STRENGTH OF CONNECTIONS IN PRECAST CONCRETE STRUCTURES

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Abstract. The available experimental and numerical results of many studies of behavior of reinforced concrete connections for different stages of loading, up to fracture loading, are presented and analyzed in this paper. The problem of beam-column connection (or plate-wall connection) in prefabricated monolithic structures is emphasized. Fracture mechanisms of RC structures, the theoretical basis for their analysis, and the use of fracture mechanics in RC structures were also considered, as well as the mathematical models of prefabricated connections. In order to formulate an adequate mathematical model for calculating the connections, the dominant parameters influencing the behavior of these connections were analyzed. A failure model for the prefabricated wall–monolithic RC plate connection was formulated. In building the model, the results of implemented experimental and numerical research of prefabricated connection in the MMS system from 2007 were used. Experiences with the implementation of the aforementioned construction system in structures in Tuzla, in the 1980’s last century, were additionally used. The proposed mathematical models provide a sufficiently accurate failure assessment of prefabricated reinforced concrete connections.

Key words: reinforced concrete prefabricated connections, experimental and numerical research, reliability, failure mechanism, mathematical model.

1. INTRODUCTION

Modelling the mechanism of behaviour of reinforced concrete (RC) elements and connections is a complex mathematical problem. In fact, reinforced concrete is a composite, highly heterogeneous material with its influential parameters having a stochastic nature, which can only be treated through reliability analysis based on theory of probability. In the analysis of reinforced concrete structures it is necessary to take into consideration the
changes in properties of reinforced concrete structures initiated by the deterioration processes in concrete, which is presented in details in the papers [7] i [8]. Theoretical considerations of modelling the behaviour of RC elements are constrained to certain simplifications and the introduction of a series of assumptions due to formulate a mathematical model of acceptable complexity and accuracy. Experimental studies are the basis for the development of theoretical settings and mathematical models in the civil engineering. Mechanisms of behaviour of reinforced concrete monolithic elements and joints are more discussed than those in prefabricated structures. Moreover, in the proposed mathematical models, the adhesion properties of concrete and reinforcement, the crack initiating mechanisms and the condition immediately before the fracture were treated. The modern approach to the analysis of structures is reduced primarily to the concept of ductile structures, which was presented in [23]. Therefore, current Codes for the RC elements and structures are primarily related to provisions for achieving sufficient ductility, which especially refers to structures loaded with horizontal variable load (wind and seismic loads).

When calculating monolithic RC structures, an integral structural work is assumed, with stiffly connected structural elements and the requirement that the failure of structural elements occurs before the failure of their connection. In Europe, such calculation concept is used also in the numerical modelling of connections of precast and monolithic RC elements.

The actual mechanism of behaviour of prefabricated connections can be determined only experimentally. For this reason, many authors have implemented their experiments in individual prefabricated systems, in order to define calculation models. This experimental database and the associated numerical analyses provide guidelines for the description of working mechanisms and failure mechanisms in prefabricated connections. Therefore, this paper provides an overview of typical studies, although the problem of unification of models for the purpose of description of behaviour of prefabricated concrete structure connections still remains. Here, some particularities and representative research results were analysed that indicate the behavioural properties of prefabricated joints and connections, which were the basis for the definition of certain aspects of methodology for the analysis of such connections. Thus, the comparative analysis is commonly applied, i.e. the method of comparison with a similar monolithic structure. In this work, the results of own experimental and numerical studies of the plate-wall connection in prefabricated monolithic types of structures was analyzed. Prefabricated connections in MMS systems were investigated in 2007. In order to formulate an adequate mathematical model for the calculation of connections, the dominant parameters influencing the behaviour of these connections were only analyzed. The proposed mathematical models provide a sufficiently accurate failure assessment of prefabricated reinforced concrete connections.

2. LITERATURE REVIEW

In [18] Mehlhorn and Schwing summarized the performed research. Based on research, joints and connections were classified, mechanisms of load transfer through connections were described and techniques for calculating and modelling by FEM were proposed. In the procedure of modelling the link elements were introduced which describe the stiffness of connections. The paper [13] provides an analytical examination of behaviour of wall-plate connections, which are used in North America, from the aspect
of effects of strength of the mortar applied while making monolith connection. In [31] Tassios and Tsoukantas analyzed the basic mechanism of the first crack and the limit state of large-panel connections under static and dynamic loading. In order to define the stress transfer in prefabricated reinforced concrete elements, Guillaud and Morlier [11] carried out an experimental study of prefabricated concrete-filled connections with a variety of shapes and positions of prefabricated joints and joint infill. They processed the results by modifying the terms for monolith structures with correction factors, which take into account the actual behaviour of the connection. Modelling plastic deformation by the use of finite elements is formulated by Ramm and Kompfner in [26]. They defined material nonlinearity using plate models according to thickness of the elements. Concrete was modelled with shell elements and reinforcement was idealized with smear layers. A multi linear, elastic and plastic material model with isotropic hardening was applied to steel.

In 1985 CEB has published guidelines for the design of prefabricated wall connections [2]. Noguchi and Watanabe performed an analytical research on nonlinear behaviour of beam-column connections [20], to clarify the mechanism of shear resistance and develop methods for a rational design of connections. Paulay in paper [24] sets equilibrium conditions in the beam-column connection. In this paper he sets two principal mechanisms of resistance to shear forces in beam-column connection. Tsoukantas presented a paper [32] where the behaviour of prefabricated connections under seismic load was analyzed, and the behaviour of monolith connections compared. He proposed analytical expressions for predicting the response of connections exposed to seismic actions.

In paper [14] Jirasek presented modified matrices for the stiffness of beam element, as constituents of frames, where the effects of connection yielding were taken into account. The yielding of connections is introduced through modification of stiffness matrix. In London, at the 12th European Conference on Earthquake Engineering, Pampanini, Calvi and Moratti presented their experimental studies [21], aiming to determine the seismic vulnerability of RC beam-column connections. In [12], the authors treated the results of experimental investigations on 24 models of internal and external beam-column connections. Research-based models for the prediction of fracture/failure stress were developed.

In papers [4] and [5] authors are classified the experimental studies on four different types of prefabricated connections. The paper [4] gives an overview of experimental results, while [5] presents the analytical expressions for the determination of bearing capacity of the underlying prefabricated connections, taking into account the yielding of connections. In [1] are provided theoretical models for the calculation of plastic rotation capacity of connections in RC structures. Uma and Jain presented in [33], a critical review of recommendations for the design and construction of details in beam-column connections for RC frames exposed to seismic loads.

Experiences regarding the comparative experimental and numerical studies conducted in Tuzla in the period from 2005 to 2007 were presented in papers [34] and [35].

3. FRACTURE MECHANISMS IN RC STRUCTURES

Nonlinear behaviour of concrete is the result of two different micro structural changes occurring in materials: plastic flow and the development of micro-cracks and micro-cavities. With the application of the theory of plasticity, problems where the material is primarily under compression are successfully addressed. In cases where the tension-
compression plays an important role, the theory of plasticity is applied on the field of compression, while damage mechanics and fracture mechanics are applied on model areas subjected to tension.

By monitoring the crack developing mechanism, the predominant character of fracture is determined. Fracture mechanics considers the numerical models of crack developing and fracture state mechanisms. It is based on the principle that all materials include initial defects in the form of cracks and cavities, which may affect the bearing capacity of the structure. The problem analysis in fracture mechanics consists of the analysis of stress distribution around the crack and the crack development.

The effect of size of the structural element in fracture mechanics is mainly not taken into account by current standards. According to the criterion of allowable stress, or the theory of plasticity, the ultimate nominal stress is the same regardless the beam height, e.g. the failure of structures does not depend on its size. Fracture mechanics foresees the effect of the structure size on the fracture load, as well as on the ductility, and is associated with the energy that is released and results with stress redistribution in the fracture zone [16]. The analysis of RC structures is mainly based on the theory of nonlinear fracture mechanics, which takes into account the behaviour of descending branch of the stress-deformation diagram. An important feature is the size of the nonlinear fracture zone (the zone of softening) at the root of the current incision or the actual crack (Fig. 1). The first nonlinear theory of concrete fracture mechanics is proposed by Hilleborg and Peterson in 1976 and is known as the fictitious crack model (FCM, Fig. 2).

While in the theory of elasticity, material strength and yielding are criteria defining the fracture, in fracture mechanics material failure occurs as the result of crack instability. Griffith, one of the first theoreticians of fracture mechanics, found that the fracture strength in the case of brittle fracture is inversely proportional to square root of the crack size.
Figure 3 show the development of cracks in concrete, where two zones can be distinguished: the open crack zone and the damaged zone in front of an open crack, which is called the fracture process zone (FPZ), and plays an important role in the crack development analysis [9]. Within this zone, several micro-defect mechanisms occur, including micro-cracks in the cement stone, failure of adhesion between the cement and the aggregate, crack deflection and crack branching. All these mechanisms contribute to the development of fracture energy.

The approach which is often used in the crack analysis in concrete is the use of tensile strength of concrete and its tensile capacity [10]. When a homogeneous simple concrete element, subjected to uniaxial load, is divided into finite elements, the stress-strain ratio can be defined by the material softening curves (Fig. 4). The area beneath the curve presented in Figure 4 is defined as the fracture energy ($G_F$), i.e. the energy required to form a complete crack. One of the methods for estimating the fracture energy is proposed in the CEB-FIB Model Code 1990 [3], where it is presented as a function of compressive strength of the concrete and the maximum aggregate size. However, recent studies have shown that the value of fracture energy is relatively independent of concrete strength and the size of aggregate [10].

Hillerborg proposed the expression for the characteristic length of the crack zone:

$$l_{ch} = G_F E_c / f_{ct}^2$$  \(1\)

Hillerborg proposed the expression for the characteristic length of the crack zone:
The mechanism of crack initiation in RC elements depends, among other things, on the position of reinforcement in relation to the crack. In addition to the properties of concrete that influence the crack initiation mechanism, such as the wedging action of aggregates in the crack, crack roughness, material structure at the site of crack initiation, and so on. The crack growth is influenced also by the manner of wedging of reinforcement into the crack, as well as the conditions of adhesiveness between the reinforcement and concrete that are mediated through the mechanisms of adhesion, mechanical interaction and friction [30].

The issue of building a mathematical model that defines the actual behaviour of RC structures requires the modelling of viscous elasto-plastic materials (concrete).

In [25] a general numerical model is presented for the calculation of crack width in RC concrete elements loaded with an eccentric longitudinal force. Thus, cross sections have arbitrary shape and arrangement of reinforcement and position of the axial force, with the possibility of sections coupling in several stages. Three numerical models were developed, depending on the adhesion stress diagram and reinforcement slipping (τ - Δ).

To define the mechanism of bearing capacity requires defining the slides function:

$$\Delta = 2 \frac{w_{i}^i}{(s_{i}^i)^2} x^i$$

where:

- $w_{i}^i$ - crack width at the observed rod
- $s_{i}^i$ - the theoretic minimum length along which adhesion stresses are transferred
The sliding of rod at the crack location can be determined according the following expression:

$$\Delta = \frac{w_i}{2}$$

$$\frac{w_i}{2}$$

(3)

**Fig. 7.** Geometric characteristics of two consecutive cracks, after [25]

In [27], the modelling of shear force at the node $V_j$ is proposed, which represents an internal horizontal force through the middle of height of the joint core (Fig. 8).

**Fig. 8.** Definition of the horizontal shear force in the RC beam-column connections, after [27]

The question remains as to define a reliable value of $T$ and $T'$ due to the effects of the horizontal shear force in the joint, so that this value is estimated through the proposed numerical models.
Shear deformations in the joint occur due to rotation of its elements. They are concentrated on the border between the diagonal crack and the crack due to bending, which is already formed as a result of action shear stress or stress due to bending from the previous loading cycle, as described in Figure 9. The beam-column joint is usually with such cracks, except in certain special cases when there is a loss in the adhesion in the reinforcement in the beam.

Englekirk in [6] explained the mechanism of load transfer through the external beam joint and the column (Fig.10).

In his research, Shiohara presented a mathematical model of monolithic RC beam-column connection for the case of internal and external joints, depending on the modes of failure (joint or beam failure) [28]. In his paper he recommends the beam-column connection designed as the shear deformation at the joint is less than in the connected elements. The mathematical models were developed accordingly. In these models, two sets of critical areas were considered which are connected by means of two independent types of strain, called J-model and B-model (Fig. 11).
The proper approach in the development of mathematical models of cracked concrete structures is the modelling the bearing capacity of non cracked concrete part of the structure and the reinforcement in the cracked part of the structure through modelling the adhesion of concrete and reinforcement. In addition, global analyses are performed using approximate engineering models with assumed stress-deformation conditions, while for the purpose of local analysis, the mechanism of crack development and the stress state in the vicinity of the crack needs to be defined.
4. MATHEMATICAL MODELS OF FAILURE OF PREFABRICATED CONNECTIONS

Mathematical models of prefabricated connections are more complex and less defined because the model is almost impossible to unify, i.e. the working mechanism can be experimentally defined only for a specific connection. Therefore, the study of working mechanism of prefabricated connections is reduced to comparisons with monolithic connections, i.e. the definition of yielding connections with respect to monolithic connections. A real computational model in the analysis of connection of prefabricated elements implies the introduction of semi-rigid connections.

In [4], on the example of prefabricated construction of skeletal structures of residential and commercial buildings, the yielding of connections is analyzed through the consideration and definition of the following relevant parameters:

- Ultimate bending moment $M_U$.
- Rotational stiffness $S$.
- Ultimate capacity of rotational ductility $\Phi_U$.

An ideally rigid connection is based on the assumption that in case of deformation, the angle closed by the beam and the column, or the wall and the floor structures, in the connection remains unchanged after deformation. In experiments carried out in the papers [4] and [5], prefabricated connections are found to have an area within which a relative rotation of the beam (plate) occurs in relation to rotation of the column (wall). This area is called the joint area (zone), the length of which usually corresponds to height of the structural element (beam or plate). When analyzing and processing the experimental data, the task to separate the relative rotation within the joint from the rotation due to deformability of the beam (plate) is highly complex, and sometimes impossible.

According to [4], the overall rotation of the prefabricated joint consists of:

a) rotation of the plate-wall connecting surface;
b) rotation due to deformation of the plate end within the joint zone;
c) rotation of the wall within the joint zone.
Rotational stiffness can be defined by the following expression:

\[ S_t = \frac{M_{RC}}{\phi_c} \]

where:
\( \phi_c \) – relative rotation of the end due to \( M_{RC} \)

The experimental results presented in [11] show that the crack initiation mechanism depends on the ratio between the elasticity modulus of the concrete in the prefabricated elements and the concrete in the joint (\( m_E \)). If \( m_E < 0.9 \) or \( m_E > 1.12 \), cracks occur in the plate-joint connection. If \( m_E = 0.9-1.12 \), the crack occurs either in the joint or in plates.

If the expression \( \beta = \varepsilon_r/\varepsilon_f \) denotes the ratio of deformations in the zone of restrain at the moment of crack initiation, then the following conclusion can be drawn [11]:
- if \( \beta < 5-8 \), an elastic type of crack occurs, possibly due to compression of concrete;
- if \( \beta > 10 \), a plastic type of crack occurs due to the reinforcement or its anchoring;
- if \( \beta = 8-12 \), the crack occurs either because of the concrete or the steel, therefore cross section can be optimized.

In modelling the plastic behaviour, it is assumed that the inelastic behaviour is located within the discrete critical areas (on the contact surfaces of joint elements). In the case of monolithic joints, cracks due to bending are expected in the structural elements; in the case of prefabricated systems, the inelastic behaviour is concentrated at the contact surface between the elements, while structural elements are supposed to remain in the elastic state and can handle a limited damage. Joints can be modelled by using inelastic rotational springs of corresponding hysteresis behaviour, while the elastic rheological elements (springs) are used to model the structural elements.

In [22], an analytical model of non-linear behaviour of structural elements of joints is proposed. The equivalent rotation of spring defines the relative rotation in the joint, and is adopted to describe the joint behaviour in the linear and non-linear range. As shown in Figure 14, beam and column elements connected in the joint are modelled as one-dimensional frame elements with the concentration of inelasticity in the critical area of the joint, defined using the appropriate moment-curvature curves.
For the analysis of floor slab-wall connections in the CIB Report [2], the calculation models are shown in Figure 15.

**Fig. 14.** The proposed analytical model of behaviour of joint with a rotational spring, after [22]

Flexural stiffness $K_{sup}$ takes into account the joint stiffness, as well as stiffness of the associated walls and slabs.

$$K_{sup} = \frac{l}{\frac{l}{K_{wb} + K_{wt}} + \frac{l}{K_r}}$$  \hspace{1cm} (5)

**Fig. 15.** Calculation model for the analysis of floor slab-wall connections, after [2]

In the period from 2002 to 2004, Kim, Stanton and MacRae developed a connection model, where, using the GAP link elements, they modelled crack initiation in RC or
prefabricated concrete (Fig. 16). It was necessary for the model to describe the behaviour of the analyzed samples realistically, including the position of neutral axis, changes in strength due to the variation of normal forces in the beam, as well as crack initiation in the beam in the vicinity of the beam-column contact [17]. In the selected model, the behaviour of the connection is assumed as rigid. The transfer of shear forces between the beam and column is modelled with axial stiff elements that transmit vertical forces between joints b5-c5 and b6-c6. Horizontal deformation occurs freely between these joints.

A group of authors [29] proposed the simulation of contact zone in the connection with "multi spring" contact elements. Figure 17 shows the principle of setting the springs in case of application of 10 contact springs. Contact springs are pressed and have no tension stiffness. They are distributed along the full height of the contact surface.

Fig. 16. A mathematical model of a prefabricated beam-column joint, according to [17]

Fig. 17. The principle of placing the contact elements with ten contacts springs [29]
5. THE PREFABRICATED PLATE-MONOLITHIC WALL CONNECTION

Based on the analyzed and previously presented experimental and numerical researches with the proposed mathematical models of monolithic and prefabricated RC connections, the analysis of a specific prefabricated connection of RC prefabricated plate and RC monolithic wall was performed in the 1980's.

Results of experimental and numerical studies, presented in [34], [35] and [36] were used to formulate the proposed mathematical model of failure (ultimate limit state) of the underlying prefabricated connection. Working diagrams (M-\(\Phi\)) of comparative monolithic and prefabricated models, shown in Figure 18a and 18b were obtained experimentally.

Fig. 18. a) Comparative momentum-relative rotation curves; b) Extension of the experimental curve for the prefabricated model
Working diagrams shown in the previous figures are used for modelling the connection zone of a monolithic RC wall and a prefabricated RC plate. Yielding of the tested prefabricated connection compared to the monolithic connection is defined through the relative rotation, as shown in Figure 19. On that occasion, the stiffness matrix is modified through the introduction of joint yielding.

For modelling the structural failure, the concept of residual stiffness is introduced. In [15], a numerical analysis of prefabricated structures from the aspect of variable properties of materials in the joint is presented, as well as from the aspect of variable loading characteristics. Horizontal and vertical joints are analyzed. In this analysis, the value of residual stiffness, as shown in Figure 20, is introduced.

**Fig. 19. Values of relative shift of the prefabricated connection**

**Fig. 20. Determination of residual stiffness [15]**
For the specific example of prefabricated RC wall - monolithic RC wall connection, residual stiffness is defined as shown in Figure 21.

![Fig. 21. Residual stiffness of the prefabricated connection - $S_0^R$.](image)

The mathematical model is developed using the exact method of displacement (beam elements) and the finite element method.

![Fig 22. a) Prefabricated model with BEAM elements; b) Prefabricated model with SHELL elements](image)

Yielding of the connection is modelled by using the LINK elements. Models with BEAM elements use a single-node connection, i.e. a single LINK element and the serial connection of two LINK elements. In models with SHELL elements, a multi-node connection is used, i.e. a parallel connection with multiple LINK elements (Fig. 22).
The results of comparative analysis of specific mathematical models are presented in Figure 23.

Fig. 23. Comparison of the calculated support momentum - external load curves, obtained by numerical analysis.

From the above comparative diagrams, it is obvious that through the modification of stiffness matrix and the matrix of equivalent loading, i.e. the introduction of LINK elements, it is possible to model the impact of the connections working mechanism on the behaviour of the structure. It is important to note that the definition of the connections working mechanism requires experimental research in order to define a reliable model. In modelling the working diagram in the area of maximum loading and in the descending part of the diagram (the failure area), numerical models can be improved through the combination of several serial or parallel connected LINK elements.

6. CONCLUSIONS

Based on the literature which was studied, it can be said that the innovation of standard, theoretical considerations and research are nowadays focused on the explanation, understanding and defining the response of structures on seismic loading. Scientific principles were traditionally applied by their reduction to mathematical expressions, but two features of structural behaviour are particularly difficult to reduce to algorithmic form: ductility and shearing. Ductility is the key structural feature that is exposed to earthquake. Ductility, as the ability of post-elastic behaviour, serves to reduce seismic energy.

Shear strength and the development of effective shear transfer mechanism are very important for the development of an adequate flow of seismic forces. The complex mechanism of shear transfer in monolithic structures was reduced to the beam theory, which in some cases, especially in prefabricated concrete structures, is not appropriate. Understanding mechanism of shear transfer and its limit states is crucial for the understanding and definition of stress flow in prefabricated concrete systems.

The theory of elasticity is the basis for the development and use of most design procedures. In some cases, the material behaviour is primarily elastic and the assumptions of the
theory of elasticity are correct. Where the assumption of elastic behaviour is not acceptable, the idealized model of plasticity (ultimate load) should be used. The effective transfer of shearing is crucial for the proper behaviour of structure, particularly in structures exposed to cyclic loading and structures in the region of plasticity. Degradation if investigated precast connection start at load intensity grater then serviceability load, and it is insignificant up to ultimate load. Yielding of connection rigidity increase in the range over 1.33 $P_{service}$ [34].

REFERENCES


Čvrstoća veza u montažnim betonskim konstrukcijama

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Ključne reči: armiranobetonski montažni spojevi, eksperimentalna i numerička istraživanja, pouzdanost, mehanizam otkaza, matematički model.