THERMO ELASTIC-PLASTIC TRANSITION IN A THIN ROTATING DISC WITH INCLUSION

by

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Stresses for the elastic-plastic transition and fully plastic state have been derived for a thin rotating disc with shaft at different temperatures and results have been discussed and depicted graphically. It has been observed that the rotating disc with inclusion and made of compressible material requires lesser angular speed to yield at the internal surface and higher percentage increase in angular speed to become fully plastic as compared to disc made of incompressible material. With the introduction of thermal effect, the rotating disc with inclusion required lesser angular speed to yield at the internal surface. Rotating disc made of compressible material with inclusion requires higher percentage increase in angular speed to become fully-plastic as compared to disc made of incompressible material. Thermal effect also increases the values of radial and circumferential stresses at the internal surface for fully-plastic state.

Key words: stress, displacement, rotating disc, angular speed, inclusion, temperature

Introduction

Rotating discs are an essential part of the rotating machinery structure, e.g., rotors, turbines, compressors, flywheel, and computer’s disc drive. The stress analysis of thin rotating discs made of isotropic material has been discussed extensively by Timoshenko and Goodier [1] in the elastic range and by Chakrabarty [2] and Heyman [3] for the plastic range. Their solutions for the problem of fully plastic state do not involve the plane stress condition, that is to say, one can obtain the same stresses and angular speed necessary for fully plastic stress of the disc without using the plane stress condition (i.e., $T_{zz} = 0$). Gupta and Shukla [4] obtained a different solution for the fully plastic state by using Seth’s transition theory [5] and plane stress condition. This theory does not require any assumptions like an yield condition or incompressibility condition and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. It utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deformed field and has been successfully applied to a large number of problems [4, 7-14, 16].
Seth [6] has defined the generalized principal strain measure as:

\[ e_{ii} = \frac{\hat{e}_{ii} - 2}{0} \left[ 1 - 2 \hat{e}_{ii} \right]^{\frac{n}{2}} \text{d} \hat{e}_{ii} = \frac{1}{n} [1 - (1 - 2 \hat{e}_{ii})], \quad (i = 1, 2, 3) \quad (1) \]

where \( n \) is the measure and \( \hat{e}_{ii} \) are the Almansi finite strain components [6].

In this paper, we investigate the problem of “thermo elastic-plastic transition in a thin rotating disc with shaft” by using Seth’s transition theory. Results have been discussed and depicted graphically.

**Governing equations**

We consider a thin annular disc with central bore of radius \( a \) and outer radius \( b \) (fig. 1). The disc, produced of material of constant density, is mounted on a rigid shaft.

The disc is rotating with angular speed \( \omega \) about a central axis perpendicular to its plane. The thickness of disc is assumed to be constant and is taken sufficiently small so that the disc is effectively in a state of plane stress, that is, the axial stress \( T_{zz} \) is zero. The temperature at the central bore of the disc is \( \Theta \).

The displacement components in cylindrical polar co-ordinates are given by [6]:

\[ u = r(1 - \beta), \quad v = 0, \quad w = dz \quad (2) \]

where \( \beta \) is position function, depending on \( r = (x^2 + y^2)^{1/2} \) only, and \( d \) is a constant.

The finite strain components are given by Seth [6] as:

\[ \hat{e}_{rr} = \frac{\partial u}{\partial r} - \frac{u}{r}, \quad \hat{e}_{\theta\theta} = \frac{u}{r}, \quad \hat{e}_{zz} = \frac{1}{2} \left( \frac{\partial w}{\partial z} \right)^2 \quad (3) \]

\[ \hat{e}_{\theta r} = \hat{e}_{\theta z} = \hat{e}_{zz} = 0 \]

---

**Figure 1. Geometry of rotating disc**
where $\beta' = \frac{d\beta}{dr}$ and meaning of superscripts A is Almansi. Substituting eqs. (3) in eq. (1), the generalized components of strain are:

$$
e_{rr} = \frac{1}{n} \left[ 1 - (r\beta' + \beta)^n \right]$$
$$
e_{\theta\theta} = \frac{1}{n} (1 - \beta^n)$$
$$
e_{zz} = \frac{1}{n} \left[ 1 - (1 - d\beta)^n \right]$$
$$
e_{r\theta} = e_{0z} = e_{zz} = 0$$

(4)

The stress-strain relation for thermo elastic isotropic material are given by [17]:

$$T_{ij} = \lambda \delta_{ij} I_1 + 2\mu e_{ij} - \xi \Theta \delta_{ip}$$
$$\quad \text{ (i, j = 1, 2, 3)}$$

(5)

where $T_{ij}$ are the stress components, $\lambda$ and $\mu$ are Lame’s constants, $I_1 = e_{kk}$ is the first strain invariant, $\delta_{ij}$ is the Kronecker’s delta, $\xi = \alpha(3\lambda + 2\mu)$, $\alpha$ being the coefficient of thermal expansion, and $\Theta$ is the temperature. Further, $\Theta$ has to satisfy:

$$\nabla^2 \Theta = 0$$

or

$$\frac{d^2 \Theta}{dr^2} + \frac{1}{r} \frac{d \Theta}{dr} = 0$$

which has solutions:

$$\Theta = k_1 (\log r + k_2)$$

(6)

where $k_1$ and $k_2$ are constants of integration and can determined from the boundary condition.

Equation (5) for this problem become:

$$T_{rr} = \frac{2\lambda \mu}{\lambda + 2\mu} (e_{rr} + e_{r0}) + 2\mu e_{rr} - \frac{2\mu \xi \Theta}{\lambda + 2\mu}$$
$$T_{00} = \frac{2\lambda \mu}{\lambda + 2\mu} (e_{rr} + e_{00}) + 2\mu e_{r0} - \frac{2\mu \xi \Theta}{\lambda + 2\mu}$$
$$T_{r\theta} = T_{0z} = T_{rr} = T_{zz} = 0$$

(7)

Substituting eqs. (4) in eq. (5), the strain components in terms of stresses are obtained as [15]:
where \( E \) is the Young’s modulus and \( C \) is the compressibility factor of the material in terms of Lame’s constant, and are given by \( E = \mu(3\lambda + 2\mu) / (\lambda + \mu) \) and \( C = 2\mu / (\lambda + 2\mu) \).

Substituting eqs. (4) in eqs. (7), we get:

\[
T_{rr} = \frac{2\mu}{n} \left\{ 3 - 2C - \beta^n \left[ 1 - C + (2 - C) \left( \frac{\mu}{\beta} + 1 \right)^n \right] + \frac{nC\zeta_0}{2\mu\beta^n} \right\} \\
T_{\theta\theta} = \frac{2\mu}{n} \left\{ 3 - 2C - \beta^n \left[ 2 - C + (1 - C) \left( \frac{\mu}{\beta} + 1 \right) ^n \right] + \frac{nC\zeta_0}{2\mu\beta^n} \right\} \\
\]

and

\[
T_{r\theta} = T_{\theta r} = T_{rr} = T_{zz} = 0
\]

All equations of equilibrium are satisfied except:

\[
\frac{d}{dr} (rT_{rr}) - T_{\theta\theta} + \rho \omega^2 r^2 = 0
\]

where \( \rho \) is the density of the material of the rotating disc.

The temperature satisfying Laplace eq. (6) with boundary condition:

\[
\Theta = \Theta_0 \quad \text{at} \quad r = a, \\
\Theta = 0 \quad \text{at} \quad r = b
\]

where \( \Theta_0 \) is constant, given by [17]:

\[
k_1 = \frac{\Theta}{\log \frac{a}{b}} \quad \text{and} \quad k_2 = -\log b
\]

Substituting \( k_1 \) and \( k_2 \) form eq. (6), we get:

\[
\Theta_1 = \frac{\Theta_0 \log \frac{r}{b}}{\log \frac{a}{b}}
\]
Using eqs. (9) and (11) in eq. (10), we get a non-linear differential equation in $\beta$ as:

$$
(2 - C) n \beta^{n+1} P (P + 1)^{n-1} \frac{dP}{d\beta} = \frac{n \rho \alpha^2 r^2}{2\mu} - \frac{n C \Theta_0}{2\mu} + \beta^n [1 - (P + 1)^n - n P [1 - C + (2 - C)(P + 1)^n]}
$$

(12)

where $\Theta_0 = \Theta_0' \log (a/b)$ and $n \beta' = \beta P$ ($P$ is function of $\beta$ and $\beta$ is function of $r$).

From eq. (12), the turning points of $\beta$ are $P = -1$ and $\pm \infty$.

The boundary conditions are:

$$
u = 0 \text{ at } r = a \text{ and } T_{rr} = 0 \text{ at } r = b
$$

(13)

**Solution through the principal stresses**

For finding the plastic stress, the transition function is taken through the principal stress (see Seth [7, 8], Hulsurkar [9], and Gupta et al. [10-14, 16]) at the transition point $P \rightarrow \pm \infty$. We take the transition function $R$ as:

$$
R = \frac{n}{2\mu} (T_{00} - C \zeta \Theta) = (3 - 2C) - \beta^n [2 - C + (1 - C)(P + 1)^n] - \frac{n \zeta \Theta}{\mu}
$$

(14)

Taking the logarithmic differentiation of eq. (14) with respect to $r$ and using eq. (12), we get:

$$
\frac{d(\log R)}{dr} = -\frac{\beta^n \frac{1 - C}{2 - C} \left[ \frac{1 - (P + 1)^n - n (1 - C) P + n \rho \alpha^2 r^2}{2\mu \beta^n} + \frac{n \zeta \Theta_0 (3 - 2C)}{\mu (4 - 2C) \beta^n} \right] + (2 - C) n P \beta^n}{r \left[ 3 - 2C - \beta^n [2 - C + (1 - C)(P + 1)^n] - \frac{n \zeta \Theta}{2\mu} \right]}
$$

(15)

Taking the asymptotic value of eq. (15) at $P \rightarrow \pm \infty$ and integrating, we get:

$$
R = K_1 r^{-\frac{1}{2 - C}}
$$

(16)

where $K_1$ is a constant of integration, which can be determine by boundary condition.

From eqs. (14) and (16), we have:
Substituting eq. (17) in eq. (10) and integrating, we get:

\[
T_{00} = \frac{2\mu}{n} A_1 r \frac{1}{2-c} + \frac{C_2 \Theta_0 \log \frac{r}{b}}{\log \frac{a}{b}} \tag{17}
\]

Substituting eq. (17) in eq. (10) and integrating, we get:

\[
T_{00} = \frac{2\mu(2-C)}{n(1-C)} A_1 r \frac{1}{2-c} + \frac{C_2 \Theta_0 \log \frac{r}{b}}{\log \frac{a}{b}} - \frac{C_2 \Theta_0 \frac{\rho \alpha^2 r^2}{3}}{\log \frac{a}{b}} + \frac{K_2}{r} \tag{18}
\]

where \(K_2\) is a constant of integration, which can be determine by boundary condition.

Substituting eq. (17) and (18) in second equation of eqs. (8), we get:

\[
\beta = \sqrt{1 - \frac{2(1-C)}{E(2-C)} \left[ \frac{\rho \alpha^2 r^2}{3} + \frac{\alpha E \Theta_0 (2-C)}{\log \frac{a}{b}} + \frac{2(2-C) \alpha E \Theta_0 \log \frac{r}{b}}{\log \frac{a}{b}} - B_1 \right]} \tag{19}
\]

where \(C_2 = \alpha E(2-C)\).

Substituting eq. (19) in eq. (2), we get:

\[
u = r - r \sqrt{1 - \frac{2(1-C)}{E(2-C)} \left[ \frac{\rho \alpha^2 r^2}{3} + \frac{\alpha E \Theta_0 (2-C)}{\log \frac{a}{b}} + \frac{2(2-C) \alpha E \Theta_0 \log \frac{r}{b}}{\log \frac{a}{b}} - B_1 \right]} \tag{20}\]

where \(E = 2\mu(3-2C)/(2-C)\) is the Young’s modulus in term of compressibility factor.

Using boundary condition (13) in eqs. (18) and (20), we get:

\[
K_1 = \frac{\rho \alpha^2 n(1-C)(b^3 - a^3)}{6 \mu(2-C)b^{2-c}} + \alpha E \Theta_0 n(1-C)(b-a) \frac{1-c}{2 \mu \log \frac{a}{b} b^{2-c}} - \alpha E \Theta_0 \frac{n a}{\mu b^{2-c}} \tag{21}\]

\[
K_2 = \frac{\rho \alpha^2 a^3}{3} + \alpha E \Theta_0 (2-C) a \frac{1-c}{\log \frac{a}{b}} + \frac{2(2-C) \alpha E \Theta_0 a}{1-C} \tag{22}\]
Substituting eqs. (21) and (22) in eqs. (17), (18), and (20), respectively, we get the transitional stresses and displacement as:

\[
T_{00} = \frac{\rho \omega^2 (1-C)(b^3-a^3)}{3r(2-C)} \left( \frac{r}{b} \right)^{1-C} + \alpha E \Theta_0 (2-C) \left[ \frac{\log \frac{L}{h}}{\log \frac{a}{b}} - \frac{2a}{(2-C)r} \left( \frac{r}{b} \right)^{1-C} + \frac{(1-C)(b-a)}{r(2-C) \log \frac{a}{b}} \right] \tag{23}
\]

\[
T_{rr} = \frac{\rho \omega^2}{3r} \left( b^3-a^3 \right) \left( \frac{r}{b} \right)^{1-C} - r^3 + a^3 + \frac{\alpha E \Theta_0 (2-C)}{\log \frac{a}{b}} \left[ \log \frac{r}{b} + b-a \left( \frac{r}{b} \right)^{1-C} + a \frac{1}{r} \right] + \frac{2\alpha E \Theta_0 (2-C)}{1-C} \left[ \frac{a}{r} - \frac{a}{r} \left( \frac{r}{b} \right)^{1-C} \right] \tag{24}
\]

and

\[
u = r - r \left[ \frac{\rho \omega^2 (r^3-a^3)}{3r} + \alpha E \Theta_0 (2-C)(r-a) \right] + \frac{2(2-C)\alpha E \Theta_0}{1-C} \left( \frac{\log \frac{r}{b} - a}{\log \frac{a}{b}} \right) \tag{25}
\]

From eqs. (23) and (24), we get:

\[
|T_{rr} - T_{00}| = \frac{\rho \omega^2}{3} \left[ b^3-a^3 \left( \frac{r}{b} \right)^{1-C} - r^2 + a^3 \frac{1}{r} \right] + \alpha E \Theta_0 \left[ \frac{2a}{r(1-C)r} \left( \frac{r}{b} \right)^{1-C} + b-a \left( \frac{r}{b} \right)^{1-C} + \frac{2-C}{\log \frac{a}{b}} \frac{a}{r} \right] \tag{26}
\]
From eq. (26), it is seen that \( |T_n - T_{00}| \) is maximum at the internal surface (that is at \( r = a \)), therefore yielding of the disc takes place at the internal surface of the disc and eq. (26) can be written as:

\[
|T_n - T_{00}|_{r=a} = \frac{\rho \omega^2 (b^3 - a^3)}{3(2-C)a} \left( \frac{a}{b} \right)^{1-C/2-C} + \\
\alpha E \Theta_0 \left[ \frac{b-a}{a \log \frac{a}{b}} \left( \frac{a}{b} \right)^{1-C/2-C} - \frac{2}{1-C} \left( \frac{a}{b} \right)^{1-C/2-C} + \frac{2(2-C)}{1-C} \right] = Y
\]

where \( Y \) is the yielding stress.

The angular speed \( \Omega_i \) necessary for initial yielding is given by:

\[
\Omega_i^2 = \frac{\rho \omega^2 b^2}{Y} = \frac{3(2-C)}{Y} \left( \frac{a}{b} \right)^{2-C} - \\
\frac{3(2-C) \alpha E \Theta_0}{Y} \left[ \frac{1-a}{b \log \frac{a}{b}} \frac{2}{1-C} - \frac{2}{1-C} \right] \tag{27}
\]

and

\[
\omega_i = \frac{\Omega_i}{b} \sqrt{\frac{Y}{\rho}}
\]

The disc becomes fully plastic \((C \to 0)\) at the external surface and eq. (26) becomes:

\[
|T_n - T_{00}|_{r=b} = \frac{\rho \omega^2 (b^3 - a^3)}{6b} + \alpha E \Theta_0 \left[ \frac{b-a}{b \log \frac{a}{b}} - \frac{2}{b} \right] = Y
\]

where \( E = 3\mu \).

The angular \( \Omega_f \) speed required for fully plastic state is given by:

\[
\Omega_f^2 = \frac{\rho \omega_f^2 b^2}{Y} = \frac{6}{1-a^3/b^3} + \frac{6 \alpha E \Theta_0}{Y} \left( \frac{1-a}{b \log \frac{a}{b}} - \frac{2}{b} \right) \tag{28}
\]
where

\[ \omega_f = \frac{\Omega_f}{b} \sqrt{\frac{Y}{\rho}} \]

We introduce the following non-dimensional components:

\[ R = \frac{r}{b}, R_0 = \frac{a}{b}, \sigma_r = \frac{T_0}{Y}, \sigma_0 = \frac{\alpha E \Theta_0}{Y}, \bar{u} = \frac{u}{b}, \Theta_i = \frac{\rho \omega_i b^2}{Y}, \text{ and } \Omega_i^2 = \frac{\rho \omega_i^2 b^2}{Y} \]

Elastic-plastic transitional stresses, angular speed and displacement from eq. (23)-(25), and (27) in non-dimensional form become:

\[
\sigma_0 = \frac{\Omega_1^2}{3R} \left[ \frac{(1 - R_0^3)(1 - C)}{2 - C} \right] + \frac{1}{2 - C} (1 - R_0) \left[ \frac{1 - C}{R_0} \right] + \frac{\Theta_i(2 - C)}{R_0} \left[ \log R + \frac{1 - R_0}{R} \right] + \frac{2 \Theta_i(2 - C)}{1 - C} \left( \frac{R_0}{R} - \frac{R_0}{R} \right) \]

\[ \sigma_i = \frac{\Omega_1^2}{3R} \left[ \frac{(1 - R_0^3)}{2 - C} \right] + \left[ \log R + \frac{1 - R_0}{R} \right] + \frac{2 \Theta_i(2 - C)}{1 - C} \left( \frac{R_0}{R} - \frac{R_0}{R} \right) \]

\[ \Omega_i^2 = \left[ \frac{3(2 - C)}{1 - R_0^3} \right] - \left[ \frac{3 \Theta_i(2 - C)}{1 - R_0^3} \right] \left[ \frac{1 - R_0}{1 - C} + \frac{2(2 - C)}{R_0} \right] \]

and

\[ \bar{u} = \sqrt{R - R} \left[ 1 - \frac{2Y}{E} \frac{1 - C}{2 - C} \right] + \frac{\Omega_1^2}{3R} \left( \frac{R^3 - R_0^3}{1 - R_0^3} \right) + \frac{\Theta_i(2 - C)(R - R_0)}{R_0} + \frac{\Theta_i(2 - C)}{1 - C} \left( \frac{\log R}{R} - \frac{R_0}{R} \right) \]

Stresses, displacement, and angular speed for fully plastic state \((C \to 0)\), are obtained from eqs. (29), (30), (32), and (28) as:
\[
\sigma_0 = \frac{\Omega_1^2}{3R} \left( \frac{1 - R_0^3}{2} \right)^\frac{1}{\sqrt{R}} + 2\Theta_1 \left( \frac{1 - R_0}{2R \log R_0} \sqrt{R} + \frac{\log R}{\log R_0} - \frac{R_0}{R} \sqrt{R} \right)
\]  
\(33\)

\[
\sigma_t = \frac{\Omega_1^2}{3R} \left[ (1 - R_0^3) \sqrt{R} - R^3 + R_0^3 \right] + \frac{2\Theta_1}{\log R_0} \left( \log R + \frac{1 - R_0}{r} \sqrt{R} + \frac{R_0}{R} - 1 \right) + 4\Theta_1 \left( \frac{R_0}{R} - \frac{R_0}{R_0} \sqrt{R} \right)
\]  
\(34\)

\[
n = R - R \sqrt{1 - \frac{Y}{E} \frac{\Omega_1^2}{3R} (R^3 - R_0^3) + \frac{2\Theta_1}{R \log R_0} (R - R_0) + 4\Theta_1 \left( \frac{\log R}{\log R_0} - \frac{R_0}{R} \right)}
\]  
\(35\)

and

\[
\Omega_1^2 = \frac{6}{1 - R_0^3} \left[ 6\Theta_1 \left( \frac{1 - R_0}{R_0} \sqrt{R} - 2R_0 \right) \right]
\]  
\(36\)

**Particular case**

When there is no thermal effect (\(\Theta_1 = 0\)), the transitional stresses from eq. (29) to (32) become:

\[
\sigma_0 = \frac{\Omega_1^2}{3R} \left[ (1 - R_0^3)(1 - C) \frac{1 - C}{2 - C} \right]
\]  
\(37\)

\[
\sigma_t = \frac{\Omega_1^2}{3R} \left[ (1 - R_0^3) (2 - C) - R^3 + R_0^3 \right]
\]  
\(38\)

\[
n = R - R \sqrt{1 - \frac{Y}{E} \frac{1 - C}{2 - C} \left[ \frac{\Omega_1^2}{3R} (R^3 - R_0^3) \right]}
\]  
\(39\)

where

\[
\Omega_1^2 = \frac{3(2 - C)}{1 - R_0^3} R_0^\frac{2 - C}{3}
\]  
\(40\)

For fully plastic state stresses, displacement, and angular speed from eq. (33) to (36) becomes:
\[ \sigma_0 = \frac{\Omega_i^2}{3R} \frac{1 - R_0^3}{2} \sqrt{R} \] (41)

\[ \sigma_r = \frac{\Omega_i^2}{3R} \left[ (1 - R_0^3) \sqrt{R - R^3 + R_0^3} \right] \] (42)

\[ \bar{u} = R - R \sqrt{1 - \frac{Y \Omega_i^2}{E} (R^3 - R_0^3)} \] (43)

where

\[ \Omega_i^2 = \frac{\rho \omega_i^2 b^2}{Y} = \frac{6}{\left| 1 - R_0^3 \right|} \] (44)

These equations are same as obtained by Gupta and Pankaj [15].

**Results and discussion**

For calculating the stresses, angular speed, and displacement based on the above analysis, the following values have been taken: \( C = 0.00, 0.25, 0.5, \) and \( 0.75, \) \( E/Y = 1/2 \) and \( 2, \) \( \Theta_0 = 0 \) and \( 700 \, ^\circ F, \) \( \alpha = 5.0 \cdot 10^{-5} \, \text{deg F}^{-1} \) (for methyl methacrylate) [18], \( \Theta_1 = \alpha E \Theta/\gamma = 0, 0.0175, \) and \( 0.07 \) for \( E/Y = 1/2 \) and \( 2, \) and \( \Theta_0 = 0 \) and \( 700 \, ^\circ F, \) respectively.

Curves have been drawn in fig. 2 between angular speed \( \Omega_i^2 \) required for initial yielding and various radii ratios \( R_0 = a/b \) for \( C = 0, 0.25, 0.5, \) and \( 0.75 \) at \( \Theta_1 = 0, 0.0175, \) and \( 0.07. \) It has been observed that in the absence of thermal effect the rotating disc made of incompressible material with inclusion require higher angular speed to yield at the internal surface as compared to disc made of compressible material and a much higher angular speed is required to yield with the increase in radii ratio. With the introduction of thermal effects, lesser angular speed is required to yield at the internal surface. It can also be seen from tab. 1 that for compressible material higher percentage increased in angular speed is required to become fully plastic as compared to rotating disc made of incompressible material.

In figs. 3 and 4, curves have been drawn for stresses and displacement with respect to radii ratio \( R = r/b \) for elastic-plastic transition and fully plastic state, respectively. It has been seen that temperature has a quite effect on radial and circumferential stresses \( i. \ e. \) with the introduction of thermal effect it decrease the value of radial and circumferential stress at the internal surface for transitional state, whereas from fig. 4, it can been seen that thermal effect increases the values of radial and circumferential stress at the internal surface for fully-plastic state.
Figure 2. Angular speed required for initial yielding at the internal surface of the rotating disc

Table 1. Angular speed required for initial yielding and fully plastic state

<table>
<thead>
<tr>
<th>Different values of temperatures, $\Theta_1$</th>
<th>Compressibility of material, $C$</th>
<th>Angular speed required for initial yielding, $\Omega_i^2$</th>
<th>Angular speed required for fully-plastic state, $\Omega_f^2$</th>
<th>Percentage increase in angular speed $\left(\frac{\Omega_f^2}{\Omega_i^2} - 1\right) \times 100$ [%]</th>
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<td>$6.857143$</td>
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Figure 3a. Stresses at the elastic plastic transition state

Figure 3b. Stresses at the elastic plastic transition state
Figure 3c. Stresses at the elastic plastic transition state

Figure 4. Stresses at the fully plastic state
Acknowledgment

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Nomenclature

- $a, b$ – internal and external radii of the disc, [m]
- $C$ – compressibility factor, [--]
- $K_1, K_2, k_1, k_2$ – constants of integration, [--]
- $T_{ij}, e_{ij}$ – stress [kgm$^{-1}$s$^{-2}$] and strain rate tensor
- $u, v, w$ – displacement components, [m]
- $Y$ – yield stress, [kgm$^{-1}$s$^{-2}$]

Greek letters

- $\Theta$ – temperature, [$^\circ$F]
- $\sigma_r$ – radial stress component ($T_{rr}/Y$), [--]
- $\sigma_\theta$ – circumferential stress component ($T_{\theta\theta}/Y$), $\Theta_1 = \alpha \Theta_0 Y$, [--]
- $\rho$ – density of material, [kgm$^{-3}$]
- $\Omega^2$ – $\rho a^2 b^2 / E$ (speed factor), $R = r/b$, $R_0 = a/b$ (radii ratio), [--]
- $\omega$ – angular speed of rotation, [s$^{-1}$]

References


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