CONSTRUCTIVE PROCEDURE FOR DETERMINATION OF ABSOLUTE CONIC FIGURE IN GENERAL COLLINEAR SPACES

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Abstract

When they are collinear, projective spaces set with five pairs of bivocally associated points are general. To map quadrics (II degree surfaces), in these spaces, the absolute conic was used. Geometrical position of all the absolute points in the infinitely distant plane of one space, i.e. an absolute conic of space cannot be graphically represented. To the infinitely distant planes are associated by the vanishing planes, and the absolute conics are associated by the conic in the vanishing planes, that is, figures of the absolute conics. Prior to mapping the quadrics, it is necessary to constructively determine the characteristics parameters such as the vanishing planes, axes and centers of space, and then the figures of the absolute conics, in the vanishing planes of both spaces. In order to constructively determine the figure of the absolute conic in the second space, a sphere in the first space was used, which maps into a rotating ellipsoid in the second space. The center of the sphere is on the axis of the first space, and the infinitely distant plane intersects it along the absolute conic. The associated rotational ellipsoid, whose center is on the axis of the second space is intersected by the vanishing plane of the first space along the imaginary circumference $a_I$, whose real representative is circumference $a_z$. The circumference $a_I$ is the figure of the absolute conic of the first space. General collinear spaces are presented in a pair of Monge’s projections.

1 Introduction

In order to simplify the mapping, it is necessary to determine the characteristic parameters in the general collinear (GC) spaces $\theta^1$ and $\theta^2$, set by five pairs of bivocally associated points $A_1, B_1, C_1, D_1, E_1$ and $A_2, B_2, C_2, D_2, E_2$, (fig.1) which are presented in a pair of Monge’s projections. The projective geometry methods, with the aid of cross ratio on the associated sequences of points, firstly determine the vanishing planes $N_1$ and $M_2$, which are associated to the infinitely distant planes

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Constructive procedure for determination of absolute conic in general collinear spaces

In the paper, the constructive determination of the figure of absolute conic in the space \(\theta^1\) was presented, and the same procedure can be used for determination of the figure of absolute conic in space \(\theta^2\). Only first Monge’s projections and planes of transformation for both spaces were presented.

In the space \(\theta^1\) the sphere \(s_1\) (fig. 2) has been selected, which intersects its infinitely distant plane \(M_1^\infty\) along the absolute conic \(a_{11}\), and the quadric associated to it in the space \(\theta^2\) is a rotational ellipsoid \(s_2\) which intersects the vanishing plane \(M_2\) along the imaginary circumference \(a_{12}\), whose real representative is the circumference \(a_{22}\), which is associated to the absolute conic of the space \(\theta^1\). In order to determine the figure of the absolute conic of the first space in the vanishing plane of the second space, it is necessary to firstly map the sphere into the rotational ellipsoid, and then determine its intersection with the vanishing plane \(M_2\).

The sphere \(s_1\) in the space \(\theta^1\) has been selected to map as simple as possible into the associated quadric in the space \(\theta^2\). Regarding that the points on the axes of space are mutually associated, the point \(O_1\) has been chosen as a center of the sphere, and one diameter \(1_12_1\), on the axis of the space \(\theta^1\), because with the aid of cross ratio on the associated axes \(\lambda = (O_1W_1^\infty1_21) = (O_2W_21_22)\), these points can be more simply mapped, an in this way one diameter of the associated quadric
Figure 1: determination of characteristic parameters in GC spaces
has been determined, that is the axis of the ellipse has been determined (contours of the rotational ellipsoid), that is lying on the axis $o_2$ of space $\theta^2$.

The point $O_2$ is the pole of the rotational ellipsoid $s_2$ in respect to the vanishing plane $M_2$ and it is associated to the center of the sphere $O_1$ which is a pole, in respect to the infinitely distant plane $M_1^\infty$. To the point $L_1$ which is a pole in respect to the vanishing plane $N_1$ in the space $\theta^1$ the point $L_2$ has been associated, which is a pole in respect to the infinitely distant plane $N_2^\infty$ of the space $\theta^2$. The point $L_2$ is the center of the rotational ellipsoid and it is obtained with the aid of dual relationship $\lambda = (O_1W_1^\infty L_1X_1) = (O_2W_2L_2X_2^\infty)$.

In order to accomplish association of the remaining points in both spaces, which are on the axes of space, the vanishing planes are brought into a ray position through transformation, so in these projections, the quadric contours are conics, the sphere $s_1$ is a circumference, and the rotational ellipsoid $s_2$ is an ellipse. The planes parallel to the vanishing plane $N_1$ in the space $\theta^1$ map in the planes parallel to the vanishing plane $M_2$ in space $\theta^2$ and in these projections they appear as rays.

The intersection of the straight lines parallel to the vanishing plane $N_1$, with the sphere $s_1$ in the space $\theta^1$ are circumferences, and for the mapping in the space $\theta^2$ two were selected, one through the center $O_1$, sphere $s_1$ of the diameter $3_14_1$, and the other through the pole $L_1$ of the diameter $5_16_1$. In the space $\theta^2$ the associated circumferences with the centers $O_2$ and $L_2$, are sought, whose diameters must be obtained by mapping the end points of the circumferences $3_1$, $A_1$, and $5_1, 6_1$ whose centers are $O_1$ and $L_1$, and their true size is obtained by rotation.

Through the point $3_1$, of the circumference $3_14_1$, in the space $\theta^1$ (fig. 2) a perpendicular line to the vanishing plane $N_1$ has been placed, and their piercing points through the planes $A_1B_1E_1$ and $A_1B_1D_1$, have been determined, those being $L_{j1}$ and $K_{j1}$ points. With the aid of cross ratio on the associated sequences of points, the points $L_{j2}$ and $K_{j2}$ in the space $\theta^2$ have been determined. From the dual relationship $\lambda = (3_1W_1^\infty L_{j1}K_{j1}) = (3_2W_2L_{j2}K_{j2})$, the point $3_2$, has been determined, which lies on the plane through the point $O_2$ that is parallel to the vanishing plane $M_2$. The radius of the associated circumference $3_24_2$, $O_23_2^\infty$ has been determined in the true size, by rotation, and the point symmetrical to $3_2^\infty$ is $o4_2^\infty$ in respect to $O_2$. The chord of the ellipse (contours of rotational ellipsoid $s_2$) $3_2^\infty4_2^\infty$ is the polar line for the pole $W_2$ in the vanishing plane $M_2$.

In the same manner the points $5_2^\infty$ and $6_2^\infty$ have been found, that determine the diameter (axis) of the ellipse $s_2$ (contour of rotational ellipsoid). Through the point $5_1$, of the circumference $5_16_1$, a perpendicular line to the vanishing plane $N_1$ in the space $\theta^1$ (fig. 2) has been set, and their piercing points through the planes $A_1B_1E_1$ and $A_1B_1D_1$, have been determined, those being the points $S_1$ and $C_1$. With the aid of cross ratio on the associated sequences of points, the points $S_2$ and $C_2$ in the space $\theta^2$ have been determined. From the dual relationship $\lambda = (5_1W_1^\infty S_1C_1) = (5_2W_2S_2C_2)$, the point $5_2$, has been determined, lying in the plane through the point $L_2$ parallel to the vanishing line $M_2$. The radius of the associated circumference $5_26_2$, $L_25_2^\infty$ has been determined in the true size, by rotation, and the point symmetrical to the point $5_2^\infty$ is the point $6_2^\infty$ in respect to $L_2$. The axis of the ellipse $s_2$ (contours of rotational ellipsoid) $5_2^\infty6_2^\infty$ is the polar line for the pole $X_2^\infty$ in
Figure 2: determination of the figure of absolute conic in space $\theta^2$
the infinitely distant plane $N_2^\infty$.

The contour of the rotational ellipsoid $s_2$ in the space $\theta^2$ in the transformation plane, which is associated to the sphere $s_1$ in the space $\theta^1$ has been determined with the aid of the axes $1_22_2$ and $5_2^66_2^6$ of the ellipse, $3_2^74_2^7$ being its chord. Intersection of the rotational ellipsoid $s_2$ with the vanishing plane $M_2$ is the imaginary circumference $a_{12}$, whose real representative is circumference $a_{22}$, and it has been associated to the absolute conic $a_{11}$ of the space $\theta^1$.

Constructive procedure for determination of the imaginary circumference $a_{12}$ is determination of the conjugate imaginary double points of elliptical involutory sequences of pencils of straight lines $W_2$ in the vanishing plane $M_2$. Symmetrical points of the mentioned elliptical involutory sequences determine the circumference $a_{22}$, the real representative of the imaginary circumference $a_{12}$. In order to determine the symmetrical points on the straight line $M_2$ (ray position of the vanishing plane $M_2$), induced by the ellipse $s_2$ (contour of rotational ellipsoid), which is associated to the absolute sequence on the infinitely distant straight line of the infinitely distant plane $M_1^\infty$, it is necessary first to determine the symmetrical points $S_2^A$ and $S_2^B$ of the elliptic involutory sequence induced by the affine circumference $s_2^A$ to the given ellipse $s_2$. The affine circumference to the given ellipse (contour of the ellipsoid) has a diameter equal to one axis of the ellipse $1_22_2$ which is situated on the axis of space, and it is simultaneously the affinity axis of these two conics. With the aid of affinity, the symmetrical points of the elliptical involutory sequence of the $M_2$ induced by the ellipse $s_2$ (contour of ellipsoid) $S_2$ and $S^2$ can be determined.

In this way, the diameter of the circumference $a_{22}$, $S_2\overrightarrow{S^2}$, has been determined, which is a real representative of the imaginary circumference $a_{12}$, whose center is the point $W_2$, which is the center of the space $\theta^2$. In the general position of the space, in a pair of Monge’s projections, this imaginary circumference represented by the real representative appear as ellipse (fig.2).

3 Conclusion

In general collinear spaces, figures of the absolute conics of space can be constructively determined by the intersection of the quadrics (II degree surfaces) with the vanishing planes in other space, which are associated to the sphere in the first space which is intersected by the infinitely distant plane along the absolute conic. In the first space, the sphere is a quadric intersecting the infinitely distant plane of its space along the absolute conic. The quadric associated to a sphere in the first space, can be in the second space sphere or rotating (ellipsoid, two-sheet hyperboloid and paraboloid) or triaxial (ellipsoid, two-sheet hyperboloid and paraboloid) depending on the position of the sphere in respect to the axis and the vanishing plane in the space. Of these quadrics, rotating ellipsoid was chosen, because its intersection with the vanishing plane in the second space is imaginary circumference, which has a real representative, whose center coincides with the center of this space. This imaginary circumference is the figure of the absolute conic of the first space.
References


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