GENERAL CONFORMAL ALMOST SYMPLECTIC
N-LINEAR CONNECTIONS
IN THE BUNDLE OF ACCELERATIONS

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Abstract

The aim of this paper\(^1\) is to find the transformation for the coefficients of an N-linear connection on \(E = \text{Osc}^2 M\), by a transformation of nonlinear connections, to define in the bundle of accelerations the general conformal almost symplectic N-linear connection notion and to determine the set of all general conformal almost symplectic N-linear connections on \(E\). We treat also some special classes of general conformal almost symplectic N-linear connections on \(E\).

1 Introduction

The literature on the higher order Lagrange spaces geometry highlights the theoretical and practical importance of these spaces: \([4] \text{–} [7]\).

Motivated by concrete problems in variational calculation, higher order Lagrange geometry, based on the \(k\)-osculator bundle notion, has witnessed a wide acknowledgment due to the papers: \([4] \text{–} [7]\), published by Radu Miron and Gheorghe Atanasiu.

The geometry of \(k\)-osculator spaces presents not only a special theoretical interest, but also an applicative one.

Due to its content, the present paper continues a trend of interest with a long tradition in the modern differential geometry, i.e. the study of remarkable geometrical structures.

In the present paper we find the transformations for the coefficients of an N-linear connection on \(E = \text{Osc}^2 M\), by a transformation of nonlinear connections, (§2).

We define the general conformal almost symplectic N-linear connection notion on \(E\), (§3).

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We determine the set of all these connections and we treat also some special classes of general conformal almost symplectic \(N\)-linear connections on \(E\), (§4).

This paper is a generalization of the papers: [10] − [13]. Concerning the terminology and notations, we use those from: [4], [9], which are essentially based on M. Matsumoto’s book: [2].

2 The set of the transformations of \(N\)-linear connections in the 2-osculator bundle

Let \(M\) be a real \(n\)-dimensional \(C^\infty\)-manifold and let \((\text{Osc}^2M, \pi, M)\) be its 2-osculator bundle, with \(E = \text{Osc}^2M\) the total space.

The local coordinates on \(E\) are denoted by: \((x^i, y^{(1)i}, y^{(2)i})\), briefly: \((x, y^{(1)}, y^{(2)})\).

If \(N\) is a nonlinear connection on \(E\), with the coefficients \(N_{(1)i}^j, N_{(2)i}^j\), then let \(D\) be an \(N\)-linear connection on \(E\), with the coefficients \(D\Gamma(N) = (L^i_{jk}, C_{(1)jk}^i, C_{(2)jk}^i)\).

If \(\overline{N}\) is another nonlinear connection on \(E\), with the coefficients \(\overline{N}_{(1)i}^j(x, y^{(1)}, y^{(2)}), \overline{N}_{(2)i}^j(x, y^{(1)}, y^{(2)})\), then there exist a uniquely determined tensor fields \(A_{(\alpha)i}^j \in \tau^1(E), (\alpha = 1, 2)\) such that:

\[
(2.1) \quad \overline{N}_{(\alpha)i}^j = N_{(\alpha)i}^j - A_{(\alpha)i}^j, \quad (\alpha = 1, 2).
\]

Conversely, if \(N_{(\alpha)i}^j\) and \(A_{(\alpha)i}^j\) \((\alpha = 1, 2)\) are given, then \(\overline{N}_{(\alpha)i}^j, (\alpha = 1, 2)\), given by (2.1) is a nonlinear connection.

Let us suppose that the mapping \(N \to \overline{N}\) is given by (2.1).

According to Cap.III, §3.3, [4], we have:

\[
D_{(\alpha)i}^j \frac{\delta}{\delta y^{(\alpha)i}} = L^i_{jk} \frac{\delta}{\delta y^{(\alpha)j}}, \quad D_{(\beta)i}^j \frac{\delta}{\delta y^{(\beta)j}} = C_{(\beta)jk}^i \frac{\delta}{\delta y^{(\alpha)j}},
\]

\[
(\beta = 1, 2; \alpha = 0, 1, 2; y^{(0)i} = x^i) \quad \text{and} \quad \frac{\overline{\partial}}{\overline{\partial}x^i} = \overline{\frac{\partial}{\partial x^i}} - \overline{N}_{(1)i}^j \frac{\partial}{\partial y^{(1)j}} - \overline{N}_{(2)i}^j \frac{\partial}{\partial y^{(2)j}}, \quad \overline{\frac{\partial}{\partial y^{(1)i}}} = \frac{\partial}{\partial y^{(1)i}} - \overline{N}_{(1)i}^j \frac{\partial}{\partial y^{(1)j}} - \overline{N}_{(2)i}^j \frac{\partial}{\partial y^{(2)j}} = \overline{\frac{\partial}{\partial y^{(2)i}}}.
\]

It follows first of all that the transformation (2.1) preserve the coefficients \(C_{(2)jk}^i\).
Taking in account the fact that:

\[ \frac{\delta}{\delta x^i} = \frac{\delta}{\delta x^i} + A_{(1) i} \frac{\partial}{\partial y^{(1)i}} + A_{(2) i} \frac{\partial}{\partial y^{(2)i}}, \quad \frac{\delta}{\delta x^j} = \frac{\delta}{\delta x^j} = \frac{\delta}{\delta x^j} + A_{(1) j} \frac{\partial}{\partial y^{(1)j}} + A_{(2) j} \frac{\partial}{\partial y^{(2)j}}, \]

it follows:

\[ D \frac{\delta}{\delta x^i} \frac{\delta}{\delta y^{(2)j}} = D \frac{\delta}{\delta y^{(1)i}} \frac{\partial}{\partial y^{(2)j}} = \tilde{L}_{jk} \frac{\partial}{\partial y^{(2)j}} = D \left( \frac{\delta}{\delta x^i} + A_{(1) i} \frac{\partial}{\partial y^{(1)i}} + A_{(2) i} \frac{\partial}{\partial y^{(2)i}} \right) \frac{\partial}{\partial y^{(2)j}} = \]

\[ = D \frac{\delta}{\delta x^i} \frac{\partial}{\partial y^{(2)j}} + A_{(1) i} \frac{\partial}{\partial y^{(1)i}} \frac{\partial}{\partial y^{(2)j}} + A_{(2) i} \frac{\partial}{\partial y^{(2)j}} \frac{\partial}{\partial y^{(2)j}} = \]

\[ = L^{i}_{jk} \frac{\partial}{\partial y^{(2)j}} + A_{(1) i} \frac{\partial}{\partial y^{(1)i}} C^{i}_{(1)jl} \frac{\partial}{\partial y^{(2)j}} + A_{(1) i} \frac{\partial}{\partial y^{(2)j}} N^{m}_{(1) i} C^{i}_{(2)jm} \frac{\partial}{\partial y^{(2)j}} + A_{(2) i} \frac{\partial}{\partial y^{(2)j}} C^{i}_{(2)jl} \frac{\partial}{\partial y^{(2)j}} = \]

\[ = \left( L^{i}_{jk} + A_{(1) i} \frac{\partial}{\partial y^{(1)i}} C^{i}_{(1)jl} + A_{(2) i} \frac{\partial}{\partial y^{(2)j}} C^{i}_{(2)jl} \right) \frac{\partial}{\partial y^{(2)j}}. \]

Therefore the change we are looking for is:

\[
\begin{align*}
\frac{\delta}{\delta y^{(2)j}} \frac{\partial}{\partial x^{i}} & = \tilde{\Gamma}^{i}_{jk} + \left( L^{i}_{jk} + A_{(1) i} \frac{\partial}{\partial y^{(1)i}} C^{i}_{(1)jl} + A_{(2) i} \frac{\partial}{\partial y^{(2)j}} C^{i}_{(2)jl} \right) \frac{\partial}{\partial y^{(2)j}}, \\
& = \tilde{\Gamma}^{i}_{jk} + C^{i}_{(1)jk} + A_{(1) i} \frac{\partial}{\partial y^{(1)i}} C^{i}_{(2)jl} + A_{(2) i} \frac{\partial}{\partial y^{(2)j}} C^{i}_{(2)jl}.
\end{align*}
\]

So, we have proved:

**Proposition 2.1** The transformation (2.1) of nonlinear connections imply the transformations (2.2) for the coefficients \( D\Gamma(N) = (L^{i}_{jk}, C^{i}_{(1)jk}, C^{i}_{(2)jk}) \) of the N-linear connection \( D \).

Now, we can prove:

**Theorem 2.1** Let \( N \) and \( \tilde{N} \) be two nonlinear connections on \( E \), with the coefficients \( (N^{i}_{(1) j}, N^{i}_{(2) j}), (\tilde{N}^{i}_{(1) j}, \tilde{N}^{i}_{(2) j}) \)-respectively. If \( D\Gamma(N) = (L^{i}_{jk}, C^{i}_{(1)jk}, C^{i}_{(2)jk}) \) and \( D\Gamma(\tilde{N}) = (\tilde{L}^{i}_{jk}, \tilde{C}^{i}_{(1)jk}, \tilde{C}^{i}_{(2)jk}) \) are two \( N \)-, respectively \( \tilde{N} \)-linear connections on the differentiable manifold \( E \), then there exists only one quintet of tensor fields \( (A_{(1) i}^{j}, A_{(2) i}^{j}, B^{i}_{jk}, D_{(1) i}^{j}, D_{(2) i}^{j}) \) such that:
\[ \begin{align*}
N_{(\alpha)}^i{}_j &= N_{(\alpha)}^i{}_j - A_{(\alpha)}^i{}_j, \quad (\alpha = 1, 2),
L^i_{jk} &= L^i_{jk} + A_{(1)}^i{}_k C_{(1)}^i{}_j + A_{(1)}^i{}_k N_{(1)}^m C_{(2)}^i{}_m + A_{(2)}^i{}_k C_{(2)}^i{}_j - B^i_{jk},
C_{(1)}^i{}_{jk} &= C_{(1)}^i{}_{jk} + A_{(1)}^i{}_k C_{(2)}^i{}_j - D_{(1)}^i{}_{jk},
C_{(2)}^i{}_{jk} &= C_{(2)}^i{}_{jk} - D_{(2)}^i{}_{jk}.
\end{align*} \tag{2.3} \]

**Proof.** The first equality (2.3) determines uniquely the tensor fields \( A_{(\alpha)}^i{}_j \), \( (\alpha = 1, 2) \), since \( C_{(\alpha)}^i{}_{jk} \), \( (\alpha = 1, 2) \) are tensor fields, the second equation (2.3) determines uniquely the tensor field \( B^i_{jk} \). Similarly the third and the fourth equation (2.3) determine the tensor fields \( D_{(1)}^i{}_{jk} \) and \( D_{(2)}^i{}_{jk} \) respectively.

We have immediately:

**Theorem 2.2** If \( D\Gamma(N) = (L^i_{jk}, C_{(1)}^i{}_{jk}, C_{(2)}^i{}_{jk}) \) are the coefficients of an \( N \)-linear connection \( D \) on \( E \) and \((A_{(1)}^i{}_j, A_{(2)}^i{}_j, B^i_{jk}, D_{(1)}^i{}_{jk}, D_{(2)}^i{}_{jk})\) is a quintet of tensor fields on \( E \), then: \( D\Gamma(\overline{N}) = (\overline{L}^i_{jk}, \overline{C}_{(1)}^i{}_{jk}, \overline{C}_{(2)}^i{}_{jk}) \) given by (2.3) are the coefficients of an \( \overline{N} \)-linear connection \( \overline{D} \) on \( E \).

The tensor fields \((A_{(1)}^i{}_j, A_{(2)}^i{}_j, B^i_{jk}, D_{(1)}^i{}_{jk}, D_{(2)}^i{}_{jk})\) are called the difference tensor fields of \( D\Gamma(N) \) to \( D\Gamma(\overline{N}) \) and the mapping \( D\Gamma(N) \rightarrow D\Gamma(\overline{N}) \) given by (2.3) is called a transformation of \( N \)-linear connection to \( \overline{N} \)-linear connection, [2].

### 3 The notion of general conformal almost symplectic \( N \)-linear connection in the bundle of accelerations

Let \( M \) be a real \( n = 2n' \)-dimensional \( C^\infty \)-manifold and let \((Osc^2 M, \pi, M)\) be its 2-osculator bundle. The local coordinates on the total space \( E = Osc^2 M \) are denoted by \((x^i, y^{(1)}{}^i, y^{(2)}{}^i)\).

We consider on \( E \) an almost symplectic \( d \)-structure, defined by a \( d \)-tensor field of the type \((0, 2)\), let us say \( a_{ij}(x^i, y^{(1)}{}^i, y^{(2)}{}^i) \), alternate:

\[ a_{ij}(x, y^{(1)}, y^{(2)}) = -a_{ji}(x, y^{(1)}, y^{(2)}), \tag{3.1} \]

and nondegenerate:
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We associate to this $d$-structure Obata’s operators:

$$
\Phi^{ir}_{sj} = \frac{1}{2}(\delta^i_r \delta^s_j - a_{sj} a^{ir}), \Phi^{*ir}_{sj} = \frac{1}{2}(\delta^i_r \delta^s_j + a_{sj} a^{ir}),
$$

where $(a^{ij})$ is the inverse matrix of $(a_{ij})$:

$$a_{ij} a^{jk} = \delta^k_i.$$

Obata’s operators have the same properties as the ones associated with the metrical $d$-structure on $E$, [8].

Let $\mathcal{A}_2(E)$ be the set of all alternate $d$-tensor fields of the type $(0,2)$ on $E$. As is easily shown, the relation for $b_{ij}, c_{ij} \in \mathcal{A}_2(E)$ defined by:

$$b_{ij} \sim c_{ij} \iff \{\exists \rho(x, y^{(1)}, y^{(2)}) \in \mathcal{F}(E) | b_{ij} = e^{2\rho} c_{ij}\}$$

is an equivalent relation on $\mathcal{A}_2(E)$.

**Definition 3.1** The equivalent class: $\hat{a}$ of $\mathcal{A}_2(E)/\sim$, to which the almost symplectic $d$-structure $a_{ij}$ belongs, is called a conformal almost symplectic $d$-structure on $E = \text{Osc}^2 M$.

Every $a'_{ij} \in \hat{a}$ is a $d$-tensor field alternate and nondegenerate, expressed by:

$$a'_{ij} = e^{2\rho} a_{ij}.$$

Obata’s operators are defined for $a'_{ij} \in \hat{a}$ by putting $(a''_{ij}) = (a'_{ij})^{-1}$. Since equation (3.6) is equivalent to:

$$a''_{ij} = e^{-2\rho} a^{ij},$$

we have

**Proposition 3.1** Obata’s operators depend on the conformal almost symplectic $d$-structure $\hat{a}$, and do not depend on its representative $a'_{ij} \in \hat{a}$.

Let $N$ be a nonlinear connection on $E$ with the coefficients $(N_{(1)}^{i}_{j}, N_{(2)}^{i}_{j})$ and let $D$ be an $N$-linear connection on $E$ with the coefficients in the adapted basis $\{\frac{\partial}{\partial x^i}, \frac{\partial}{\partial y^{(1)}}, \frac{\partial}{\partial y^{(2)}}\} : D\Gamma(N) = (L_{jk}^i, C_{(1)jj}^i, C_{(2)jj}^i)$.
Definition 3.2 An N-linear connection $D$ on $E$, is said to be a general conformal almost symplectic N-linear connection on $E$, if it verifies the following relations:

\[(3.8) \quad a_{ijk} = K_{ijk}, \ a_{ij} \ |_{k} = Q_{(\alpha)ijk}, \ (\alpha = 1, 2),\]

where $K_{ijk}, \ Q_{(\alpha)ijk}, \ (\alpha = 1, 2)$, are tensor fields of the type $(0, 3)$, having the properties of antisymmetry in the first two indices:

\[(3.9) \quad K_{ijk} = -K_{jik}, \ Q_{(\alpha)ijk} = -Q_{(\alpha)jik}, \ (\alpha = 1, 2),\]

\[|, \ denote \ the \ h-\ and \ \nu_{\alpha}-covariant \ derivatives, \ (\alpha = 1, 2), \ with \ respect \ to \ D\Gamma(N).\]

Particularly, we can give:

Definition 3.3 An N-linear connection $D$ on $E$, for which there exists a 1-form $\omega$ in $\mathcal{X}^*(\text{Osc}^2M), (\omega = \tilde{\omega}_i dx^i + \tilde{\omega}_{(1)i} \delta y^{(1)i} + \tilde{\omega}_{(2)i} \delta y^{(2)i})$ such that:

\[(3.10) \quad a_{ijk} = 2\tilde{\omega}_k a_{ij}, \ a_{ij} \ |_{k} = 2 \tilde{\omega}_{(\alpha)k} a_{ij}, \ (\alpha = 1, 2),\]

where $|, \ denote \ the \ h-\ and \ \nu_{\alpha}-covariant \ derivatives \ (\alpha = 1, 2)$ with respect to $D\Gamma(N)$, is said to be compatible with the conformal almost symplectic structure $\hat{a}$, or a conformal almost symplectic N-linear connection on $E$ with respect to the conformal almost symplectic structure $\hat{a}$, corresponding to the 1-form $\omega$, and is denoted by: $D\Gamma(\omega)$.

For any representative $a'_{ij} \in \hat{a}$ we have:

Theorem 3.1 For $a'_{ij} = e^{2\rho}a_{ij}$, a conformal almost symplectic N-linear connection with respect to $\hat{a}$, corresponding to the 1-form $\omega$, $D\Gamma(\omega)$ satisfies:

\[(3.11) \quad a'_{ijk} = 2\tilde{\omega}'_k a_{ij}, \ a'_{ij} \ |_{k} = 2 \tilde{\omega}'_{(\alpha)k} a_{ij}, \ (\alpha = 1, 2),\]

where $\omega' = \omega + d\rho$.

Since in Theorem 3.1, $\omega' = 0$ is equivalent to $\omega = d(-\rho)$, we have:
Theorem 3.2 A conformal almost symplectic N-linear connection with respect to $\hat{a}$, corresponding to the 1-form $\omega$, $D\Gamma(\omega)$, is an almost symplectic N-linear connection with respect to some $a'_{ij} \in \hat{a}$, (i.e. $a'_{ij} | _k = 0$, $\left(\alpha = 1, 2\right)$), if and only if $\omega$ is exact.

4 The set of all general conformal almost symplectic N-linear connections in the bundle of accelerations

Let $N$ and $\tilde{N}$ be two nonlinear connections on $E = \text{Osc}^2 M$, with the coefficients $(N^{(1)}_i j, N^{(2)}_i j)$ and $(N^{(1)}_i j, N^{(2)}_i j)$ respectively.

Let $D \Gamma (N) \equiv \left(0 \overset{0}{L}_{jk}, 0 \overset{0}{0}_{(1)jk}, 0 \overset{0}{C}_{(2)jk}\right)$, be the coefficients of an arbitrary fixed $N$-linear connection on $E$. Then any N-linear connection on E, with the coefficients $D\Gamma(N) = \left(L_{jk}, C_{(1)jk}, C_{(2)jk}\right)$, can be expressed in the form (2.3), taking $D\Gamma(\tilde{N})$ for $D\bar{\Gamma}(\tilde{N})$ and $D \Gamma (N)$ for $D\Gamma(N)$, where $(A_{(1)}^i j, A_{(2)}^i j, B_{jk}, D_{(1)jk}, D_{(2)jk})$ is the difference tensor fields of $D \Gamma (N)$ to $D\Gamma(N)$.

In order that $D\Gamma(N)$ is a general conformal almost symplectic N-linear connection on E, that is (3.8) holds for $D\Gamma(N)$, it is necessary and sufficient that $B_{jk}, D_{(1)jk}, D_{(2)jk}$ satisfy:

$$
\Phi^{ir}_{sj} D^{s}_{rk} = -\frac{1}{2}a^{im}_{\alpha} \left[ a_{mj} | _k + A_{(1)_l}^l a_{mj} \right] \left| _l \right.
$$

$$
\left. + (A_{(2)_l}^l + A_{(1)_l}^l N^{(2)}_{r} | _l a_{mj} \right| _l - K_{mjk}],
$$

$$
\left(4.1\right)
$$

$$
\Phi^{ir}_{sj} D^{s}_{(1)rk} = -\frac{1}{2}a^{im}_{\alpha} \left[ a_{mj} | _k + A_{(1)_l}^l a_{mj} \right] \left| _l - Q^{(1)_{mjk}},
$$

$$
\Phi^{ir}_{sj} D^{s}_{(2)rk} = -\frac{1}{2}a^{im}_{\alpha} \left[ a_{mj} | _k - Q^{(2)_{mjk}},
$$

where $\bar{\Gamma}$ and $\tilde{\Gamma}$, $(\alpha = 1, 2)$, denote the h- and $v_{\alpha}$-covariant derivatives,
(α = 1, 2), with respect to $D_0^0 \Gamma (N)$.

Thus, we have:

**Proposition 4.1** Let $D_0^0 \Gamma (N)$ be a fixed $N$-linear connection on $E$. Then the set of all general conformal almost symplectic $N$-linear connections, $D\Gamma(N)$ is given by (2.3), where $B_{jk}^i$, $D_{(\alpha)jk}^i$, ($\alpha = 1, 2$), are arbitrary tensor fields satisfying (4.1). Especially, if $D_0^0 \Gamma (N)$ is a general conformal almost symplectic $N$-linear connection, then (4.1) becomes:

\[
\begin{align*}
\Phi^{*ir} B_{stk}^s &= -\frac{1}{2} a^{im}[A_{(1) k}^l a_{mj}^l]_t + (A_{(2) k}^l + A_{(1) k}^r N_{(1)}^l a_{mj}^l)_t, \\
\Phi^{*ir} D_{(1)rk}^s &= -\frac{1}{2} a^{im} A_{(1) k}^l a_{mj}^l, \\
\Phi^{*ir} D_{(2)rk}^s &= 0,
\end{align*}
\]

(4.2)

From Theorem 5.4.3[4], however, the system (4.1) has solutions in $B_{jk}^i$, $D_{(\alpha)jk}^i$, ($\alpha = 1, 2$). Substituting in (2.3) from the general solution we have:

**Theorem 4.1** Let $D_0^0 \Gamma (N)$ be a fixed $N$-linear connection on $E$. The set of all general conformal almost symplectic $N$-linear connections $D\Gamma(N)$ is given by:
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\[ L_{jk}^i = L_{jk}^i + X_{(1) \ l}^i \ C_{(1)jl}^i + X_{(1) \ l}^i N_{(1) \ m}^i C_{(2)jm}^i + X_{(2) \ l}^i k C_{(2)jl}^i + \]
\[ + \frac{1}{2} a^{im}[a_{mj}^0 + X_{(1) \ l}^i k a_{mj}^1] + (X_{(2) \ l}^i k + X_{(1) \ l}^i N_{(1) \ r}^i) a_{mj}^2 \]
\[ - K_{mjk} + \Phi^{ir}_{sj} X_{rsk}^s, \]
\[ C_{(1)jk}^i = C_{(1)jk}^i + C_{(2)jl}^i X_{(1) \ l}^i k + \frac{1}{2} a^{im}(a_{mj}^0 + X_{(1) \ l}^i k a_{mj}^1) - Q_{(1)mjk} + \Phi^{ir}_{sj} Y_{rsk}^s, \]
\[ C_{(2)jk}^i = C_{(2)jk}^i + \frac{1}{2} a^{im}(a_{mj}^0 + Q_{(2)mjk} + \Phi^{ir}_{sj} Y_{rsk}^s), \]

where \( N_{(\alpha)}^i \ j = N_{(\alpha)}^i \ j - X_{(\alpha)}^i \ j, X_{(\alpha)}^i \ j, X_{(\alpha)jk}^i, Y_{(\alpha)jk}^i, (\alpha = 1, 2) \) are arbitrary tensor fields, and \( 0, 1 \), denote the \( h \)-and \( v_{\alpha} \)-covariant derivatives, \( (\alpha = 1, 2) \), with respect to \( D_{0}^{0} \Gamma (N) \).

If we take a general conformal almost symplectic N-linear connection as \( D_{0}^{0} \Gamma (N) \), in Theorem 4.1, then (4.3) becomes:

\[ L_{jk}^i = L_{jk}^i + X_{(1) \ l}^i k C_{(1)jl}^i + X_{(1) \ l}^i k N_{(1) \ m}^i C_{(2)jm}^i + X_{(2) \ l}^i k C_{(2)jl}^i + \]
\[ + \frac{1}{2} a^{im}[X_{(1) \ l}^i k Q_{(1)mjl} + (X_{(2) \ l}^i k + X_{(1) \ l}^i k N_{(1) \ r}^i) Q_{(2)mjk} + \Phi^{ir}_{sj} X_{rsk}^s, \]
\[ C_{(1)jk}^i = C_{(1)jk}^i + C_{(2)jl}^i X_{(1) \ l}^i k + \frac{1}{2} a^{im}Q_{(2)mjl} + \Phi^{ir}_{sj} Y_{rsk}^s, \]
\[ C_{(2)jk}^i = C_{(2)jk}^i + \Phi^{ir}_{sj} Y_{rsk}^s, \]

where \( N_{(\alpha)}^i \ j = N_{(\alpha)}^i \ j - X_{(\alpha)}^i \ j, X_{(\alpha)}^i \ j, X_{(\alpha)jk}^i, Y_{(\alpha)jk}^i, (\alpha = 1, 2) \) are arbitrary tensor fields and \( 0, 1 \), denote the \( h \)-and \( v_{\alpha} \)-covariant derivatives, \( (\alpha = 1, 2) \), with respect to \( D_{0}^{0} \Gamma (N) \).
Observations 4.1.

(i) If we consider \( X_{(\alpha)}^i j = X_{(\alpha)}^i jk = Y_{(\alpha)}^i jk = 0, \) \((\alpha = 1, 2),\) then from (4.3) we obtain the set of all general conformal almost symplectic N-linear connections, corresponding to the same nonlinear connection \( N, [13]. \)

(ii) If we take \( K_{ijk} = 2a_{ij} \tilde{\omega}_k, Q_{(\alpha)}ijk = 2a_{ij} \dot{\omega}_{(\alpha)k}, \) \((\alpha = 1, 2),\) such that \( \omega = \tilde{\omega}_i dx^i + \dot{\omega}_{(1)i}^0 \delta y_{(1)i} + \dot{\omega}_{(2)i}^i \delta y_{(2)i} \) is a 1-form in \( \mathcal{X}^0(Osc^2 M), \) and if we preserve the nonlinear connection \( N, (i.e. N = \bar{N}), \) then from (4.3) we obtain the set of all conformal almost symplectic N-linear connections, corresponding to the same nonlinear connection \( N, [12]. \)

(iii) If we consider \( K_{ijk} = 0, Q_{(\alpha)}ijk = 0, \) \((\alpha = 1, 2),\) and if we preserve the nonlinear connection \( N, (i.e. N = \bar{N}), \) then from (4.3) we obtain the set of all almost symplectic N-linear connections, corresponding to the same nonlinear connection \( N, [11]. \)

(iv) Finally, if we preserve the nonlinear connection \( N, (i.e. N = \bar{N}) \) from (4.4), we obtain the transformations of general conformal almost symplectic N-linear connections, corresponding to the same nonlinear connection \( N, [13]. \)

References


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