RETAILER’S OPTIMAL ORDERING POLICIES FOR EOQ MODEL WITH IMPERFECTIVE ITEMS UNDER A TEMPORARY DISCOUNT

Wen Feng LIN
Department of Aviation Service and Management, China University of Science and Technology, Taipei, Taiwan
linwen@cc.cust.edu.tw

Horng Jinh CHANG
Graduate Institute of Management Sciences, Tamkang University, Tamsui, Taiwan
chj@mail.tku.edu.tw

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Abstract: In this article, we study inventory models to determine the optimal special order and maximum saving cost of imperfective items when the supplier offers a temporary discount. The received items are not all perfect and the defectives can be screened out by the end of 100% screening process. Three models are considered according to the special order that occurs at regular replenishment time, non-regular replenishment time, and screening time of economic order quantity cycle. Each model has two sub-cases to be discussed. In temporary discount problems, in general, there are integer operators in objective functions. We suggest theorems to find the closed-form solutions to these kinds of problems. Furthermore, numerical examples and sensitivity analysis are given to illustrate the results of the proposed properties and theorems.

Keywords: Economic Order Quantity, Temporary Discount, Imperfective Items, Inventory.

MSC: 90B05.

1. INTRODUCTION

The economic order quantity (EOQ) model is popular in supply chain management. The traditional EOQ inventory model supposed that the inventory parameters (for example: cost per unit, demand rate, setup cost or holding cost) are constant during sale period. Schwarz [32] discussed the finite horizon EOQ model, in which the costs of the model were static and the optimal ordering number could be found during the finite horizon. In real life, there are many reasons for suppliers to offer a temporary price...
discount to retailers. The retailers may engage in purchasing additional stock at reduced price and sell at regular price later. Lev and Weiss [22] considered the case where the cost parameters may change, and the horizon may be finite as well as infinite. However, the lower and upper bounds they used did not guarantee that the boundary conditions could be met. Tersine and Barman [35] incorporated quantity and freight discounts into the lot size decision in a deterministic EOQ system. Ardalan [3] investigated optimal ordering policies for a temporary change in both price and demand, where demand rate was not constant. Tersine [34] proposed a temporary price discount model, the optimal EOQ policy was obtained by maximizing the difference between regular EOQ cost and special ordering quantity cost during sale period. Martin [26] revealed that Tersine’s [34] representation of average inventory in the total cost was flawed, and suggested the true representation of average inventory. But Martin [26] sacrificed the closed-form solution in solving objective function, and used search methods to find special order quantity and maximum gain. Wee and Yu [38] assumed that the items deteriorated exponentially with time and temporary price discount purchase occurred at the regular and non-regular replenishment time. Sarkar and Kindi [31] proposed five different cases of the discount sale scenarios in order to maximize the annual gain of the special ordering quantity. Kovalev and Ng [21] showed a discrete version of the classic EOQ problem, they assumed that the time and product were continuously divisible and demand occurred at a constant rate. Cárdenas-Barrón [6] pointed out that there were some technical and mathematical expression errors in Sarkar and Kindi [31] and presented the closed form solutions for the optimal total gain cost. Li [23] presented a solution method which modified Kovalev and Ng’s [21] search method to find the optimal number of orders. Cárdenas-Barrón et al. [8] proposed an economic lot size model where the supplier was offered a temporary discount, and they specified a minimum quantity of additional units to purchase. García-Laguna et al. [16] illustrated a method to obtain the solution of the classic EOQ and economic production quantity models when the lot size must be an integer quantity. They obtained a rule to discriminate between the situation in which the optimal solution is unique and the situation when there are two optimal solutions. Chang et al. [13] used closed-form solutions to solve Martin’s [26] EOQ model with a temporary sale price and Wee and Yu’s [38] deteriorating inventory model with a temporary price discount. Chang and Lin [12] deal with the optimal ordering policy for deteriorating inventory when some or all of the cost parameters may change over a finite horizon. Taleizadeh et al. [33] developed an inventory control model to determine the optimal order and shortage quantities of a perishable item when the supplier offers a special sale. Other authors also considered similar issues, see Abad [1], Khouja and Park [20], Wee et al. [37], Carreras-Barrón [5], Andriolo et al. [2], etc.

In traditional EOQ model, the assumption that all items are perfect in each ordered lot is not pertinent. Because of defective production or other factors, there may be a percentage of imperfect quantity in received items. Salameh and Jaber [30] investigated an EOQ model which contains a certain percentage of defective items in each lot. The percentage is a continuous random variable with known probability density function. Their model assumes that shortage of stock is not allowed. Cárdenas-Barrón [7] modified the expression of optimal order size in Salameh and Jaber [30]. Goyal and Cárdenas-Barrón [17] presented a simple approach to determine Salameh and Jaber’s [30] model. Papachristos and Konstantaras [29] pointed out that the proportion of the imperfects is a random variable, and that the sufficient condition to avoid shortage may not really
prevent occurrence in Salameh and Jaber [30]. Wee et al. [39] and Eroglu and Ozdemir [15] extended imperfect model by allowing shortages backordered. Maddah and Jaber [25] proposed a new model and used renewal-reward theorem to derive the exact expression for the expected profit per unit time in Salameh and Jaber [30]. Hsu and Yu [18] investigated an EOQ model with imperfective items under a one-time-only sale, where the defective rate is known. However, Hsu and Yu’s [18] representation of holding cost is true whenever the ratio of special order quantity to economic order quantity is an integer value. Ouyang et al.[27] developed an EOQ model where the supplier offers the retailer trade credit in payment, products received are not all perfect, and the defective rate is known. Wahab and Jaber[36] extended Maddah and Jaber [25] by introducing different holding cost for the good and defective items. Chang [10] present a new model for items with imperfect quality, where lot-splitting shipments and different holding costs for good and defective items are considered. Other authors also considered similar issues, see Chang [9], Chung and Huang [14], Chang and Ho [11], Lin [24], Khan et al. [19], Bhowmick and Samanta [4], Ouyang et al.[28], etc.

In this article, we extend Hsu and Yu [18], considering that the end of special order process is not coincident with the regular economic order process. We also propose theorems to find closed form solutions when integer operators are involved in objective function. The remainder of this paper is organized as follows. In Section 2, we described the notation and assumptions used throughout this paper. In Section 3, and Section 4, we establish mathematical models and propose theorems to find maximum saving cost and optimal order quantity. In Section 5, we give numerical examples to illustrate the proposed theorems and the results. In Section 6, we summarize and conclude the paper.

2. NOTATION AND ASSUMPTIONS

Notation:

\( \lambda \) the demand rate

\( c \) the purchasing cost per unit

\( b \) the holding cost rate per unit/per unit time

\( a \) the ordering cost per order

\( p \) the defective percentage for each order

\( w \) the screening cost per unit

\( s \) the screening rate, \( s > \lambda \).

\( k \) the discount price of purchasing cost per unit

\( Q_r \) the order size for purchasing cost \( c \) per unit
$Q_{sj}$ the special order quantity, $j = 1, 2, \ldots, 6$.

$T_p$ EOQ model’s optimal period under regular price

$T_{sj}$ special order model’s optimal period under reduced price, $j = 1, 2, \ldots, 6$.

$TC_s^{(j)}$ the total cost corresponding to special order policy, $j = 1, 2, \ldots, 6$.

$TC_n^{(j)}$ the total cost without special order, $j = 1, 2, \ldots, 6$.

$D^{(j)}(Q_{sj})$ the saving cost for Case (1) to Case (6), $j = 1, 2, \ldots, 6$.

$q_{j0}$ the remnant stock level at time $T$, $j = 1, 2, \ldots, 6$.

$\left\lceil \right\rceil$ integer operator, integer value equal to or greater than its argument

$\left\lfloor \right\rfloor$ integer operator, integer value equal to or less than its argument

* the superscript representing optimal value

Assumptions:
1. The demand rate is constant and known.
2. The rate of replenishment is infinite.
3. Based on past statistics, the defective rate is small and known.
4. For shortage is not allowed, the sufficient condition is $\lambda/s < 1 - p$.
5. In Model 1, the purchasing cost for the first regular order quantity is $c - k$.
6. The defective items are withdrawn from inventory when all order quantities are inspected.
7. The time horizon is infinite.

3. MODEL FORMULATION

When suppliers offer a temporary discount to retailers, retailers typically respond with ordering additional items to take advantage of the price reduction. Saving cost is the difference between total cost when special order is taken and total cost when special cost is not taken. According to the time that supplier offers a temporary reduction to retailers, there are three models to be discussed. Model 1 considers the case when special order occurs at regular replenishment time. Model 2, special order occurs at non-regular replenishment time and before the end of screening time. Model 3, special order occurs at non-regular replenishment time and after the end of screening time.

3.1. Model 1

According to the special period length ends before or after screening time of last regular EOQ period length, we have following two sub-cases to be discussed. (i) Case (1)
\( t_s \leq T_s < t_n + Q_p/s \), as shown in Fig. 1. (ii) Case (2) : \( t_s + Q_p/s < T_{s2} < t_{s1} \), as shown in Fig. 2. The procurement cost for special order policy is \( a + (c-k)Q_p \), the screening cost is \( wQ_{ja} \), and the holding cost is \( (c-k)b(1-p)^2/2\lambda + p/sQ_p^2 \), where \( j = 1, 2 \). The total cost corresponding to special order policy during \( 0 \leq t \leq T_s \), \( j = 1, 2 \), is

\[
TC^{(s)}_i(Q_p) = a + (c-k)Q_p + wQ_{ja} + (c-k)b \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] Q_p^2
\]

(1)

In Model 1, the first order is taken using regular EOQ at reduced price \( c-k \), others are taken at regular price \( c \). For Case (1), the total cost without special order during the identical period length \( T_{s1} \) is

\[
TC^{(s)}_a(Q_i) = a \left[ \frac{T_{s1}}{T_p} \right] + (c-k)Q_p + c(Q_{ja} - Q_p) + wQ_{ja} + (c-k)b \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] Q_p^2
\]

\[+ cb \left( \frac{T_{s1}}{T_p} - 1 \right) \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] Q_p^2 + \frac{1}{2}(Q_p + q_i)(T_{s1} - T_s) \]

(2)

For Case (2), the total cost without special order during the identical period length \( T_{s2} \) is

\[
TC^{(s)}_a(Q_{s2}) = a \left[ \frac{T_{s2}}{T_p} \right] + (c-k)Q_p + c(Q_{ja} - Q_p) + wQ_{ja}
\]

\[+ (c-k)b \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] Q_p^2 + cb \left[ \frac{T_{s2}}{T_p} - 1 \right] \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] Q_p^2 - \frac{q_i^2}{2\lambda} \]

(3)

The saving cost of Case (1) and Case (2) is

\[
D^{(s)}(Q_p) = TC^{(s)}_a(Q_i) - TC^{(s)}_a(Q_{s2}) \quad j = 1, 2
\]

(4)

Since \( T_q = Q_p/(1-p)/\lambda \), \( T_p = Q_p/(1-p)/\lambda \), \( T_s = \left[ \frac{T_{s1}}{T_p} \right] T_p \), \( q_j = \left[ \frac{T_{s2}}{T_p} \right] Q_p - Q_{ja} \)

and \( a = cb[(1-p)^2/2\lambda + p/s]Q_p^2 \), where \( j = 1, 2 \), we get

\[
D^{(s)}(Q_p) = -(c-k)b \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] Q_p^2 + kQ_p + \frac{1}{2}(1+\frac{k}{c})a + a \frac{Q_{ja}}{Q_p}
\]

\[+ a \frac{Q_{ja}}{Q_p} + \frac{1-p}{2\lambda} cb \left[ \frac{Q_{ja}}{Q_p} Q_p + Q_p - Q_{ja} \right] \left( Q_{ja} - \frac{Q_{ja}}{Q_p} Q_p \right) \]

(5)
\[ D^{(2)}(Q_{p}) = -(c-k)b \left[ \frac{(1-p)^2 + p}{2\lambda} + \frac{p}{s} \right] Q_{p}^2 + kQ_{p} - \left[ \frac{kQ_{p} + (1 + \frac{k}{c})a}{2\lambda} \right] + 2a \left[ \frac{Q_{p}}{Q_{p}} \right] \]

\[ = \frac{cb}{2\lambda} \left[ \frac{Q_{p}}{Q_{p}} \left( Q_{p} - Q_{p}^2 \right) \right] \]

In Model 1, if the defective percentage for each order is zero, the screening rate quickly tends to infinity and the screening cost is zero, Model 1 is the same as Martin (1994) model. Martin (1994) considered the ordering cost is \( \frac{1}{s}pQ_{p}aQ \), in this paper, the ordering cost is \( \frac{1}{s}pQ_{p}aQ \).

3.2. Model 2

According to the special period length ends before or after screening time of last regular EOQ period length, we have following two sub-cases to be discussed. (i) Case (3): \( t_n < T_{3} < t_n + Q_{p} / s \), as shown in Fig. 3. (ii) Case (4): \( t_n + Q_{p} / s < T_{4} < t_{n+1} \), as shown in Fig. 4. Retailer places an economic order quantity \( Q_{p} \) at \( t = 0 \), the remnant stock level at \( t = T \) is \( q_{j\alpha} \), \( j = 3,4 \). Because supplier offers a temporary discount at \( t = T \), retailer additionally places a special order quantity \( Q_{p} \), \( j = 3,4 \). The procurement cost is \( 2a + cQ_{p} + (c-k)Q_{p} \), the screening cost is \( w(Q_{p} + Q_{p}) \), \( j = 3,4 \), and the holding cost is

\[ c \left[ \frac{(1-p)^2 + p}{2\lambda} \right] Q_{p}^2 + (c-k)b \left[ \frac{Q_{p} + Q_{p}}{s} - \frac{Q_{p} - q_{j\alpha}}{\lambda} \right] \]

\[ + \frac{(Q_{p} + Q_{p} - Q_{p} - Q_{p})^2}{2\lambda} - \left[ Q_{p}p \left( \frac{Q_{p}}{s} - \frac{Q_{p} - q_{j\alpha}}{\lambda} \right) + \frac{(q_{j\alpha} - Q_{p})^2}{2\lambda} \right] \]

The total cost corresponding to special order policy during \( 0 \leq t \leq T_{j} \), \( j = 3,4 \), is

\[ TC^{(1)}(Q_{p}) = 2a + cQ_{p} + (c-k)Q_{p} + w(Q_{p} + Q_{p}) + cb \left[ \frac{(1-p)^2 + p}{2\lambda} \right] Q_{p}^2 \]

\[ + (c-k)b \left[ \frac{Q_{p} + Q_{p}}{s} - \frac{Q_{p} - q_{j\alpha}}{\lambda} \right] + \frac{(Q_{p} + Q_{p} - Q_{p} - Q_{p})^2}{2\lambda} \]

\[ - \left[ Q_{p}p \left( \frac{Q_{p}}{s} - \frac{Q_{p} - q_{j\alpha}}{\lambda} \right) + \frac{(q_{j\alpha} - Q_{p})^2}{2\lambda} \right] \]

For Case (3) and Case (4), if there is no temporary price discount occurs, the total cost without special order during the identical period length \( T_{j} \), \( j = 3,4 \), is
\[ TC^{33}_{n}(Q_{s}) = a \left[ \frac{T_{p}}{T_{r}} \right] + c(Q_{s} + Q_{r}) + w(Q_{s} + Q_{r}) \]
\[ + cb \left[ \frac{T_{s}}{T_{r}} \left( \frac{(1-p)^2}{2}\frac{p}{s} + \frac{1}{2}(Q_{p} + q_{s1}(T_{s3} - t_{s})) \right) \right] \]

\[ TC^{33}_{n}(Q_{s}) = a \left[ \frac{T_{p}}{T_{r}} \right] + c(Q_{s} + Q_{r}) + w(Q_{s} + Q_{r}) \]
\[ + cb \left[ \frac{T_{s}}{T_{r}} \left( \frac{(1-p)^2}{2}\frac{p}{s} + \frac{1}{2}(Q_{p} - q_{s2}) \right) \right] \]

Since \( a = cb \left( \frac{1-p}{2} \frac{p}{s} \right) \), \( T_{o} = (Q_{o} + Q_{o}) \left( 1 - p \right) \lambda \), \( T_{p} = Q_{p} \left( 1 - p \right) \frac{1}{\lambda} \), \( T_{n} = \left[ \frac{T_{p}}{T_{r}} \right] T_{n} \) and \( q_{s1} = \left[ \frac{T_{p}}{T_{r}} \right] Q_{p} - Q_{p} - Q_{q} \), where \( j = 3, 4 \), the saving cost of Case (3) and Case (4) is

\[ D^{33}(Q_{s}) = - (c-k)b \left[ \frac{(1-p)^2}{2} \frac{p}{s} Q_{s} + k - (c-k)b \left[ \frac{2pQ_{o}}{s} + \frac{p(2-p)Q_{p}}{\lambda} + q_{s1} \right] \right] Q_{s} \]
\[ - a + \left[ \frac{Q_{s}}{Q_{p}} \right] + a \left[ \frac{Q_{s}}{Q_{p}} \right] + \frac{1-p}{2} cb \left[ \frac{Q_{s}}{Q_{p}} \right] Q_{p} - Q_{s} \right] \left( Q_{s} - \left[ \frac{Q_{s}}{Q_{p}} \right] Q_{p} \right) \]

\[ D^{44}(Q_{s}) = - (c-k)b \left[ \frac{(1-p)^2}{2} \frac{p}{s} Q_{s} + k - (c-k)b \left[ \frac{2pQ_{o}}{s} - \frac{p(2-p)Q_{p}}{\lambda} + q_{s1} \right] \right] Q_{s} \]
\[ - a + 2a \left[ \frac{Q_{s}}{Q_{p}} \right] - \frac{1-p}{2} cb \left[ \frac{Q_{s}}{Q_{p}} \right] Q_{p} - Q_{s} \right] ^2 \]

### 3.3. Model 3

According to the special period length ends before or after screening time of last regular EOQ period length, we have following two sub-cases to be discussed. (i) Case (5) : \( t_{n} \leq T_{s3} < t_{n} + Q_{p} / s \), as shown in Fig. 5. (ii) Case (6) : \( t_{n} + Q_{p} / s < T_{s6} < t_{n+1} \), as shown in Fig. 6. Retailer places an economic order quantity \( Q_{p} \) at \( t = 0 \), the remnant stock level at \( t = T \) is \( q_{j0} \), \( j = 5, 6 \). Because supplier offers a temporary discount at \( t = T \), retailer additionally places a special order quantity \( Q_{s} \), \( j = 5, 6 \). The procurement cost is \( 2a + cQ_{p} + (c-k)Q_{s} \), the screening cost is \( w(Q_{p} + Q_{s}) \), \( j = 5, 6 \), and the holding cost is...
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\[ cb \left[ \frac{(1-p)^2}{\lambda} + \frac{p}{s} \right] Q_p^2 + (c-k)b \left[ Q_q p - \frac{Q_a + q_{jt} - Q_p p^2}{2\lambda} - q_{jt}^2 \right] \quad j = 5,6 \]

The total cost corresponding to special order policy during \( 0 \leq t \leq T_j \), \( j = 5,6 \), is

\[ TC_{s}^{(j)}(Q_q) = 2a + cQ_p + (c-k)Q_q + w(Q_p + Q_q) + cb \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] Q_p^2 \]
\[ + (c-k)b \left[ \frac{pQ_a^2}{s} + \frac{Q_a + q_{jt} - Q_p p^2}{2\lambda} \right] q_{jt}^2 \quad j = 5,6 \] (12)

For Case (5) and Case (6), if there is no temporary price discount occurs, the total cost without special order during the identical period length \( T_j \), \( j = 5,6 \), is

\[ TC_{a}^{(s)}(Q_q) = a \left[ \frac{T_a}{T_p} \right] + c(Q_{st} + Q_p) + w(Q_{st} + Q_p) \]
\[ + cb \left[ \frac{T_{st}}{T_p} \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] Q_p^2 + \frac{1}{2} (Q_p + q_{st})(T_{st} - t_a) \right] \] (13)

\[ TC_{a}^{(s)}(Q_q) = a \left[ \frac{T_a}{T_p} \right] + c(Q_{st} + Q_p) + w(Q_{st} + Q_p) \]
\[ + cb \left[ \frac{T_{st}}{T_p} \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] Q_p^2 - q_{st}^2 \right] \] (14)

Since \( a = cb(1-p)^2/2\lambda + p/s \), \( T_a = (Q_q + Q_p)(1-p)/\lambda \), \( T_p = Q_p(1-p)/\lambda \),
\( t_a = \frac{T_a}{T_p} \), and \( q_{st} = \left[ \frac{T_{st}}{T_p} \right] Q_p - Q_p - Q_q \), where \( j = 5,6 \), the saving cost of Case (5) and Case (6) is

\[ D^{(s)}(Q_q) = -(c-k)b \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] Q_p^2 + \left[ k - \frac{1-p}{\lambda} (c-k) b q_{st} \right] Q_s - a + a \left[ \frac{Q_{st}}{Q_p} \right] \]
\[ + a \left[ \frac{Q_{st}}{Q_p} \right] + \left[ \frac{Q_{st}}{Q_p} \right] Q_p - Q_s \left[ Q_s - \left[ \frac{Q_{st}}{Q_p} \right] Q_p \right] \] (15)
\[ D^{(j)}(Q_{m}) = -(c-k)b \left( \frac{1-p}{2} \right)^2 + \frac{p}{s} Q_{m} + \left[ k - \frac{1-p}{2} (c-k)bq_{ao} \right] Q_{m} - a + 2a \left[ \frac{Q_{m}}{Q_{p}} \right] - \frac{cb}{2\lambda} \left[ \left( \frac{Q_{m}}{Q_{p}} \right) Q_{p} - Q_{m} \right]^2 \]  

(16)

4. THEORETICAL RESULTS

In this section, we suggest properties of \( D^{(j)}(Q_{m}) \), \( j=1,2,\cdots,6 \), and give theorems to solve the proposed models.

Property 4-1

\( D^{(j)}(Q_{m}) \) is a piecewise continuous function in which jump values at \( Q_{m} = mQ_{p} \) are

\[
\lim_{a \to 0^+} D^{(j)}((m+a)Q_{p}) - \lim_{a \to 0^-} D^{(j)}((m-a)Q_{p}) = \begin{cases} 2a - (1-p)cbQ_{p}^2 / 2\lambda & \text{if } j = 1,3,5 \\ 2a - cbQ_{p}^2 / 2\lambda & \text{if } j = 2,4,6 \end{cases}
\]  

(17)

where \( m \) is a non-negative integer.

Proof of Property 4-1 is given in appendix.

Property 4-2

Let \( m \) is a non-negative integer, and

\[
\omega_i = \begin{cases} 0 & i=1,2 \\ \sigma & i=3,4 \\ \tau & i=5,6 \end{cases}
\]  

(18)

\[
\sigma = (c-k)b \left[ \frac{2pQ_{p}}{s} - \frac{p(2-p)Q_{p}}{\lambda} + \frac{q_{ao}}{\lambda} \right] \quad i = 3,4
\]  

(19)

\[
\tau = \frac{(1-p)(c-k)bq_{ao}}{\lambda} \quad i = 5,6
\]  

(20)

\[
m_{li} = \frac{c(k-\omega_i)Q_{p}}{2(c-k)a} - 1 \quad i=1,2,\cdots,6
\]  

(21)

\[
m_{li} = \frac{c(k-\omega_i + (1-p)cbQ_{p} / \lambda)Q_{p}}{2(c-k)a} \quad i=1,3,5
\]  

(22)

\[
m_{li} = \frac{c(k-\omega_i + cbQ_{p} / \lambda)Q_{p}}{2(c-k)a} \quad i=2,4,6
\]  

(23)
(a) $D^{(i)}(Q_m)$ is an increasing function of $Q_m$ between $mQ_p$ and $(m+1)Q_p$ when $m < \left\lceil m_{kr} \right\rceil$, where $i = 1, 2, \cdots, 6$.

(b) $D^{(i)}(Q_m)$ is a decreasing function of $Q_m$ between $mQ_p$ and $(m+1)Q_p$ when $m > \left\lceil m_{kr} \right\rceil$, where $i = 1, 2, \cdots, 6$.

(c) $D^{(i)}(Q_m)$ is a concave function of $Q_m$ between $mQ_p$ and $(m+1)Q_p$ when $\left\lceil m_{kr} \right\rceil < m < \left\lfloor m_{kr} \right\rfloor$, where $i = 1, 2, \cdots, 6$.

Proof of Property 4.2 is given in appendix.

**Theorem 1**

Let

$$\Delta_i = (c-k) \left[ \frac{(1-p)^2}{2 \lambda} + \frac{p}{s} \right] + \frac{1-p}{2 \lambda} c$$

$$Q_m(m) = \frac{k - \omega_i + (1-p)c b Q_{p} (m+1)/ \lambda}{2 \Delta_i b} \quad i = 1, 3, 5$$

$$DM_{i}(m) = \frac{-(1-p)(c-k)a}{2 \lambda \Delta_i} (m+1)^2 + \frac{(1-p)k c Q_{p} + 2a}{2 \lambda \Delta_i} (m+1)$$

$$+ \frac{k^2}{4 \Delta_i b} - 2a + \frac{1-p}{2 \lambda} \frac{c b Q_{p}^2 - k (Q_r + \frac{a}{c})}{2 \lambda \Delta_i}$$

$$h_{kr}(m) = \lim_{a \to \infty} D^{(i)} \left( (m + \alpha) Q_p \right) = -a \left( 1 - \frac{k}{c} \right) m^2 + (k Q_p + 2a)m - k(Q_p + \frac{a}{c})$$

$$h_{kr}(m) = \lim_{a \to \infty} D^{(i)} \left( (m + \alpha) Q_p \right) = -a \left( 1 - \frac{k}{c} \right) m^2 + (k Q_p + 2a)m - k(Q_p + \frac{a}{c}) \quad i = 3, 5$$

$$\epsilon_i = \frac{[(k - \omega_i)c Q_{p} - 3a + 3a][1-p] + 4a \lambda (c-k) \left[ (1-p)^{2} / 2 \lambda + p / s \right]}{2(1-p)(c-k)a} \quad i = 1, 3, 5$$
For \( i = 1, 3, 5 \), if \( z_i \) is not an integer, let \( m_n = \lfloor z_i \rfloor \). If \( z_i \) is an integer, let \( m_n = \lceil z_i \rceil + 1 \). The special order quantity \( Q\alpha \) and maximum value of \( D^{(i)}(Q\alpha) \) can be found in the following:

(a) When \( m_n \leq [m_{kl}] \)

\[
Q\alpha = \begin{cases} 
Q\alpha([m_n]) & \text{if } T_{n} - t_n < Q_p / s \\
Q\alpha([m_n]) / Q_p Q_p + \lambda Q_p / s(1 - p) & \text{if } T_{n} - t_n \geq Q_p / s 
\end{cases}
\]

\[
D^{(i)}(Q\alpha) = \begin{cases} 
DM_i([m_n]) & \text{if } T_{n} - t_n < Q_p / s \\
\max \{D^{(i)}(Q\alpha), DM_i([m_n] + 1)\} & \text{if } T_{n} - t_n \geq Q_p / s 
\end{cases}
\]

(b) When \( [m_{kl}] < m_n < [m_{kr}] \)

\[
Q\alpha = \begin{cases} 
Q\alpha([m_n]) & \text{if } T_{n} - t_n < Q_p / s \\
Q\alpha([m_n]) / Q_p Q_p + \lambda Q_p / s(1 - p) & \text{if } T_{n} - t_n \geq Q_p / s 
\end{cases}
\]

\[
D^{(i)}(Q\alpha) = \begin{cases} 
DM_i([m_n]) & \text{if } T_{n} - t_n < Q_p / s \\
\max \{DM_i([m_n] - 1), D^{(i)}(Q\alpha), DM_i([m_n] + 1)\} & \text{if } T_{n} - t_n \geq Q_p / s 
\end{cases}
\]

(c) When \( m_n \geq [m_{kr}] \)

\[
Q\alpha = \begin{cases} 
Q\alpha([m_n]) & \text{if } T_{n} - t_n < Q_p / s \\
Q\alpha([m_n]) / Q_p Q_p + \lambda Q_p / s(1 - p) & \text{if } T_{n} - t_n \geq Q_p / s 
\end{cases}
\]

\[
D^{(i)}(Q\alpha) = \begin{cases} 
\max \{DM_i([m_n] - 1), D^{(i)}(Q\alpha), h_a([m_n] + 1)\} & \text{if } T_{n} - t_n < Q_p / s \\
\max \{DM_i([m_n] - 1), D^{(i)}(Q\alpha), h_a([m_n] + 1)\} & \text{if } T_{n} - t_n \geq Q_p / s 
\end{cases}
\]

Proof of Theorem 1 is given in the appendix.

**Theorem 2**

Let \( \Delta_2 = (c - k) \left[ \frac{(1 - p)^2}{2\lambda} + \frac{p}{s} \right] + \frac{1}{2\lambda} c \)

\[
Q\alpha(m) = \frac{k - \omega + cbQ_p(m + 1) / \lambda}{2\Delta_2b} \quad i = 2, 4, 6
\]
\[DM_1(m) = \frac{-(c-\lambda)a}{2\lambda\Delta_2} (m+1)^2 + \left(\frac{kcQ_p}{2\lambda\Delta_2} + 2a\right)(m+1) + \frac{k^2}{4\lambda\Delta_2} - a - k(Q_p + \frac{a}{c})\] (39)

\[DM_i(m) = \frac{-(c-\lambda)a}{2\lambda\Delta_2} (m+1)^2 + \left(\frac{(k-\omega_i)cQ_p}{2\lambda\Delta_2} + 2a\right)(m+1) + \frac{(k-\omega_i)^2}{4\lambda\Delta_2} - a \quad i = 4,6\] (40)

\[z_i = \frac{(k-\omega_i)cQ_p - ca + 3ka + 4a\lambda(c-k)}{2(c-k)a} \left(1 - p \right)^2 / 2\lambda + p / s \] (41)

For \(i = 2,4,6\), if \(z_i\) is not an integer, let \(m_i = \left\lfloor z_i \right\rfloor\). If \(z_i\) is an integer, let \(m_i = \left\lfloor z_i \right\rfloor + 1\). The special order quantity \(Q_a\) and maximum value of \(D^{(i)}(Q_a)\) can be found in the following:

(a) When \(m_i \leq \left\lfloor m_i \right\rfloor\)

\[Q_a = \begin{cases} Q_a(\left\lfloor m_i \right\rfloor) & \text{if } Q_p / s < T_a - t_a < T_p \\ Q_a(\left\lfloor m_i \right\rfloor) / Q_p + \lambda Q_p / s(1 - p) & \text{if } Q_p / s \geq T_a - t_a \end{cases}\] (42)

\[D^{(i)}(Q_a) = \max \left\{ D^{(i)}(Q_a), DM_i(\left\lfloor m_i \right\rfloor + 1) \right\} \text{ if } Q_p / s \geq T_a - t_a \] (43)

(b) When \(\left\lfloor m_i \right\rfloor < m_i < \left\lfloor m_i \right\rfloor + 1\)

\[Q_a = \begin{cases} Q_a(\left\lfloor m_i \right\rfloor) & \text{if } Q_p / s < T_a - t_a < T_p \\ Q_a(\left\lfloor m_i \right\rfloor) / Q_p + \lambda Q_p / s(1 - p) & \text{if } Q_p / s \geq T_a - t_a \end{cases}\] (44)

\[D^{(i)}(Q_a) = \max \left\{ DM_i(\left\lfloor m_i \right\rfloor - 1), D^{(i)}(Q_a), DM_i(\left\lfloor m_i \right\rfloor + 1) \right\} \text{ if } Q_p / s \geq T_a - t_a \] (45)

(c) When \(m_i \geq \left\lfloor m_i \right\rfloor + 1\)

\[Q_a = \begin{cases} Q_a(\left\lfloor m_i \right\rfloor) & \text{if } Q_p / s < T_a - t_a < T_p \\ Q_a(\left\lfloor m_i \right\rfloor) / Q_p + \lambda Q_p / s(1 - p) & \text{if } Q_p / s \geq T_a - t_a \end{cases}\] (46)
\[ D^{(i)}(Q^*_n) = \max \left\{ DM_i(\lfloor m_{st} \rfloor), D^{(i)} \left( \left\lfloor Q_n (\lfloor m_{st} \rfloor + 1) / Q_p \right\rfloor Q_p + \lambda Q_p / s (1 - p) \right) \right\} \]

if \( Q_n / s < T_{st} - t_s < T_p \)

if \( Q_n / s \geq T_{st} - t_s \)  \( (47) \)

Proof of Theorem 2 is the same as Theorem 1. For \( j = 1, 2, \ldots, 6 \), comparing \( D^{(j)}(Q^*_n) \) each other in Model 1 to Model 3, we can find maximum saving cost \( D^{(j)}(Q^*_n) \) and special order quantity \( Q^*_n \) in each Model.

5. NUMERICAL EXAMPLES

In this section, we use the same cost parameters of Hsu and Yu (2009) to illustrate the theorems proposed. The sensitivity analysis of major parameters on the optimal solutions will also be carried out.

**Example 1.** Given \( a = \$800 \) /order, \( b = 0.1 \), \( c = \$12 / \) unit, \( s = \$24000 \) units/yr, \( \lambda = \$8000 \) units/yr, \( q_0 = q_0 = 900 \) units, \( q_0 = q_0 = 200 \) units, \( w = \$2 / \) unit, \( p = 0.1 \) and \( k = \$4 / \) unit in Model 1. In Case (1), we find \([m_{st}] = 41, [m_{st}] = 42, [m_{st}] = 43 \) and \( m_{st} = 43 \), then \( m_{st} \geq \lfloor m_{st} \rfloor \). Because \( Q_n = Q_n(\lfloor m_{st} \rfloor) = 46721 \) satisfies \( 0 \leq T_{st} - t_s < T_p \), the maximum saving cost of Case (1) is \( D^{(1)}(Q^*_n) = \max \{ DM_i(\lfloor m_{st} \rfloor), h_{st}(\lfloor m_{st} \rfloor + 1) \} = 93553.2 \), then the special order quantity is \( Q^*_n = (\lfloor m_{st} \rfloor + 1)Q_p = 47431 \) units. The result is shown in Fig. 7. In Case (2), we find \([m_{st}] = 41, [m_{st}] = 43, [m_{st}] = 44 \) and \( m_{st} = 43 \), then \( [m_{st}] < m_{st} < [m_{st}] \). Owing to \( Q_{n2} = Q_{n2}(m_{st}) = 47462 \) does not satisfy \( Q_n / s < T_{st} - t_s < T_p \), we take \( Q_{n2} = Q_{n2}(m_{st}) = 47460 \) into Eq.(6) and obtain \( D^{(2)}(47840) = 93525.1 \). The maximum saving cost of Case (2) is \( D^{(2)}(Q^*_n) = \max \{ DM_i(42), D^{(2)}(47840), D^{(2)}(48943) \} = 93525.8 \), then the special order quantity is \( Q^*_n = Q_{n2}(42) = 46766 \) units. The result is shown in Fig. 8. Comparing \( D^{(1)}(Q^*_n) \) with \( D^{(2)}(Q^*_n) \) in Model 1, we can find maximum saving cost of Model 1 is \( D^{(1)}(Q^*_n) = 93553.2 \) and special order quantity is \( Q^*_n = 47431 \) units. The optimal ordering policies for Model 1 to Model 3 under different discounts are represented in Table 1.

From Table 1, we can obtain following results: (a) Ordering quantity and saving cost increase as discount price increases. This implies that when supplier offers more temporary discount, retailers will order more quantity to save cost. (b) The rankings of
special order quantity $Q_5' > Q_3' > Q_1'$ are not consistent with saving cost $D^{(5)}(Q_5') > D^{(3)}(Q_3') > D^{(1)}(Q_1')$ for the same discount. The reason is the purchasing cost for the first economic order quantity in Model 1 is $(c-k)Q_1'$, but the purchasing cost in Model 2 and Model 3 are $cQ_a$. The difference $kQ_a$ influences the ranking of saving cost. The largest saving cost in three models is $D^{(5)}(Q_5')$. The reason is the defective items are withdrawn from inventory before special order occurs. The holding cost does not involve defective items.

**Example 2.** The sensitivity analysis is performed to study the effects of changes of major parameters on the optimal solutions. All the parameters are identical to Example 1 except the given parameter. The following inferences can be made based on Table 2.

(a) Higher values of screening rate $s$ cause a higher value of special order quantity $Q_a$ and maximum saving cost $D^{(i)}(Q_a')$, $i=1,3,5$. It implies that the retailer should take some actions to increase the item’s screening rate in order to save more cost.

(b) Higher values of holding cost rate $b$ and purchasing cost $c$ cause a lower value of special order quantity $Q_a$ and maximum saving cost $D^{(i)}(Q_a')$, $i=1,3,5$. Hence, in order to increase saving cost, the retailer should have low holding cost rate and purchasing cost.

(c) Higher values of remnant stock level $q_{i0}$ cause a lower value of special order quantity $Q_a$ and maximum saving cost $D^{(i)}(Q_a')$, $i=3,5$. It implies when remnant stock level is high, it don’t need to orders more special order quantity. It induces low saving cost.

6. CONCLUSION

In this article, we developed an inventory model to determine the optimal special order and maximum saving cost of imperfective items for retailers who use economic order quantity model and are faced with a temporary discount. According to the time that supplier offers a temporary reduction to retailers, we discuss three models in this article. Each model has two sub-cases to be discussed. In temporary discount problems, the ordering number is an integer variable, there are integer operators in objective function. It is hard to find closed-form solutions of their extreme values. A distinguishing feature of the proposed theorems is that they can easily apply to find closed-form solutions of temporary discount problems. The results in numerical examples and sensitivity analysis of key model parameters indicate following insights: (a) Both ordering quantity and saving cost increase as discount price increases; (b) For the same discount, Case (5) has larger saving cost than others. This means, in Case (5), retailers earn maximum saving cost; (c) Higher values of screening rate induce special order quantity and higher saving cost; (d) Higher values of holding cost rate and purchasing cost cause a lower value of special order quantity and saving cost.

The further advanced research will extend the proposed models in several ways. For example, we can extend the imperfect model by allowing shortages the horizon may be finite. Also we can consider the demand rate as not been constant.
REFERENCES

Proof of Property 4-1:

We prove property of $D^{(1)}(Q_{m})$ only, others are similar to the proof of $D^{(1)}(Q_{m})$. Let $\alpha \rightarrow 0$.

$$\lim_{{\alpha \rightarrow 0}} D^{(1)}\left((m+\alpha)Q_p\right) = -a\left(1-\frac{k}{c}\right)\alpha^2 + (kQ_p + 2a)m - k(Q_p + \frac{a}{c})$$  \hspace{1cm} (A1)
\[
\lim_{a \to 0} D^{(i)} \left( (m-a)Q_p \right) = -a(1 - \frac{k}{c})m^2 + (kQ_p + 2a)m - k(Q_p + \frac{a}{c}) - 2a + \frac{1}{2\lambda} cbQ_p^2
\]  
(A2)

\[
\lim_{a \to 0} D^{(i)} \left( (m+a)Q_p \right) - \lim_{a \to 0} D^{(i)} \left( (m-a)Q_p \right) = 2a - \frac{1}{2\lambda} cbQ_p^2
\]  
(A3)

This implies \( D^{(i)}(Q_{al}) \) is a piecewise continuous function in which jump values at \( Q_{al} = mQ_p \) are \( 2a - (1 - p)cbQ_p^2 / 2\lambda \).

Proof of Property 4-2:

We prove property of \( D^{(i)}(Q_{al}) \) only, others are similar to the proof of \( D^{(i)}(Q_{al}) \). During \( mQ_p < Q_{al} < (m+1)Q_p \),

\[
D^{(i)}(Q_{al}) = \left\{ (c-k)b \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] + \frac{1}{2\lambda} cb \right\} Q_{al}^2 + \left\{ k + \frac{1}{2\lambda} cbQ_p(m+1) \right\}
\]  
(A4)

\[
\frac{dD^{(i)}(Q_{al})}{dQ_{al}} = \left\{ (c-k)b \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] + \frac{1}{2\lambda} cb \right\} Q_{al} + k + \frac{1}{2\lambda} cbQ_p(m+1)
\]  
(A5)

\[
\frac{d^2D^{(i)}(Q_{al})}{dQ_{al}^2} = \left\{ (c-k)b \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] + \frac{1}{2\lambda} cb \right\} < 0
\]  
(A6)

This implies \( D^{(i)}(Q_{al}) \) is a concave function during \( mQ_p < Q_{al} < (m+1)Q_p \). From Eq. (A5), if \( D^{(i)}(Q_{al}) \) has \( \frac{dD^{(i)}(Q_{al})}{dQ_{al}} = 0 \) property during \( mQ_p < Q_{al} < (m+1)Q_p \), it will be happened at

\[
Q_{al}(m) = \frac{k + (1-p)cbQ_p(m+1) / \lambda}{2(c-k)b(1-p)^2/2\lambda+p/s+(1-p)cb/\lambda}
\]  
(A7)

Since \( Q_{al}(m) \) should satisfy \( mQ_p < Q_{al} < (m+1)Q_p \), we have

\[
m_{al} = \frac{ckQ_p}{2(c-k)a} - 1 < m < \frac{c[k + (1-p)cbQ_p / \lambda]Q_p}{2(c-k)a} = m_{al}
\]  
(A8)

Owing to \( m \) is an integer, the region of \( m \) should change to \( \left[ m_{al} \right] < m < \left[ m_{al} \right] \).

According to concavity of \( D^{(i)}(Q_{al}) \) during \( mQ_p < Q_{al} < (m+1)Q_p \), \( D^{(i)}(Q_{al}) \) is an increasing function of \( Q_{al} \) for \( m < \left[ m_{al} \right] \) and a decreasing function of \( Q_{al} \) for \( m > \left[ m_{al} \right] \). 

Proof of Theorem 1
We prove property of \( D^{(1)}(Q_i) \) only, others are similar to the proof of \( D^{(1)}(Q_i) \).

Taking \( Q_i(m) \) into Eq. (A4) and let

\[
\Delta_i = (c-k) \left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right] + \frac{1-p}{2\Delta_i} c
\]

We have

\[
DM_i(m) = \frac{-(1-p)(c-k)a}{2\Delta_i} (m+1)^2 + \frac{(1-p)kcQ_p}{2\Delta_i} + 2a \tag{A9}
\]

\[
+ \frac{k^2}{4\Delta_i} - 2a + \frac{1-p}{2\Delta_i} cbQ_p^2 - k(Q_r + \frac{a}{c})
\]

The first and second derivatives of \( DM_i(m) \) respect to \( m \) are respectively

\[
\frac{dDM_i(m)}{dm} = \frac{-(1-p)(c-k)a}{2\Delta_i} (m+1) + \frac{(1-p)kcQ_p}{2\Delta_i} + 2a \tag{A10}
\]

\[
\frac{d^2DM_i(m)}{dm^2} = -\frac{(1-p)(c-k)a}{2\Delta_i} < 0 \tag{A11}
\]

It means that \( DM_i(m) \) is a concave function of \( m \). Owing to \( m \) is an integer, by \( DM_i(m) - DM_i(m+1) \geq 0 \) and let

\[
z_i = \left\lfloor \frac{(kcQ_p - ca + 3ka)(1-p) + 4a\lambda(c-k)\left[ \frac{(1-p)^2}{2\lambda} + \frac{p}{s} \right]}{2(1-p)(c-k)a} \right\rfloor
\]

then \( m = \left\lfloor z_i \right\rfloor \) is the value that maximizes \( DM_i(m) \). By \( DM_i(m) - DM_i(m-1) \geq 0 \), then \( m = \left\lfloor z_i + 1 \right\rfloor \) is the value that maximizes \( DM_i(m) \). To sum up, if \( z_i \) is not an integer, let \( m_i = \left\lfloor z_i \right\rfloor = \left\lfloor z_i + 1 \right\rfloor \); otherwise, let \( m_i = \left\lfloor z_i \right\rfloor \) and \( m_i = \left\lfloor z_i \right\rfloor + 1 \). The maximum value of \( DM_i(m) \) is \( DM_i(m_i) \).

(a) When \( m_i \leq \left\lfloor m_{sl} \right\rfloor \), it means \( DM_i(m) \) is a decreasing function of \( m \) during \( \left\lfloor m_{sl} \right\rfloor \leq m \leq \left\lfloor m_{sl} \right\rfloor \). Hence, the maximum value of \( DM_i(m) \) during \( \left\lfloor m_{sl} \right\rfloor \leq m \leq \left\lfloor m_{sl} \right\rfloor \) is \( DM_i\left( \left\lfloor m_{sl} \right\rfloor \right) \) and the special order quantity is
\( Q_i = Q_i \left( \left[ m_{lt} \right] \right) \). Case (1) is justified only in the condition \( 0 \leq T_{st} - t_s < Q_p / s \). If \( Q_{si} \) is not satisfied \( 0 \leq T_{st} - t_s < Q_p / s \), the maximum value of \( D^{(i)}(Q_{si}) \) will happen at \( T_{st} = t_s + Q_p / s \), i.e., \( Q_{si} = \left[ Q_{si} \left( \left[ m_{lt} \right] \right) / Q_p \right] Q_p + \lambda Q_p / s(1 - p) \). Because \( D^{(i)}(Q_{si}) \) has positive jumps at break points, \( DM_i \left( \left[ m_{lt} \right] + 1 \right) \) maybe greater than \( D^{(i)}(Q_{si}) \). So both \( D^{(i)}(Q_{si}) \) and \( DM_i \left( \left[ m_{lt} \right] + 1 \right) \) should be compared to determine the global maxima.

(b) When \( \left[ m_{lt} \right] < m_i < \left[ m_{rt} \right] \), it means \( DM_i(m) \) is a concave function of \( m \) during \( \left[ m_{lt} \right] \leq m \leq \left[ m_{rt} \right] \). Hence, the maximum value of \( DM_i(m) \) during \( \left[ m_{lt} \right] \leq m \leq \left[ m_{rt} \right] \) is \( DM_i(m_{si}) \) and the special ordering quantity is \( Q_{si} = Q_i(m_{si}) \). If \( Q_{si} \) is not satisfied \( 0 \leq T_{st} - t_s < Q_p / s \), the maximum value of \( D^{(i)}(Q_{si}) \) will happen at \( T_{st} = t_s + Q_p / s \), i.e., \( Q_{si} = \left[ Q_{si} \left( m_{si} \right) / Q_p \right] Q_p + \lambda Q_p / s(1 - p) \). In this time, \( D^{(i)}(Q_{si}) \) may be smaller than \( DM_i(m_{si} - 1) \) or \( DM_i(m_{si} + 1) \). So \( DM_i(m_{si} - 1) \cdot D^{(i)}(Q_{si}) \) and \( DM_i(m_{si} + 1) \) should be compared to determine the global maxima.

(c) When \( m_{si} \geq \left[ m_{rt} \right] \), \( DM_i(m) \) is an increasing function of \( m \) during \( \left[ m_{lt} \right] \leq m \leq \left[ m_{rt} \right] \). Because \( D^{(i)}(Q_{si}) \) has positive jumps at break points, \( h_{ir} \left( \left[ m_{ir} \right] + 1 \right) \) maybe greater than \( DM_i(\left[ m_{ir} \right]) \). We need to check whether \( Q_{si} = Q_i(\left[ m_{ir} \right]) \) is satisfied \( 0 \leq T_{st} - t_s < Q_p / s \) or not. If \( Q_{si} \) is not satisfied the condition, the maximum value of \( D^{(i)}(Q_{si}) \) will happen at \( T_{st} = t_s + Q_p / s \), i.e., \( Q_{si} = \left[ Q_{si} \left( m_{ir} \right) / Q_p \right] Q_p + \lambda Q_p / s(1 - p) \). In this time, \( D^{(i)}(Q_{si}) \) may be smaller than \( DM_i(\left[ m_{ir} \right] - 1) \) or \( h_{ir} \left( \left[ m_{ir} \right] + 1 \right) \). So \( DM_i(\left[ m_{ir} \right] - 1) \cdot D^{(i)}(Q_{si}) \) and \( h_{ir} \left( \left[ m_{ir} \right] + 1 \right) \) should be compared to determine the global maxima. \( \square \)

<table>
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<tr>
<th>Table 1: The optimal ordering policies for three Models under different discounts</th>
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Table 2: Sensitivity analysis of some parameters on the optimal solutions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
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<td>(c)</td>
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Figure 1: Case (1) diagram

Figure 2: Case (2) diagram
Figure 7: The saving cost $D^{(1)}(Q_{s})$ of example 1

Figure 8: The saving cost $D^{(2)}(Q_{s})$ of example 1