Equipotential Surface Identification of Punctual Electricaly System Charges Using Measurement Results

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Abstract: Identification of equipotential surfaces, based on the measurement results and by application of Least Dispersion Volume Method (LDVM) is described in this paper. In a concrete example of two punctual electric charges a suitable methodology for determining corresponded parameters, which characterize the observed equipotential surfaces is presented. Error analyses that take into account the tested parameters are given.

Keywords: Electromagnetic, Least Dispersion Volume Method, equipotential surfaces, Least Square Method.

1 Introduction

Determining time-space dependencies of electromagnetic quantities in environment, such as for example conductive, semi-conductive, electrical and magnetic, represents a crucial problem in theoretical and practical electromagnetic.

During the last twenty years significance efforts have been made for realization of measurement equipment intended for evaluation of electromagnetic quantities. However, the accuracy of these instruments has not been at satisfactory level. By combining hardware and software techniques, that are typical for embedded systems, it is possible in a efficient manner to by-pass the above mentioned problems.

By performing a huge amount of measurement, for a short time interval, provide us to store complete characteristics of interdependencies among the electromagnetic quantities. Typical such examples are distribution of electrical field and potential in term of space coordinates of their interdependencies. Thanks to the

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results obtained in the area of error theory and mathematical statistics, without limited accuracy of the conducted measurements, it is possible now to increase the measuring accuracy by involving filtering. Filtering is performed by involving data-manipulation in a software way. These kinds of data-manipulations are performed by specialized measuring equipments called intelligent instrumentation. The filtering process of the measurement results, is realized by involving special mathematical apparatus implemented in a form of an algorithm embedded into the intelligent instrumentation.

Least Square Method (LSM) is well known mathematical procedure intended for data-manipulation with the measurement results during identification of their functional dependencies [1–3]. In this paper a suitable method, called Least Dispersive Volume Method (LDVM), by which it is possible to determine the shape of equipotential surfaces for two punctual charges is described.

The Identification criterion of LDVM from mathematical point of view is different in respect to LSM. It has some other properties and possibilities. However, in cases when both methods can be applied the obtained results, from aspect of error accuracy evaluation during measurement, are practically identical [4–7].

But in the described method, the procedure which relates to identification of multi-dimensional surface \( f(x_1, x_2, \ldots, x_m) = 0 \) has advantages in respect to LSM. The LSM criterion is based on minimal average value of the tested quantities. For instance, \( \Delta x_{1i} = x_{1i}(x_{2i}, x_{3i}, \ldots, x_{mi}) - x_{1i}, \ i = 1, n \), errors \( \Delta x_{2i}, \Delta x_{3i}, \ldots, \Delta x_{mi} \) of quantities \( x_2, x_3, \ldots, x_m \) are not taken into account. In practice this situation is not real.

The criterion LDVM is based on minimization of error product \( \Delta x_{1i} \Delta x_{2i} \cdots \Delta x_{mi} \) of tested quantities \( x_1, x_2, \ldots, x_m \), that exist in \( f(x_1, x_2, \ldots, x_m) = 0 \). This function corresponds to equation of a surface for which we apply the LDVM criterion. Accordingly, we have that for LDVM all measurement quantities \( x_j, j = 1, m \) have equal treatments during the results generation. The accuracy of LDVM is greater in respect to LSMs accuracy, what implies that the filtering process is more efficient [4].

2 Least Dispersion Volume Method (LDVM)

Potential function \( V = V(x, y, z) \) in space point \( T(x, y, z) \) appears as a consequence of electric charges \( q_1 \) and \( q_2 \), located in points \( T_1(x_1, 0, 0) \) and \( T_2(x_2, 0, 0) \), respectively (see Fig.1). The function \( V = V(x, y, z) \) fulfill the condition [8]

\[
V = \frac{k_1}{r_1} + \frac{k_2}{r_2}
\]
where \( k_1 \) and \( k_2 \) are constants, determined by values \( q_1 \) and \( q_2 \), and environmental dielectric constant \( \varepsilon \), \( r_1 \) and \( r_2 \) are absolute values of corresponding vectors \( \vec{r}_1 \) and \( \vec{r}_2 \),

\[
\begin{align*}
k_1 &= \frac{q_1}{4\pi\varepsilon}, \\
k_2 &= \frac{q_2}{4\pi\varepsilon}, \\
r_1 &= r_1(x,y,z) = \sqrt{(x-x_{p1})^2 + y^2 + z^2}, \\
r_2 &= r_2(x,y,z) = \sqrt{(x-x_{p2})^2 + y^2 + z^2}.
\end{align*}
\]

According to the potential \( V_i \) and a distances \( r_{1i}, r_{2i}, i = 1, n \) which corresponds to the measured results, by using LDVM, it is possible to determine \( q_1 \) and \( q_2 \).

Criterion LDVM has a form

\[
\sum_{i=1}^{n} \Delta V_i \Delta r_{1i} \Delta r_{2i} = \min,
\]

where \( \Delta V_i, \Delta r_{1i}, \) and \( \Delta r_{2i}, i = 1, n \) are corresponding errors in a measurement process, of quantities \( V, r_1 \) and \( r_2 \), respectively. If the potential function \( V(r_1, r_2) \) has a first derivative in respect to distances \( r_1 \) and \( r_2 \), then the transformation criterion defined by eq. (3) can be realized. According to equation (1) we have

\[
\frac{dV}{dr_1} = -\frac{k_1}{r_1^2}, \quad \frac{dV}{dr_2} = -\frac{k_2}{r_2^2}.
\]

By using approximation of differentials that exist in eq. (4), with corresponding increments, we obtain

\[
\Delta r_{1i} = -\frac{\Delta V_i}{k_1 r_{1i}^2}, \quad \Delta r_{2i} = -\frac{\Delta V_i}{k_2 r_{2i}^2}, \quad i = 1, n
\]

where \( i \) points to the number of measured results. In this way, the potential variation \( \Delta V_i \) can be written as

\[
\Delta V_i = \frac{k_1}{r_{1i}} + \frac{k_2}{r_{2i}} - V_i,
\]

where \( V_i \) corresponds to a potential value of the \( i \)-th measurement.

By substituting eqs. (5) and (6) into (3) the criterion LDVM obtains a form

\[
\sum_{i=1}^{n} \frac{r_{1i}^2 r_{2i}^2}{k_1 k_2} (\Delta V_i)^3 = \sum_{i=1}^{n} \frac{r_{1i}^2 r_{2i}^2}{k_1 k_2} \left( \frac{k_1}{r_{1i}} + \frac{k_1}{r_{1i}^2} - V_i \right)^3 = \min.
\]
If the first derivatives of eq (7) are equated to zero, for parameters $k_1$ and $k_2$, from eq (1), we obtain

$$\sum_{i=1}^{n} r_{1i} r_{2i}^2 \left( \frac{k_1}{r_{1i}} + \frac{k_1}{r_{1i}} - V_i \right)^2 = 0, \quad (8)$$

$$\sum_{i=1}^{n} r_{1i}^2 r_{2i} \left( \frac{k_1}{r_{1i}} + \frac{k_1}{r_{1i}} - V_i \right)^2 = 0. \quad (9)$$

After short algebraic transformations, the eqs (8) and (9) take the following forms

$$\sum_{i=1}^{n} r_{1i} r_{2i} \left( \frac{k_1^2}{r_{1i}^2} + 2 \frac{k_1 k_2}{r_{1i} r_{2i}} + \frac{k_2^2}{r_{2i}^2} - 2k_2 \frac{V_i}{r_{2i}} + V_i^2 - 2k_1 \frac{V_i}{r_{1i}} \right) = 0, \quad (10)$$

$$\sum_{i=1}^{n} r_{1i}^2 r_{2i} \left( \frac{k_1^2}{r_{1i}^2} + 2 \frac{k_1 k_2}{r_{1i} r_{2i}} + \frac{k_2^2}{r_{2i}^2} - 2k_2 \frac{V_i}{r_{2i}} + V_i^2 - 2k_1 \frac{V_i}{r_{1i}} \right) = 0, \quad (11)$$

or

$$S_1 k_1^2 + S_2 k_1 k_2 + S_3 k_2^2 + S_4 k_2 + S_5 + S_6 k_1 = 0, \quad (12)$$

$$S_7 k_1^2 + S_8 k_1 k_2 + S_9 k_2^2 + S_{10} k_2 + S_{11} + S_{12} k_1 = 0, \quad (13)$$
in which the sums are determined from

\[ S_1 = \sum_{i=1}^{12} \frac{r_{12}^2}{r_{1i}^2}, \quad S_2 = 2 \sum_{i=1}^{12} r_{2i}, \quad S_3 = \sum_{i=1}^{12} r_{1i}, \]
\[ S_4 = -2 \sum_{i=1}^{12} r_{1i}r_{2i}V_i, \quad S_5 = \sum_{i=1}^{12} r_{1i}r_{2i}^2 V_i^2, \quad S_6 = -2 \sum_{i=1}^{12} r_{2i}^2 V_i, \]
\[ S_7 = \sum_{i=1}^{12} r_{2i}, \quad S_8 = 2 \sum_{i=1}^{12} r_{1i} = 2S_3, \quad S_9 = \sum_{i=1}^{12} \frac{r_{1i}^2}{r_{2i}}, \]
\[ S_{10} = -2 \sum_{i=1}^{12} r_{1i}^2 V_i, \quad S_{11} = \sum_{i=1}^{12} \frac{r_{1i}^2 r_{2i} V_i^2}{r_{2i}}, \quad S_{12} = -2 \sum_{i=1}^{12} r_{1i} r_{2i} V_i = S_4. \]

By implementing a cosine theorem to the corresponding triangles that vectors \( \vec{r}_{12}, \vec{r}_1, \vec{r}_2 \) and \( \vec{r} \) close (see Fig.1), we obtain the following relations

\[ r_{12}^2 = r_{12}^2 - 2r_1 r_2 \cos \theta_1 + r_1^2, \]  
(15)
\[ r_1^2 = r_{12}^2 + 2r_1 r_2 \cos \theta_2 + r_2^2, \]  
(16)
\[ r_2^2 = r_{02}^2 - 2r_{02} r \cos \theta + r^2, \]  
(17)
\[ r^2 = r_{02}^2 + 2r_{02} r \cos \theta_2 + r_2^2. \]  
(18)

From here we determine now

\[ \cos \theta_1 = \frac{r_1^2 + r_{12}^2 - r_2^2}{2r_1 r_{12}}, \]  
(19)
\[ \cos \theta_2 = \frac{r_1^2 - r_{12}^2 - r_2^2}{2r_1 r_{12}}, \]  
(20)
\[ \cos \theta = \frac{r^2 + r_{02}^2 - r_2^2}{2r_{02} r}, \]  
(21)
\[ r = \sqrt{r_{02}^2 + 2r_{02} r \cos \theta_2 + r_2^2}. \]  
(22)

Let the punctual charges and from Fig.1 located in a air-space, in corresponding points

\[ P_1(x_{p1}, y_{p1}, z_{p1}) = P_1(-2[m], 0, 0), \]
\[ P_2(x_{p2}, y_{p2}, z_{p2}) = P_2(2[m], 0, 0) \]

and the values of charges are

\[ q_1 = -10 \text{ nC}, \]  
(23)
\[ q_2 = 20 \text{ nC}, \]  
(24)
then in accordance to eq. (2) we have

\[
k_1 = \frac{q_1}{4\pi\varepsilon_0} = 9 \times 10^9 q_1 = -90 \text{ Nm}^2/\text{C}^2
\]

\[
k_2 = \frac{q_2}{4\pi\varepsilon_0} = 9 \times 10^9 q_2 = 180 \text{ Nm}^2/\text{C}^2
\]

(25)

\[
r_1 = \sqrt{(x + 2)^2 + y^2 + z^2}
\]

(26)

By substituting eq (23) into (1) we obtain

\[
V(r_1, r_2) = -\frac{90}{r_1} + \frac{180}{r_2}.
\]

(27)

If \(r_{1\min}\) and \(r_{2\min}\) correspond to absolute values of radius vectors \(r_1\) and \(r_2\) (see Fig.1), then according to eqs (23) and (26) we have

\[
r_{1\min} + r_{2\min} = x_2 - x_1 = 2 - (-2) = 4[m]
\]

(28)

Let the identification of equipotential surface, which pass through the top of a vector \(\hat{r}_{1\min}\) is for interest for us. Assume now that the absolute value of \(r_{1\min} = 3[m]\). Then according to condition (28) we obtain \(r_{2\min} = 1[m]\). By substituting the last results in eq (27) we determine \(V_s\) at surface \(S\), as

\[
V_s = V(r_{1\min}, r_{2\min}) = \frac{90}{3} + \frac{180}{1} = 150 \text{ V}
\]

(29)

From results (29) and eq (27) the equipotential surface has a form

\[
-\frac{90}{r_1} + \frac{180}{r_2} = 150.
\]

(30)

In eq. (30) \(r_1\) and \(r_2\) are defined by eq. (26).

The measuring point \(T(x, y, z)\), represents a space location in which the sensor of electrostatic measuring equipment is located. For a given \(V_s = 150 \text{ V}\) the following conditions have to be fulfilled:

1. equipotential surface determined by eq. (30), and
2. triangle inequality, which according to Fig. 1, has a form

\[
r_1 \leq r_{12} + r_2
\]

(31)

The simulation of the measured results \(V_{si}, r_{1i}\) and \(r_{2i}, i = 1, \ldots, n\) is performed according to eqs. (30), (31), and (19) up to (22). The simulated and calculated results are given in Table 1.
Table 1. Simulated and calculated results.

<table>
<thead>
<tr>
<th>(i)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_{1i}), m</td>
<td>3.00</td>
<td>3.18</td>
<td>3.32</td>
<td>3.54</td>
<td>3.68</td>
<td>3.86</td>
<td>3.96</td>
<td>4.18</td>
<td>4.35</td>
<td>4.47</td>
<td>4.68</td>
<td>4.85</td>
</tr>
<tr>
<td>(r_{2i}), m</td>
<td>1.00</td>
<td>1.01</td>
<td>1.02</td>
<td>1.03</td>
<td>1.04</td>
<td>1.04</td>
<td>1.05</td>
<td>1.05</td>
<td>1.06</td>
<td>1.06</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td>(\theta_{1i}), (^\circ)</td>
<td>180.0</td>
<td>148.8</td>
<td>137.5</td>
<td>123.3</td>
<td>115.3</td>
<td>105.2</td>
<td>92.53</td>
<td>77.49</td>
<td>70.31</td>
<td>55.70</td>
<td>41.84</td>
<td>8.46</td>
</tr>
<tr>
<td>(\theta_{2i}), (^\circ)</td>
<td>0.00</td>
<td>9.48</td>
<td>11.98</td>
<td>14.07</td>
<td>14.66</td>
<td>15.07</td>
<td>15.00</td>
<td>14.53</td>
<td>13.63</td>
<td>12.90</td>
<td>10.78</td>
<td>8.46</td>
</tr>
<tr>
<td>(\theta_{3i}), (^\circ)</td>
<td>0.00</td>
<td>24.70</td>
<td>28.97</td>
<td>30.96</td>
<td>30.83</td>
<td>30.14</td>
<td>28.04</td>
<td>27.13</td>
<td>24.75</td>
<td>22.94</td>
<td>18.66</td>
<td>14.19</td>
</tr>
<tr>
<td>(r_i), m</td>
<td>1.00</td>
<td>1.25</td>
<td>1.43</td>
<td>1.67</td>
<td>1.82</td>
<td>2.00</td>
<td>2.21</td>
<td>2.30</td>
<td>2.45</td>
<td>2.56</td>
<td>2.74</td>
<td>2.89</td>
</tr>
<tr>
<td>(V_i), V</td>
<td>150.0</td>
<td>149.2</td>
<td>149.9</td>
<td>149.0</td>
<td>150.3</td>
<td>149.3</td>
<td>150.4</td>
<td>149.5</td>
<td>150.4</td>
<td>149.8</td>
<td>150.6</td>
<td>149.7</td>
</tr>
</tbody>
</table>

According to Table 1 and eqs. (14) the corresponded sums,

\[
\begin{align*}
S_1 &= 3.35353, & S_2 &= 24.92, & S_3 &= 47.04, \\
S_4 &= -14690.8, & S_5 &= 1.4674 \times 10^6, & S_6 &= -3878.8, \\
S_7 &= 12.46, & S_8 &= 99.14, & S_9 &= 180.696, \\
S_{10} &= -56535.4, & S_{11} &= 4.42381 \times 10^6, & S_{12} &= -14690.8.
\end{align*}
\]  

are determining.

By substituting eq. (32) into eqs. (12) and (13), we obtain complex results for parameters \(k_1\) and \(k_2\),

\[
\begin{align*}
\hat{k}_1 &= (-91.2952 \pm j9.21492) \vee (-90.4989 \pm j3.17311), \\
\hat{k}_2 &= (180.29 \pm j2.38709) \vee (180.003 \pm j0.547151)
\end{align*}
\]  

Their absolute values are

\[
\begin{align*}
\text{abs}[\hat{k}_1] &= 91.759 \vee 90.5545, \\
\text{abs}[\hat{k}_2] &= 180.306 \vee 180.004.
\end{align*}
\]  

In eq (34) we have four results for parameters \(k_1\) and \(k_2\), i.e. four pairs of the values, defined by vectors

\[
\{k_1, k_2\} = \{-91.2952, 190.290\} \vee \\
\{-90.4989, 180.003\} \vee \\
\{-91.9590, 180.306\} \vee \\
\{-90.5545, 180.004\}
\]  

The question which arises now is: How to determine a correct solution? An answer to this question is obtained by determining the minimum according to eq. (7), and by calculating the average values of the identification errors \((\Delta V)_{av}\), \((\Delta r_1)_{av}\) and \((\Delta r_2)_{av}\).
Lj. Golubović:

The minimal value is obtained for the fourth solution in eq. (35), i.e. for

\[ \{ k_1, k_2 \} = \{-90.5545, 180.004\} \] (36)

According to the eq (5) up to (7) we obtain the following vector errors,

\[ \{ \Delta V_i, \Delta r_{1i}, \Delta r_{2i} \} = \frac{r_{1i}^2 - r_{2i}^2}{90.5545 \times 180.004} ( - \frac{90.5545}{r_{1i}} + \frac{180.004}{r_{2i}} )^3 \]

\[ = \{-0.000099937, 0.000345137, -0.000493239, 0.000147837, 0.000317825, -0.000275349, -0.000326421\}, \] (37)

\[ \{ \Delta V_i \} = \frac{90.555}{r_{1i}} + \frac{180.004}{r_{2i}} = \{-0.181, 0.54537, -0.70109, 0.18066, -0.14617, 0.32093, -0.18666, 0.21514, 0.21514, -0.24330, 0.13426, -0.14310\}, \] (38)

\[ \{ \Delta r_{1i} \} = -\frac{\Delta V_i}{k_1} = \{-0.01408, 0.06654, -0.07834, 0.03450, -0.01041, 0.06651, -0.01706, 0.07021, 0.06633, -0.03068, -0.00546, -0.00737\}, \] (39)

\[ \{ \Delta r_{2i} \} = -\frac{\Delta V_i}{k_2} = \{0.00079, -0.00338, 0.00372, -0.00147, 0.00041, -0.00243, 0.00059, -0.00223, -0.00194, 0.00087, 0.00014, 0.00018\}. \] (40)

The values of vectors \( r_{1i}, r_{2i} \) and \( V_i \) are given in Table 1. The average values of vector-errors (37)-(40) are

\[ (\Delta V \Delta r_1 \Delta r_2)_{av} = \sqrt{\frac{1}{12} \sum_{i=1}^{12} (\Delta V_i \Delta r_{1i} \Delta r_{2i})^2} = \pm 0.000279116 \] (41)

\[ (\Delta V)_{av} = \sqrt{\frac{1}{12} \sum_{i=1}^{12} (\Delta V_i)^2} = \pm 0.320108 \] (42)

\[ (\Delta r_1)_{av} = \sqrt{\frac{1}{12} \sum_{i=1}^{12} (\Delta r_i)^2} = \pm 0.04755 \] (43)

\[ (\Delta r_2)_{av} = \sqrt{\frac{1}{12} \sum_{i=1}^{12} (\Delta r_i)^2} = \pm 0.00191 \] (44)
A parametric form for equation of equipotential surface from Fig. 1 can be written in the following form.

\[ x = r(\theta) \cos \theta \quad (45) \]
\[ y = r(\theta) \sin \theta \cos \varphi \quad (46) \]
\[ y = r(\theta) \sin \theta \sin \varphi \quad (47) \]

or

\[ x = r \cos \theta(r) \quad (48) \]
\[ y = r \sin \theta(r) \cos \varphi \quad (49) \]
\[ y = r \sin \theta(r) \sin \varphi \quad (50) \]

where \( r(\theta) \) and \( \theta(r) \) are the corresponding functional dependences of radius vector \( r \) and the angle \( \theta \), respectively. The equipotential surfaces are rotational symmetric with respect to the axes and the measurement points \( T(x_i, y_i, z_i), i = 1, n \) (see Fig. 1) positioned at the corresponding circles, that have centers located along the axes \( Ox \). Locations of the measurement points are determined by coordinate \( \varphi \). When the parametric eqs. (45) up to (47) are given in a spherical coordinate system, where \( r = r(\theta) \) is not constant, we can derive the following conclusion: The obtained equipotential surface has a shape of degenerate sphere.

Coordinates \( r_i, \theta_i, i = 1, n \) of the experimental points from Table 1 in surface \( r(\theta) \) define the diagram \( \theta = \theta(r) \), which can be approximated by a power-two polynomial, i.e.

\[ \theta(r) = a_1 r^2 + a_2 r + a_2 \quad (51) \]

where parameters \( a_1, a_2 \) and \( a_3 \) can be determined during the measurement process, by using some of the methods for curve identification in a plane [2], [3], [4], [6], [7], [8].

The criterion LSM for a function defined by eq. (51), using the measurement results from Table 1, can be written in a form

\[ \sum_{i=1}^{12} (a_1 r_i^2 + a_2 r_i + a_3 - \theta)^2 = \min \quad (52) \]
If we equate first derivatives of eq. (52) with zero on obtain

\[ \sum_{i=1}^{12} (a_1 r_i^2 + a_2 r_i + a_3 - \theta) r_i^2 = 0 \]  
(53)

\[ \sum_{i=1}^{12} (a_1 r_i^2 + a_2 r_i + a_3 - \theta) r_i = 0 \]  
(54)

\[ \sum_{i=1}^{12} (a_1 r_i^2 + a_2 r_i + a_3 - \theta) = 0 \]  
(55)

After short algebraic transformations, the eqs. (53) up to (55) take forms

\[ S_{13}a_1 + S_{14}a_2 + s_{15}a_3 = S_{16} \]  
(56)

\[ S_{14}a_1 + S_{15}a_2 + s_{17}a_3 = S_{18} \]  
(57)

\[ S_{15}a_1 + S_{17}a_2 + 12a_3 = S_{19} \]  
(58)

where the sums $S_i, i = \overline{1,n}$ calculated by using Table 1, have the values

\[ S_{13} = \sum_{i=1}^{12} r_i^4 = 299.13, \quad S_{14} = \sum_{i=1}^{12} r_i^3 = 123.716, \quad S_{15} = \sum_{i=1}^{12} r_i^2 = 53.2986, \]

\[ S_{16} = \sum_{i=1}^{12} \theta_i r_i^2 = 1244.84, \quad S_{17} = \sum_{i=1}^{12} r_i = 24.32, \quad S_{18} = \sum_{i=1}^{12} \theta_i r_i = 576.265, \]  
(59)

\[ S_{19} = \sum_{i=1}^{12} \theta_i = 281.31 \]

By solving system of eqs. (56) up to (58) on obtain

\[ a_1 = -25.2198, \quad a_2 = 100.255, \quad a_3 = -67.7263. \]  
(60)

The angle $\theta$ given in angles degrees, from the radial distance $r$ in meters, according to eqs. (51) and (60) is derived

\[ \theta[^\circ] = -25.2198r^2[m] + 100.255r[m] - 67.7263. \]  
(61)

If the angle $\theta$ is defined in radians, then eq. (61) takes a form

\[ \theta[\text{rad}] = -440168r^2[m] + 1.7497798r[m] - 1.1820469. \]  
(62)

The graphic which corresponds to eq. (61) is sketched in Fig.2.
By substituting the eq. (62) into eqs. (48) up to (50) we have

\[
x = r \cos\left[-0.440168r^2 + 1.7497798r - 1.1820469\right], \ r \in (1, 3) \tag{63}
\]

\[
y = r \sin\left[-0.440168r^2 + 1.7497798r - 1.1820469\right] \cos \phi, \ r \in (1, 3), \ \phi \in (0, 2\pi) \tag{64}
\]

\[
z = r \sin\left[-0.440168r^2 + 1.7497798r - 1.1820469\right] \sin \phi, \ r \in (1, 3), \ \phi \in (0, 2\pi) \tag{65}
\]

By applying the corresponded command \textit{Parametric Plot3D} [9] to eqs. (63) up to (65), on obtain 3D graphic pictured in Fig.3.
3 Conclusion

The proposed LDVM method has advantages in respect to LSM. Namely, it equally treats the measured results of all interdependent quantities and gives a lower identification error for the considered function. For large number of cases by using the identification procedure on obtain a nonlinear system of equations that can be used for determination of the identified parameters, i.e. corresponding constants in the frame of some identification functions. By using computer and appropriate software packets the above mentioned problem can be efficiently solved. An optimal solution is obtained by a suitable choice of solutions from multidimensional vector solution. This choice is performed taking into account the criterion of average error. By combining numerical and the measurement numerical method high-accuracy and good agreement between practical and theoretical results is obtained.

References