INVENTORY MODEL WITH CASH FLOW ORIENTED AND TIME-DEPENDENT HOLDING COST UNDER PERMISSIBLE DELAY IN PAYMENTS

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Abstract: This study develops an inventory model for determining an optimal ordering policy for non-deteriorating items and time-dependent holding cost with delayed payments permitted by the supplier under inflation and time-discounting. The discounted cash flows approach is applied to study the problem analysis. Mathematical models have been derived under two different situations i.e. case I: The permissible delay period is less than cycle time for settling the account, and case II: The permissible delay period is greater than or equal to cycle time for settling the account. An algorithm is used to obtain minimum total present value of the costs over the time horizon H. Finally, numerical example and sensitivity analysis demonstrate the applicability of the proposed model. The main purpose of this paper is to investigate the optimal cycle time and optimal payment time for an item so that annual total relevant cost is minimized.

Keywords: Inventory, time-dependent, cash flow, delay in payments.

MSC: 90B05.

1. INTRODUCTION

In traditional economical ordering quantity (EOQ) model, it is assumed that retailer must pay for the items as soon as the items are received. However, in practice, the supplier may offer the retailer a delay period in paying for the amount of purchasing cost. To motivate faster payment, stimulate more sales or reduce credit expenses, the supplier also often provides its customers a cash discount. The permissible delay is an important source of financing for intermediate purchasers of goods and services. The permissible delay in payments reduces the buyer’s cost of holding stock, because it reduces the
amount of capital invested in stock for the duration of the permissible period. Thus, it is a marketing strategy for the supplier to attract new customers who consider it to be a type of price reduction. Most of the classical inventory models did not take into account the effects of inflation and time value of money. But during the last three decades, the economic situation of most of the countries has changed to such an extent due to large scale inflation and consequent sharp decline in the purchasing power of money, that it has not been possible to ignore the effects of inflation and time value of money any further. In supermarkets, it has been observed that the demand rate may go up and down if the on-hand inventory level increases or decreases. This type of situation generally arises for consumer goods type of inventory.

The economic order quantity (EOQ) model is widely used by practitioners as a decision making tool for the control of inventory. In general, the objective of inventory management deals with minimization of the inventory carrying cost. Therefore it is important to determine the optimal stock and optimal time of replenishment of inventory to meet the future demand. An inventory model with stock at the beginning and shortages allowed, but then partially backlogged was developed by Lin et al. [15]. Urban [23] developed an inventory model that incorporated financing agreements with both suppliers and customers using boundary condition. Yadav et al. [26] established an inventory model of deteriorating items with two warehouse and stock dependent demand. Wu et al. [25] applied the Newton method to locate the optimal replenishment policy for EPQ model with present value. Roy and Chaudhuri [18] established an EPLS model with a variable production rate and demand depending on price. Huang [11] developed an EOQ model to compare the interior local minimum and the boundary local minimum.

Various models have been proposed for inflation dependent inventory models. Buzacott [5] was first who developed EOQ model taking inflation into account. In the same year Misra [17] also developed EOQ model incorporating inflationary effects. Both models assume a uniform inflation rate for all the associated costs, and minimize the average annual cost to obtain expression for the EOQ. Hou and Lin [9] developed a cash flow oriented EOQ model with deteriorating items under permissible delay in payments. In this paper Hou and Lin [9] obtained optimal (minimum) total present value of costs. The model of Hou and Lin [9] was extended by Tripathi etc [22] by taking time-dependent demand rate for non-deteriorating items. Tripathi and Kumar [20] discussed EOQ model credit financing in economic ordering policies of time-dependent deteriorating items. Aggarwal et al. [2] developed a model on integrated inventory system with the effect of inflation and credit period. In this model, the demand rate is assumed to be a function of inflation. Tripathi and Misra [17] developed EOQ model credit financing in economic ordering policies of non-deteriorating items with time-dependent demand rate in the presence of trade credit using a discounted cash-flow (DCF) approach. Jaggi et al. [14] developed a model retailer’s optimal replenishment decision with credit-linked demand under permissible delay in payments. This paper incorporates the concepts of credit linked demand and developed a new inventory model under two levels of trade credit policy to reflect the real-life situation. An EOQ model under conditionally permissible delay in payments was developed by Huang [12] and obtained the retailer’s optimal replenishment policy under permissible delay in payments. Optimal retailer’s ordering policies in the EOQ model for deteriorating items under trade credit financing in supply chain were developed by Mahata and Mahata [16]. In this paper, the authors obtained a unique optimal cycle time to minimize the total variable
cost per unit time. Hou and Lin [10] considered an ordering policy with a cost minimizing procedure for deteriorating items under trade credit and time-discounting. Several other researchers have extended their approach to various interesting situations by considering the time-value of money, different inflation rates for the internal and external costs, finite replenishment rate, shortage etc. The models of Van Hees and Monhemius [24], Aggarwal [1], Bierman and Thomas [3], Sarker and Pan [19] etc. are worth mentioning in this direction. Brahmbhatt [4] developed an EOQ model under a variable inflation rate and marked-up prices. Gupta and Vart [8] developed a multi-item inventory model for a resource constant system under variable inflation rate. Chung [7] developed a model inventory control and trade credit revisited. Jaggi and Aggarwal [13] developed a model credit financing in economic ordering policies of deteriorating items by using discounted cash-flows (DCF) approach. Chen and Kang [6] discussed integrated vendor-buyer cooperative inventory models with variant permissible delay in payments.

For generality, this study develops an inventory model for non-deteriorating items under permissible delay in payments in which holding cost is a function of time. The discounted cash flow approach is also consider to build-up the model. We then establish algorithm to find the optimal order cycle, optimal order quantity, optimal total present value of the cost over the time-horizon H. Also, we provide numerical example and sensitivity analysis as illustrations of the theoretical results.

The rest of this paper is organized as follows. In section 2, we describe the notation and assumptions used throughout this study. In section 3, the model is mathematically formulated. In section 4, an algorithm is given for finding optimal solution. Numerical example is provided in section 5, followed by sensitivity analysis in section 6 to illustrate the features of the theoretical results. Finally, we draw the conclusions and the idea of future research in the last section 7.

2. NOTATIONS AND ASSUMPTIONS

The following notations are used throughout the manuscript:

- $H$: Length of planning horizon
- $n$: Number of replenishment during the planning horizon, $n = H/T$
- $T$: Replenishment cycle time
- $D$: Demand rate per unit time, units/unit time
- $Q$: Order quantity, units/cycle
- $s$: Ordering cost at time zero, $$/order
- $c$: Per unit cost of the item, $$/unit
- $h$: Holding cost per unit per unit time excluding interest charges, $$/unit/unit time
- $r$: Discount rate
- $f$: Inflation rate
- $k$: The net discount rate of inflation ($k = r - f$)
- $I_e$: The interest earned per dollar per unit time
- $I_i$: The interest charged per dollar in stocks per unit time by the supplier $I_i > I_e$
- $m$: The permissible delay in settling account
- $Z_1(n)$: The total present value of the costs over the time horizon $H$, for $m < T = H/n$
\( Z_2(n) \) : The total present value of the costs for \( m \geq T = H/n \)

\( E \) : The interest earned during the first replenishment cycle

\( E_1 \) : The present value of the total interest earned over the time horizon \( H \)

\( I(t) \) : The inventory level at time \( t \)

\( I_p \) : The total interest payable over the time horizon \( H \)

\( E_2 \) : The present value of the interest earned over the time horizon \( H \)

\( E_3 \) : The present value of the total interest earned over the time horizon \( H \)

In addition, the following assumptions are being made:

1. The demand rate \( D \) is constant and downward sloping function.
2. Shortages are not allowed.
3. Lead time is zero.
4. The net discount rate of inflation is constant.
5. The holding cost \( h \) is time-dependent i.e. \( h = h(t) = a + bt, a > 0, \) and \( 0 < b > 1. \)

### 3. MATHEMATICAL FORMULATION

The inventory level \( I(t) \) at any time \( t \) is depleted by the effect of demand only. Thus the variation of \( I(t) \) with respect to ‘\( t \)’ is governed by the following differential equation:

\[
\frac{dI(t)}{dt} = -D, \ 0 \leq t \leq T = H/n
\]

The present value of the total replenishment costs is given by:

\[
C_1 = s \sum_{i=0}^{n-1} e^{-ikT} = s \left( \frac{1 - e^{-kH}}{1 - e^{-kT}} \right), \ 0 \leq t \leq T = H/n
\]

The present value of the total purchasing costs is given by

\[
C_2 = e \sum_{t=0}^{\infty} e^{-t} = eDT \left( \frac{1 - e^{-t}}{1 - e^{-tH}} \right), \ 0 \leq t \leq T = H/n
\]

The present value of the total holding costs over the time horizon \( H \) is given by

\[
A = \sum_{i=0}^{\infty} e^{-at} \int_0^H h(t)I(t)e^{-at} \, dt
\]

\[
= \frac{D}{k} \left\{ aT + \frac{(bT-a) + (a+bT)e^{-at} + 2b}{kT} \left( e^{-at} - 1 \right) \right\} \left( 1 - e^{-at} \right)
\]
Case I. \(m < T = H/n\)

The present value of the interest payable during the first replenishment cycle is

\[
i_p = cI \int_0^T t e^{-r t} dt = cI \frac{D}{k^2} \left( \frac{(T-m) e^{-rn} + e^{-rT} - e^{-rn}}{k} \right)
\]

Thus, the present value of the total interest payable over the time horizon \(H\) is

\[
I_p = \sum_{i=0}^{m-1} i e^{-rT} = cI \frac{D}{k^2} \left\{ k(T-m) e^{-rn} + e^{-rT} - e^{-rn} \right\} \left( \frac{1-e^{-rn}}{1-e^{-rT}} \right), \quad T = H/n
\]

The present value of the interest earned during the first replenishment cycle is

\[
E = cI \int_0^T t e^{-r t} dt = cI \frac{D}{k^2} \left( 1-e^{-rT} - kT e^{-rT} \right), \quad T = H/n
\]

Therefore, the present value of the interest earned over the time horizon \(H\) is

\[
E_1 = \sum_{i=0}^{m-1} E e^{-rT} = cI \frac{D}{k^2} \left( 1-e^{-rT} - kT e^{-rT} \right) \left( \frac{1-e^{-rn}}{1-e^{-rT}} \right), \quad T = H/n
\]

Thus, the total present value of the costs over the time horizon \(H\) is

\[
Z_1(n) = C_1 + C_2 + A + I_p - E_1
\]

Case II. \(m \geq T = H/n\)

In this case, the interest earned in the first cycle is the interest during the time period \((0, H/n)\) plus the interest earned from the cash invested during the time period \((T, m)\) after the inventory is exhausted at time \(T\) and it is given by

\[
E_2 = cI \left[ \int_0^T t e^{-r t} dt + (m-T) e^{-rT} \int_0^T D dt \right] = cI \frac{1-e^{-rT}}{k^2} - \frac{T e^{-rT}}{k} + (m-T) T e^{-rT}
\]

and the present value of the total interest earned over the time horizon \(H\) is

\[
E_2 = \sum_{i=0}^{m-1} E e^{-rT} = cI \frac{1-e^{-rn}}{k^2} - \frac{T e^{-rT}}{k} + (m-T) T e^{-rT}
\]

Therefore, the total present value of the costs is given by
\[ Z_2(n) = C_1 + C_2 + A - E_3 \]  

(12)

From equations (9) and (12), it is difficult to obtain the optimal solution in explicit form. Therefore, the model will be solved approximately by using a truncated Taylor’s series for the exponential terms i.e.

\[ e^{-kT} \approx 1 - kT + \frac{k^2 T^2}{2}, e^{-km} \approx 1 - km + \frac{k^2 m^2}{2} \text{ etc.} \]  

(13)

This is a valid approximation for smaller values of KT and km etc.

With the above approximation, the present value of the cost over the time horizon \( H \) is

\[ Z_1(n) \approx \left[ \frac{s}{T} + cD + \frac{DT(a + bT)}{2} + cD(T - m)(T - m + km) - cDI_{1}(1 - kT) \right] \left[ 1 + \frac{k^2 T^2}{2} \right] (1 - e^{-\omega}) \]  

(14)

and

\[ Z_2(n) = \frac{1}{k} \left[ \frac{s}{T} + cD + \frac{DT(a + bT)}{2} - cD \left( m \left( \frac{1}{2} + mk \right) T + \frac{k}{2} (1 + mk) T + \frac{k^2 T^2}{2} \right) \right] \left[ 1 + \frac{k^2 T^2}{4} \right] (1 - e^{-\omega}) \]  

(15)

Note that the purpose of this approximation is to obtain the unique closed form value for the optimal solution. By taking first and second order derivatives of \( Z_1(n) \) and \( Z_2(n) \) with respect to ‘n’, we obtain

\[ \frac{\partial Z_1(n)}{\partial n} = \left[ s \left( \frac{1}{k} H \right) - \frac{k^2 H}{4n^2} \right] - cD \left( \frac{1}{2n^2} \right) \left( \frac{1}{n} \right) - cD \left( \frac{1}{2n^2} \right) \left( \frac{2b + ak}{2} \right) \left( \frac{H}{n} \right) + \frac{b^2 H^2}{n} \left( \frac{H}{n} \right) \]  

(16)

\[ - cD \left( \frac{1}{2n^2} \right) \left( 1 - \frac{k^2 T^2}{4n^2} \right) \left( 1 - e^{-\omega} \right) \]

\[ \frac{\partial Z_2(n)}{\partial n} = \left[ s \left( \frac{1}{k} H \right) - \frac{k^2 H}{4n^2} \right] - cD \left( \frac{1}{2n^2} \right) \left( \frac{1}{n} \right) - cD \left( \frac{1}{2n^2} \right) \left( \frac{2b + ak}{2} \right) \left( \frac{H}{n} \right) + \frac{b^2 H^2}{n} \left( \frac{H}{n} \right) \]  

(17)

and
\[
\frac{\partial^2 Z_1(n)}{\partial n^2} = \frac{H}{n^2} \left[ \frac{ks}{2} + CD \left( 1 + \frac{3kH}{n} \right) + \frac{D}{2k} \left( 2a + 6 \left( b + \frac{ak}{2} \right) \left( 1 + \frac{kH}{n} \right) + H + \frac{5bk^2H^3}{n^3} \right) \right] + cDH \left[ \frac{2}{2kn} \left( 2 + \frac{3kH}{n} + \frac{3k^2H^2}{n^2} + \frac{5k^3H^3}{n^3} \right) \right] + cIDH \left[ \frac{2}{2kn} \left( 1 - mk + \frac{3m^2k^2}{4} - \frac{m^3k^3}{4} \right) + \frac{3kH}{n} \left( 1 - mk + \frac{m^2k^2}{4} + \frac{k^2H^2}{n^2} \right) \right] \left( 1 - e^{-\alpha m} \right) > 0
\]

(18)

\[
\frac{\partial^2 Z_2(n)}{\partial n^2} = \frac{H}{n^2} \left[ \frac{ks}{2} + cD \left( 1 + \frac{3kH}{2n} \right) + \frac{D}{2k} \left( 2a + 6 \left( b + \frac{ak}{2} \right) \left( 1 + \frac{kH}{n} \right) + H + 5bk^2H^3 \right) \right] + cDH \left[ \frac{1}{2k} \left( 1 + mk \right) \left( 2 + \frac{3kH}{n} + \frac{9k^2H^2}{n^2} + \frac{5k^3H^3}{n^3} + \frac{15k^4H^4}{2n^4} \right) \right] \left( 1 - e^{-\alpha m} \right) > 0
\]

(19)

Since \( \frac{\partial^2 Z_1(n)}{\partial n^2} > 0 \) and \( \frac{\partial^2 Z_2(n)}{\partial n^2} > 0 \), for fixed \( H, Z_1(n) \) and \( Z_2(n) \) are strictly convex functions of \( n \). Thus, there exists a unique value of \( n^* \) which minimize \( Z_1(n) \) and \( Z_2(n) \). If we draw a curve between \( Z(n) \) and \( n^* \), the curve is convex.

At \( m = T = H/n \), we find \( Z_1(n) = Z_2(n) \), we have

\[
Z(n) = \begin{cases} 
Z_1(n), & \text{if } T = H/n \geq m \\
Z_2(n), & \text{if } T = H/n \leq m 
\end{cases}
\]

where \( Z_1(n) \) and \( Z_2(n) \) are as expressed in equations (14) and (15), respectively.

Based on the above discussion, the following algorithm is developed to derive the optimal \( n, T, Q \) and \( Z(n) \) values.

4. ALGORITHM

Step 1: Start by choosing positive integer \( n^* \), where \( n \) is equal or greater than one.
Step 2: If \( T = H/n \geq m \), for different \( n^* \), then we determine \( Z_1(n) \) from (14), if \( T = H/n \leq m \), for different \( n^* \), then determine \( Z_2(n) \) from (15).
Step 3: Repeat step 1 and 2 for all possible values of \( n \) with \( T = H/n \geq m \) until the minimum \( Z_1(n) \) is found from (14) and let \( n^*_1 = n \). For all possible values of \( n \) with \( T = H/n \leq m \) until the minimum \( Z_2(n) \) is found from (15) and let \( n^*_2 = n \). The \( n^*_1 \) and \( n^*_2 \), \( Z_1(n^*) \) and \( Z_2(n^*) \) values form the optimal solution.
Step 4: Select the optimal number of replenishment \( n^* \) such that

\[
Z(n^*) = \min \left\{ Z_1(n^*), \text{ if } T = H/n^* \geq m \right\}
\]

\[
Z_2(n^*), \text{ if } T = H/n^* \leq m
\]
Hence the optimal order quantity $Q^*$ is obtained by putting $T^* = H / n^*$

5. NUMERICAL RESULTS

An example is given to illustrate the results of the model developed in this study with the following data: $a = 2.0$ unit, $b = 0.5$ unit/time, $D = 600$ unit/year, $s = $ 80/order, the net discount rate of inflation, $k = $0.12$/year, the interest charged per dollar in stocks per year by the supplier, $I_c = $0.18$/year, the interest earned per $ per year, $I_e = $0.16$/year, $c = $15/unit and the planning horizon, $H = 5$ year. The permissible delay in settling the account, $m = 60$ days = 60/360 years (assume 360 days in a year). Using the solution algorithm procedure, the computational results are shown in Table 1. We find the case is the I optimal option in credit policy. The minimum total present value of costs is obtained when the number of replenishment $n^*$ is 18. With 18 replenishments, the optimal cycle time $T$ is 0.277778 years, the optimal order quantity, $Q = 166.666667$ units, and the optimal total present value of costs, $Z(n) = $ 35597.78 (approximately).

Table 1. The computational results: Variation of the optimal solution for different values of ‘n’

<table>
<thead>
<tr>
<th>Case</th>
<th>Order No. ($n$)</th>
<th>Cycle Time $T$ year</th>
<th>Order Quantity ($Q$) units</th>
<th>Total costs $Z(n)$ (approx.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>10</td>
<td>0.500000</td>
<td>300.000000</td>
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<tr>
<td></td>
<td>11</td>
<td>0.454545</td>
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<td></td>
<td>12</td>
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<tr>
<td></td>
<td>13</td>
<td>0.384615</td>
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<tr>
<td></td>
<td>14</td>
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</tr>
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<td></td>
<td>15</td>
<td>0.333333</td>
<td>200.000000</td>
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</tr>
<tr>
<td></td>
<td>16</td>
<td>0.312500</td>
<td>187.500000</td>
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<tr>
<td></td>
<td>17</td>
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<td>176.470588</td>
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<tr>
<td></td>
<td>18*</td>
<td>0.277778*</td>
<td>166.666667*</td>
<td>35597.78*</td>
</tr>
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<td>0.263158</td>
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<td>36145.33</td>
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<td></td>
<td>50</td>
<td>0.100000</td>
<td>60.000000</td>
<td>36674.21</td>
</tr>
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</table>

* Optimal solution
6. SENSITIVITY ANALYSIS

Taking all the parameters as in the above numerical example, the variation of the optimal solution for different values of net discount rate of inflation $k$ is given in Table 2.

Table 2: Variation of the optimal solution for different values of net discount rate of inflation $k$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$k$</th>
<th>0.12</th>
<th>0.15</th>
<th>0.18</th>
<th>0.21</th>
<th>0.24</th>
<th>0.27</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
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<td>$C_1$</td>
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From Table 2, all the observations can be summed up as follows:

(i) An increase in the net discount rate of inflation $k$ leads to a decrease of total replenishment cost, in total purchasing cost, in total holding cost, in total interest payable, in total interest earned, and also a decrease in total present value of the costs $C_1$, $C_2$, $A$, $I_p$, $E_1$ and $Z(n)$ respectively.

(ii) If the number of replenishment $n$ increases, then there is increase in total replenishment cost $C_1$, but total purchasing cost $C_2$, total holding cost $A$, total interest payable $I_p$, and total interest earned $E_1$ decreases, keeping net discount rate of inflation $k$ constant.
7. CONCLUSION AND FUTURE RESEARCH

This study develops an inventory model for non-deteriorating and time-dependent holding cost items over a finite planning horizon, when the supplier provides a permissible delay in payments. The model considers the effects of inflation and permissible delay in payments. The optimal solution procedure is given to obtain the optimal number of replenishment, cycle time and order quantity to minimize the total present value of costs. Numerical example is given to illustrate the model for case I and case II. The obtained results show that the case I is the optimal (minimum) option in credit policy. The minimum total present value of the costs is obtained when the number of replenishments $n$ is 18. With 18 replenishments, the optimal (minimum) order quantity $Q = 166.666667$ units and the optimal (minimum) total present value of the costs $Z = $ 35597.78 (approximately).

The model proposed in this paper can be extended in several ways. For instance, we may extend the time dependent deterioration rate. We could also consider the demand as a function of quantity as well as a function of inflation. Finally, we could generalize the model with stochastic demand when the supplier provides a permissible delay in payments and cash discount.

REFERENCES


