of inverter power electronic switches.

2 Control Algorithm Design

FDC algorithm based on feedback linearisation and vector control principles is developed in two steps. Firstly, the model of LPMSM is formulated in the $d_q$ co-ordinate system, which is coupled to the moving part of LPMSM:

$$\frac{ds_{mp}}{dt} = v_{mp} \tag{1}$$

$$\frac{dv_{mp}}{dt} = \frac{1}{M} [c(\Psi_d i_q - \Psi_q i_d) - F_{ext}] \tag{2}$$

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \frac{R_s}{L_d} & \frac{P v_{mp} L_q}{r L_q} \\ \frac{P v_{mp} L_d}{r L_q} & -\frac{r L_d}{L_q} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} - \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ L_d \\ 0 \\ 1 \\ \Psi_{PM} \end{bmatrix} + \begin{bmatrix} u_d \\ u_q \end{bmatrix} \tag{3}$$

where, $[i_d, i_q]^T$ and $[u_d, u_q]^T$ are, respectively, column vectors of the LPMSM primary part current and voltage components, $[\Psi_d, \Psi_q]^T$ is column vector of moving part magnetic flux with components $\Psi_d = L_d i_d + \Psi_{PM}$ and $\Psi_q = L_q i_q$ as direct and quadrature magnetic fluxes in the moving part coupled transformation frame. Position and velocity of the moving part are $s_{mp}$ and $v_{mp}$. Motor constant is $c = 3P/2r$, where $P$ is number of pole-pairs and $r = P \tau_p/\pi$ is a constant parameter of LPMSM having the dimensions of length, and $\tau_p = 82.5$ mm is pole pitch of the motor. The external force acting on the moving part is $F_{ext}$, $R_s$ is the phase resistance, $L_d$ and $L_q$ are the direct and quadrature phase inductances, $\Psi_{PM}$ is the permanent magnet linkage flux and $M$ is the mass of the linear motor moving part plus the equivalent mass of the driven mechanism.

2.1 Development of speed control algorithm

Secondly, the linearising function for translation speed of the moving part, which forces this speed to obey specified closed-loop linear differential equation with a prescribed time constant, $T_v$, is developed:

$$a_{mp} = \frac{dv_{mp}}{dt} = \frac{1}{T_v} (v_d - v_{mp}) \tag{4}$$

Linearisation function is obtained by equating the right hand side of (4) with the right hand side of the corresponding motor equation (2). This forces the non-linear
differential equation (2) to have the same response as the linear equation (4). Thus:

$$\frac{1}{M} [c(\Psi_d i_q - \Psi_q i_d) - F_{ext}] = \frac{1}{T_v} (v_d - v_{mp}).$$

(5)

The following part of the control law is formulated on the basis of vector control, which requires mutual orthogonality of moving part magnetic flux and primary part current vector. Following conventional approach [3] to achieve maximum flux up to nominal speed the current demand in direct axis is set to zero:

$$i_d = 0.$$  

(6)

Setting $i_d = 0$ in (5) on the assumption that the real current follows its demand, $i_d = i_{d,d}$ direct axis flux can be replaced by permanent magnet linkage flux only, $\Psi_d = \Psi_{PM}$. Solving equation (5) for force producing current component, $i_{q,d}$ yields the complete master control algorithm formulated for primary part current demands in the $d$- and $q$-axis:

$$i_{d,d} = 0$$  

(7a)

$$i_{q,q} = \frac{1}{c\Psi_{PM}} \left[ \frac{M}{T_v} (v_d - v_{mp}) + \hat{F}_{ext} \right] = \frac{Ma_{mp} + \hat{F}_{ext}}{c\Psi_{PM}}.$$  

(7b)

The numerator of the derived algorithm (7b) consists of two parts. The first one contains the demanded output acceleration and creates dynamic force during transients. The second part covers the external force, which needs to be estimated. By changing the prescribed acceleration, $a_{mp}$, in (7b) various operational modes can be realised.

a) Linear first order speed response, (the case already described), where specified settling time, $T_s$, is defined as the time taken for the error between demanded and real speed to fall and to stay below 5% ($T_s = 3T_v$):

$$a_d = \frac{1}{T_v} (v_d - v_{mp}) = \frac{3}{T_s} (v_d - v_{mp}).$$  

(8)

The derived master control law (5) yields a moving part speed response with linear, first order dynamics and unity dc gain defined as:

$$F_s(p) = \frac{v_{mp}(p)}{v_d(p)} = \frac{1}{1 + pT_v}.$$  

(9)
b) Direct acceleration control with constant acceleration (ramp), where the moving part produces an acceleration following a demanded constant one with negligible dynamic lag. The demanded acceleration is determined by a demanded of velocity, \( v_d \) and the demanded acceleration time, \( T_s \) as:

\[
a_d = \frac{v_d}{T_s} \text{sgn}(v_d - v_{mp}).
\]  

(10)

Via proper definition of smooth mathematical functions for drive acceleration as shown in [2] different operational modes can be achieved. Prescribed ideal acceleration and speed responses to described modes for \( M = 1 \, \text{kg} \) are shown in Fig. 2:

![Prescribed profiles of speed and accelerations for a) first order dynamics and b) constant acceleration - ramp.](image)

**2.2 Representation of the load**

Representation of the driven mechanical load, which is shown in Fig. 3, is another important feature of the designed control system.

This way only the mass of motor moving part, \( M_m \) is included in the forward path as a rigid body, moving *without external force and friction* [6]. Both, external force and friction are modelled in the feedback path. The justification for this is that the not completely known friction forces can be for some applications dominated, [7].

It is also important to note that it is unnecessary to provide an accurate model of the inverse load dynamics. Since the error in the value of moving mass, \( M_m \) can be represented as a dynamic load force, \( M_e \frac{dv}{dt} \), and this together with any external force acts at the same point as \( F_{ext} \), then the estimate, \( \hat{F}_{ext} \) from the observer will include both these forces and so both of them will be compensated.
3 State Estimation And Filtering

The speed of moving part and external force, which are inputs for master control algorithm are produced by the following set of observers. The first observer, which is based on primary part current equation, works in pseudo-sliding-mode, [8] and generates an unfiltered estimate of translation speed. This observer is completed with filtering one, which provides filtered speed estimate together with estimate of external force.

3.1 Pseudo-sliding-mode observer

The pseudo-sliding-mode observer for estimation of translation speed is shown in Fig. 4a. Its real time model is based on a primary part current, \( i_q \) eq., (3b), in which are purposely used only the terms without \( v_{mp} \). Term containing \( v_{mp} \) are in (11) replaced in the model by the correction input, \( v_q \). Thus the current component, \( i_q^* \) model is:

\[
\frac{di_q^*}{dt} = -\frac{R_s}{L_q}i_q^* + \frac{1}{L_q}u_q - v_q
\] (11)

where \( i_q^* \) is estimate of \( i_q \) as in a conventional observer. In sliding-mode the estimate of \( v_q \) is produced as a fast switching of \( U_{max} \), (12a). However, the useful observer output is the continuous equivalent value of the rapidly switching \( v_q \).

Equation (12a) therefore cannot directly generate \( v_{eq} \). Instead, a pseudo-sliding-mode observer is formed, [8] by replacing signum function with high gain and saturation limits (12b):

\[
\begin{align*}
v_q &= -U_{max} \text{sgn}(i_q - i_q^*) \\
v_{eq} &= -K_{sm}(i_q - i_q^*)
\end{align*}
\] (12)

where the gain, \( K_{sm} \), is made as high as possible [9] within the stability limit. For large \( K_{sm} \), the error between real and fictitious observer current is driven almost to
zero, resulting in:

\[ v_q = \frac{-p v_{mp} \Psi_{PM}}{r_L} \]  

and this may be manipulated to yield an unfiltered velocity estimate, \( v_{mp}^* \). Thus:

\[ v_{mp}^* = \frac{-r_L q v_{eq}}{p \Psi_{PM}}. \]  

### 3.2 Filtering observer

Chattering of the pseudo-sliding mode observer due to its high gain can be limited by 'filtering' function of the following observer which provides the velocity estimate and external force estimate [10]. Real time model of this observer is based on the motor force equation (2), which is augmented by the second state equation, \( dF_{ext} / dt = 0 \), with assumption of the piecewise constant external force, \( F_{ext} \). The observer correction loops are actuated by the error between the unfiltered speed estimate of pseudo-sliding-mode observer, \( v_{mp}^* \) as input and the filtered translation speed estimate, \( \hat{v}_{mp} \), as observer output. The observer is shown in Fig. 4b and corresponding real time model is:

\[ e_v = v_{mp}^* - \hat{v}_{mp}, \]  

\[ \frac{d\hat{v}_{mp}}{dt} = \frac{1}{M} \left[ c (\Psi_d i_q - \Psi_q i_d) - \hat{F}_{ext} \right] + k_v e_v, \]  

\[ \frac{d\hat{F}_{ext}}{dt} = 0 + k_F v_e. \]  

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**Fig. 4.** Block diagrams of a) pseudo-sliding mode observer and b) filtering observer.

This is a second order linear observer with a correction loop characteristic polynomial, which may be chosen via the gains, \( k_v \) and \( k_F \) to yield the desired balance of filtering between the noise from the measurements of \( i_d \) and \( i_q \) and the noise from
the pseudo-sliding-mode velocity estimate. The transfer function of the observer, shown in Fig. 4b, is:

\[
F(p) = \frac{\hat{v}_{mp}(p)}{v_{mp}(p)} = \frac{pk_v + k_F}{p^2 + pk_v + k_F}. \tag{18}
\]

The observer gains may be chosen to yield a non-oscillatory state estimation error transient with prescribed settling time, \(T_s\). Based on settling time formula, \(T_s = \frac{1}{2}(1 + n)/\omega_0\), (where \(n\) is order of the system and \(\omega_0\) are multiple poles of the desired polynomial), the observer poles can be placed at \(\omega_0 = 9/2T_s\) yielding following polynomial:

\[
(p + \frac{9}{2T_s})^2 = p^2 + \frac{9}{T_s}p + \frac{81}{4T_s^2}, \tag{19}
\]

which if compared with (18) yields observer gains:

\[
k_v = \frac{9}{T_s} \tag{20a}
\]

\[
k_F = \frac{81M}{4T_s^2}. \tag{20b}
\]

Although the external force is assumed constant in the formulation of the observer real time model, the estimate of \(\hat{F}_{ex}\) will follow a time varying external force and will do so more faithfully as \(T_s\) is reduced, but at the expense of sensitivity to any noise contaminating the speed estimate.

### 4 Derived Control Algorithms Verification

Verifications of the designed speed control algorithms of LPMSM are carried out in two steps. The simulations were followed by the experiments on the test bench in laboratory shown in Fig. 5. Parameters of the LPMSM used in simulations and for control are listed in Appendix. A sampling frequency of 12.5 kHz achieved during experiments was used also for the power electronics switches of inverter in simulation.

Two simulations shown on Fig. 6 and Fig. 7 illustrate the designed control system response to a step speed demand of \(v_{dem} = 1 \text{ ms}^{-1}\) applied for time interval \(t \in (0.51.0)\text{s}\), which is followed by another step speed demand of \(v_{dem} = -1 \text{ ms}^{-1}\) applied for time interval \(t \in [1.01.5) \text{s as shown in subplot a). Prescribed settling time or time of ramp is } T_r = T_s = 0.2s.\) External force due to significant friction in the loading mechanism was modelled as a sum of friction force, \(F_{fr} = 50\text{sign}(v)\)
and external force, which is applied at $t = 0.8s$, and drops to zero at $t = 1s$. Negative external force is applied again at $t = 1.3s$ when speed is also negative. Subplot (b) shows primary part both current components for entire measured interval. Simulations of filtering observer operation show estimated external force $\hat{F}_{ext}$ as a sum of friction force and applied external force for whole time interval, $t \in (0.415)s$ are shown in subplot c).

![Fig. 6. Simulation results for the first order dynamics.](image)

Simulation results for ramp speed demands are shown in Fig. 8 and arranged in the same pattern as for the first order dynamics. To eliminate control chattering in steady state the original algorithm, (10) with signum function, was for $v_{mp} > 0.95v_{dem}$ replaced with the first order dynamics algorithm, (7) in which short time constant was selected.

During simulations and experiments the high gain of pseudo-sliding mode observer was set to $K_{SM} = 5e3$ and settling time of the filtering observer was chosen as $T_{so} = 20ms$. From the time function of estimated translation speed and external force it is possible to observe correct function of both observers including settling time of filtering observer. As can be seen from both simulations presented the prescribed settling time, $T_s = 3T_r$, and ramp time, $T_r$ including prescribed dynamics of the speed response were reached and good agreement with theoretical predictions was achieved.

Experimental results for control of LPMSM with the first order dynamics and
ramp speed demand are shown in Fig. 8 and Fig. 9, respectively and arranged into three subplots which correspond to simulations of Fig. 6.

As can be seen from Fig. 8 the prescribed first order dynamic was achieved (95% of the demanded speed is achieved at prescribed settling time). If compared with simulation differences can be seen mainly in the external force estimates due to sudden changes of friction sign at the moment of speed reverse. This has reflection in iq current component demand of sub-plot b), which arise significantly and real motor current is not capable to follow such high demand.

Proper work of both observers (pseudo-sliding mode and filtering) demonstrates subplot c) showing the estimate of significant friction in the loading mechanism, which consists of rotational loading motor and the set of pulleys and wire. Further external force was not applied during experiments and therefore presented experimental results are marked as ’preliminary’.

Possibility to control LPMSM with the first order dynamics is very important from the position control point of view, which will follow as the next step, because it simplifies design of the position control system. Thanks to this fact the overall speed control system can be replaced with the first order delay as defined by (9), then completed for the kinematic integrator and whole position control system can be designed as the second order system.
Fig. 9 shows the measured results for ramp speed demand. As can be seen from this figure the prescribed ramp response was achieved including prescribed ramp time.

To avoid control chattering of the ramp speed response in steady state due to signum function in control law, this control law was replaced for speed \( v_{mp} > 0.95 v_{dem} \) with the first order dynamics law (14), in which short time constant, \( T_v \) was prescribed (the same way as for simulations).

Preliminary experimental results of the proposed FDC method for an electric actuator with LPMSM also confirmed good agreement with the theoretical predictions made during control algorithms development. The actuator follows prescribed dynamics fairly closely during transients and in steady-state.

5 Conclusions and Recommendations for Further Work

Presented preliminary experimental results indicate that the designed control system based on FDC for the actuator with LPMSM operates properly. It can be observed from Fig. 8 for the first order dynamics and from Fig. 9 for ramp speed demand, respectively. Based on these results can be concluded that prescribed dynamics were achieved as it was intended.

The designed pseudo-sliding mode observer and filtering observer, which provide necessary estimates of moving part translation speed and external load force including friction operate properly, have non-oscillatory character and estimate the state variables necessary for the derived control algorithms.

The obtained preliminary experimental results of speed control of LPMSM are encouraging and sufficiently promising to warrant further investigations of LPMSM position control with or without position sensor. The experimental verification of the position control system with LPMSM actuator based on the first order speed dynamic is sought as a continuation of this work.
Appendix

LPMSM parameters: \( P_N = 3.2 \) kW, number of pole-pairs \( P = 4 \), \( r = 0.105 \) m, \( R_s = 9.52 \Omega \), \( L_d = 0.8 \) H, \( L_q = 0.8 \) H and \( \Psi_{PM} = 0.467 \) Vs. DC bus voltage \( U_{DC} = 566 \) V, nominal current \( I_N = 4.4 \) A, total load mass \( M = 57.5 \) kg and nominal force \( F_N = 200 \) N.

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