NONLINEAR TRANSIENT HEAT CONDUCTION ANALYSIS OF INSULATION WALL OF TANK FOR TRANSPORTATION OF LIQUID ALUMINUM

by

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This paper deals with transient nonlinear heat conduction through the insulation wall of the tank for transportation of liquid aluminum. Tanks designed for this purpose must satisfy certain requirements regarding temperature of loading and unloading, during transport. Basic theoretical equations are presented, which describe the problem of heat conduction finite element (FE) analysis, starting from the differential equation of energy balance, taking into account the initial and boundary conditions of the problem. General 3D problem for heat conduction is considered, from which solutions for two- and one-dimensional heat conduction can be obtained, as special cases. Forming of the finite element matrices using Galerkin method is briefly described. The procedure for solving equations of energy balance is discussed, by methods of resolving iterative processes of nonlinear transient heat conduction. Solution of this problem illustrates possibilities of PAK-T software package, such as materials properties, given as tabular data, or analytical functions. Software also offers the possibility to solve nonlinear and transient problems with incremental methods. Obtained results for different thicknesses of the tank wall insulation materials enable its comparison in regards to given conditions.

Key words: Heat conduction, finite element analysis, wall insulation, Galerkin method, PAK-T, optimization

Introduction

Liquid metal transportation represents a complex task, primarily due to high temperature of the metal, rapid cooling, changes of the metal structure during the transport, time interval in which the liquid metal needs to be transferred from the foundry to the rolling mill, etc. During the last years, many researchers have studied problems associated with optimization of the thickness and composition of insulation wall of tanks and other engineering constructions, by application of numerical methods [1-6]. Tanks for liquid aluminum transportation must have a wall made of special materials in order to prevent
cooling of aluminum below the melting point and solidification point, during its transfer from the foundry to the rolling mill.

Wall of tank for liquid aluminum transportation has a multilayered structure, made of materials which have low coefficient of conduction. Part of insulation wall that is in direct contact with liquid aluminum is made of concrete. External layer of the insulation wall is made of S235JRG2 steel. High insulating, heat resistant materials are used in between these internal concrete layer and external steel layer. The first layer is made of Insulfrax High Temperature Insulation Paper [7], which is easily die cut and fabricated for a variety of thermal applications including gaskets, aerospace heat shields, mold liners, refractory back-up insulation. The next layer is made of VATRAMIL® GM 22 [8], which is high temperature resistant mass with excellent fire protection.

The objective of this paper is to optimize and obtain the best solution for thickness of concrete and VATRAMIL® GM 22 layers within the insulation wall of the tank, by application of PAK-T [9] software package, which is based on finite element (FE) method and heat conduction laws [10-14]. Requested condition is to prevent liquid aluminum to cool down below 750 °C after 8 h, as well as to keep the temperature of the outer layer made of S235JRG2 steel below 100 °C. Coefficients of conduction of insulation materials are changing with the temperature, what is taken into account.

Software package PAK-T [9] is developed based on the theory of the heat conduction and theoretical foundation is given at the beginning of the paper. Application of PAK-T in optimization of the tank wall thickness is presented further in this paper.

Basic equation of heat conduction

Differential equation of energy balance

Differential equation of energy balance equation is based on fundamental principle of energy conservation. Namely, change of material internal energy per time unit, within elementary volume $dV$, is equal to the quantity of heat energy accumulated in that same volume per time unit. Therefore, the following equation can be stated:

$$\frac{dQ}{dt} = \frac{dU}{dt}$$

(1)

where, $dQ$ and $dU$ are changes of heat and internal energy within volume $dV$ in elementary time interval $dt$. Change of internal energy can be expressed in the following form:

$$\frac{dU}{dt} = \rho c \frac{dT}{dt} dV$$

(2)

where $\rho$ is material density, $c$ - specific heat, and $T$ - temperature. By using fig. 1, expression $dQ/dt$ can be formulated as:
where \( q_x, q_y, \) and \( q_z \) are heat flux components, as variables that represent the quantity of heat flow through the surface unit per time unit; \( q \) is the heat source intensity (heat quantity per time unit and volume unit). Flux component signs have been taken into account in equation (3): plus sign corresponds to the plus flux projection onto the external normal on the surface. Also, minus flux through the surface, corresponds to heat energy accumulation in volume \( dV \). It can be considered that \( q > 0 \) if the heat source is in volume \( dV \) (at the point of material) and \( q < 0 \) in case of heat sink.

Figure 1. Elementary volume \( dV \) with heat flux components

Conduction of heat through solid bodies is defined by Fourier's law of heat:

\[
\dot{q}_i = -k_i \frac{\partial T}{\partial x_i} \quad i = 1, 2, 3 \quad \text{no summation by } i
\]

(4)

where \( k_i \), namely \( k_x, k_y, \) and \( k_z \), are coefficients of heat conduction, in case of orthotropic material. In case of isotropic material, the following is valid:

\[
k_x = k_y = k_z = k
\]

(5)

By substituting (3) and (2) into energy balance equation (1) and using (4), differential equation is obtained in the following form:

\[
-\rho c \frac{dT}{dt} + \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + q = 0
\]

(6)

or by using index notation,
Initial and boundary conditions

General solution of differential equation of heat conduction comprises indefinite functions and constants. Solution for temperature field $T(x, y, z, t)$ that satisfies given initial and boundary conditions is to be found, in scope of practical solving of the problem. There is a unique solution for given initial and boundary conditions. Initial conditions are given only for transient problems. They assume that temperature distribution is known at the initial moment $t = 0$, namely:

$$T(x, y, z, 0) = f_0(x, y, z)$$

where $f_0(x, y, z)$ is given function of material points coordinates.

In general, boundary conditions can be:

a) given temperature at $S_1$ part

$$T = T_s(x, y, z, t)$$

b) given flux at $S_2$ part

$$q_n = q_n(x, y, z, t)$$

c) given heat convection at $S_3$ part

$$q_h = h(T_0 - T_s)$$

d) given radiation at $S_4$ part

$$q_r = h_r(T_r - T_s)$$

where $T_s, T_0$ and $T_r$ are surface temperature, environment temperature, and radiation temperature, respectively.

Equation that need to be satisfied for the boundary condition b) can be written as follows, based on Fourier's law of heat (4):

$$q_n = k_x \frac{\partial T}{\partial x} n_x + k_y \frac{\partial T}{\partial y} n_y + k_z \frac{\partial T}{\partial z} n_z = \sum_{i=1}^{3} k_i \frac{\partial T}{\partial x_i} n_i$$

at $S_2$
equations that describe heat conduction law and boundary conditions, are linear dependent on temperature.

\[ \text{Figure 2. Boundary conditions for heat conduction through solid} \]

Incremental finite element equations and Galerkin method application

The derivation of the finite element balance equations is based on equations given in the previous text. Galerkin method is then applied, for derivation of FE equations. Based on differential equation of heat conduction (7), the following can be written:

\[ \rho \frac{dT}{dt} dV + \int \left[ h_1 \sum_{j=1}^{3} \frac{\partial}{\partial x_j} \left( k_j \frac{\partial T}{\partial x_j} \right) \right] dV + \int h_0 g dV = 0 \quad I = 1, 2, \ldots, N \quad (14) \]

where \( h_i \) are interpolation functions and \( N \) is number of nodes per element, and \( V \) is finite element volume. Three-dimensional (3D) isoparametric finite element is applied, as defined in [4]. Interpolation functions, geometry and number of nodes are also adopted. Temperature \( T \) in a point of element, defined in natural coordinates \( \xi, \eta, \zeta \) is given as:

\[ T(\xi, \eta, \zeta, t) = \sum_{i=1}^{N} h_i T_i \quad (15) \]

or in matrix form,

\[ T = HT \quad (16) \]

where

\[ H = \begin{bmatrix} h_1 & h_2 & \ldots & h_N \end{bmatrix} \quad (17) \]

\[ T^T = \begin{bmatrix} T^1 & T^2 & \ldots & T^N \end{bmatrix} \quad (18) \]

are row matrix of interpolation functions and column matrix of nodal temperatures, respectively. Eight-node 3D finite element with temperature at the element point and nodal temperatures is shown in fig. 3.
Applying the partial integration and Gauss theorem on second integral in (14), the following is obtained:

$$
\int_{V} \left[ h_{i} \sum_{j=1}^{3} \frac{\partial}{\partial x_{j}} (k_{j} h_{i,j}) \right] dV T^{j} = - \int_{S} \left[ \sum_{j=1}^{3} k_{j} h_{i,j} n_{j} \right] dS + \int_{S} \left[ h_{i} \sum_{j=1}^{3} \frac{\partial T}{\partial x_{j}} n_{j} \right] dS \quad (19)
$$

When element surface heat flux $q_{n}$ is given and based on (13), the following can be written:

$$
\int_{S} \left[ h_{i} \sum_{j=1}^{3} k_{j} \frac{\partial T}{\partial x_{j}} n_{j} \right] dS = \int_{S} h_{i}^{h} q_{n} dS = Q_{i}^{h} \quad (20)
$$

where $Q_{i}^{h}$ are heat flux column matrix components and $h_{i}^{s}$ are interpolation functions for nodes on surface $S_{2}$. If the heat convection is given for the surface $S_{3}$ expression is obtained as:

$$
\int_{S} \left[ h_{i} \sum_{j=1}^{3} k_{j} \frac{\partial T}{\partial x_{j}} n_{j} \right] dS = \int_{S} h_{i}^{h} (T_{o} - T_{w}) dS = -K_{i}^{h} T^{j} + Q_{i}^{h} \quad (21)
$$

Summation over repeated index $J (J=1,2,\ldots,N)$ is assumed in the previous equation. Convection matrix coefficients $K_{i}^{h}$ and convection column matrix $Q_{i}^{h}$ are expressed as follows:

$$
K_{i}^{h} = \int_{S} h_{i}^{h} h_{i}^{h} dS \quad (22)
$$

$$
Q_{i}^{h} = \int_{S} h_{i}^{h} T_{o} dS \quad (23)
$$

For radiation boundary condition according to analogy with (21) - (23), the following expression can be written:

$$
\int_{S} \left[ h_{i} \sum_{j=1}^{3} k_{j} \frac{\partial T}{\partial x_{j}} n_{j} \right] dS = \int_{S} h_{i}^{h} (T_{o} - T_{w}) dS = -K_{i}^{h} T^{j} + Q_{i}^{h} \quad (24)
$$

where the following relations are valid:
Using (19) - (26), (15) and (14), the system of equations of the following form is obtained:

$$\mathbf{C} \dot{\mathbf{T}} + \mathbf{K} \mathbf{T} = \mathbf{Q}$$  \hspace{1cm} (27)

where matrix components for $\mathbf{C}$ and $\mathbf{K}$ and column matrix $\mathbf{Q}$ are determined as:

$$C_{ij} = \int \rho c h_i h_j dV$$  \hspace{1cm} (28)

$$K_{ij} = K^k_{ij} + K^h_{ij} + K^r_{ij}$$  \hspace{1cm} (29)

$$Q_i = Q^c_i + Q^h_i + Q^r_i$$  \hspace{1cm} (30)

Coefficients of heat conduction matrix $K^k_{ij}$ are given by:

$$K^k_{ij} = \int \left( \sum_{j=1}^{3} k_j h_{i,j} h_{i,j} \right) dV = \int \left( k_{i,x} h_{i,x} h_{i,x} + k_{i,y} h_{i,y} h_{i,y} + k_{i,z} h_{i,z} h_{i,z} \right) dV$$  \hspace{1cm} (31)

where derivatives of interpolation functions are given by $h_{i,x} = \partial h_i / \partial x, \ldots, h_{i,z} = \partial h_i / \partial z$. In case of isotropic material, relations (5) are valid, and coefficients $K^k_{ij}$ can be expressed in following form:

$$K^k_{ij} = \int \left( h_{i,x} h_{i,x} + h_{i,y} h_{i,y} + h_{i,z} h_{i,z} \right) dV$$  \hspace{1cm} (32)

Convection matrix components $K^h_{ij}$ and radiation matrix components $K^r_{ij}$ are determined by (22) and (25). It should be noted that matrices $C_{ij}$ in (30) and $K_{ij}$ in (31) are symmetric. Column matrix $Q_i^c$ in (32) can be written as:

$$Q_i^c = \int h_i q dV$$  \hspace{1cm} (33)

while $Q_i^{ch}, Q_i^{hr}$ and $Q_i^r$ are determined by (20), (23) and (26).

By using interpolation matrix $H$, defined by row matrix according to (17), matrices in (27) can be written in a next form:

$$K^h = \int B^T k B dV$$  \hspace{1cm} (34)

$$K^r = \int h H^r H^* dS$$  \hspace{1cm} (35)

$$K^r = \int h_i H^* H^* dV$$  \hspace{1cm} (36)
Row matrices $H^s$ contain interpolation functions $h^s_i$ for the surfaces. The matrix $B$ in (34) has a next form like:

$$B = \begin{bmatrix} B^1 & B^2 & \ldots & B^N \end{bmatrix}$$

where, the submatrix for the node "I" consist of derivatives of interpolation functions with respect to coordinates $x, y$ and $z$:

$$B^I = \begin{bmatrix} h_{l,x} \\ h_{l,y} \\ h_{l,z} \end{bmatrix}$$

Equation (27) represents energy balance equation for 3D finite element transient heat conduction. Total number of equations is equal to number of nodes. One temperature value corresponds to each node. In case of 2D, previous expressions remain unchanged, except that integrals over a volume $V$ are practically reduced to integrals over surface $S$ of the finite element, as shown in [10]. Interpolation functions $h_i$ have appropriate forms for 2D problem.

**Solving of the balance equation of the structure**

Matrix equation of the structure balance has the form (27) in case of transient problems. We observe transient heat conduction in case when material constants $c, k, h$ depends on the temperature or radiation boundary condition is used. The implicit iterative scheme for solution of system equations (29) is developed as follows. We set up a condition to satisfy (27) at the final step and consequently, this equation is written in the following form:

$$C^{(0)} + \Delta t \dot{T} + K^{(0)} + \Delta t \dot{T} = \Delta t \dot{T} \frac{Q^{(0)}}{Q^{(0)}}$$

where $C^{(0)}$ and $K^{(0)}$ are matrices that correspond to the known temperatures $^T\dot{T}$, at the beginning of the procedure, and $\Delta t \dot{T} \frac{Q^{(0)}}{Q^{(0)}}$ is the column matrix that corresponds to time $^t \Delta t$ (at time functions) and known temperatures $^T\dot{T}$. Then the following approximation for rate of temperature change can be written:

\[ t+\Delta t \Delta T = \frac{\Delta T^{(1)}}{\Delta t} \]  
\[ t+\Delta t T = t+\Delta T^{(1)} \]  

where \( \Delta T^{(1)} \) are increments of the temperature for the first iteration. Therefore, solution is obtained from (44) as:

\[ \Delta T^{(1)} = \left( \hat{K}^{(0)} \right)^{-1} t+\Delta t \hat{Q}^{(0)} \]  

where

\[ \hat{K}^{(0)} = K^{(0)} + \frac{1}{\Delta t} C^{(0)} \]  

and

\[ t+\Delta t \hat{Q}^{(0)} = t+\Delta T^{(0)} - K^{(0)} t+T \]  

By substitution of temperatures:

\[ t+\Delta t T^{(1)} = t+T + \Delta T^{(1)} \]  

into (44), where instead of \( C^{(0)}, K^{(0)} \) and \( t+\Delta T^{(0)} \) we put \( C^{(1)}, K^{(1)} \) and \( t+\Delta T^{(1)} \), it may be, therefore, concluded that the left hand side is not equal to the right hand side. Equation (44) is then transformed into following form:

\[ \frac{1}{\Delta t} C^{(1)} \left( t+\Delta T^{(1)} + T + \Delta T^{(2)} \right) + K^{(1)} \left( t+\Delta T^{(1)} + \Delta T^{(2)} \right) = t+\Delta T^{(1)} \]  

namely, for the \( i \)-th iteration

\[ \frac{1}{\Delta t} C^{(i-1)} \left( t+\Delta T^{(i-1)} + T + \Delta T^{(i)} \right) + K^{(i-1)} \left( t+\Delta T^{(i-1)} + \Delta T^{(i)} \right) = t+\Delta T^{(i-1)} \]  

which represents the known temperature from the last iteration. Obviously, the solution of (52) for temperature increment \( \Delta T^{(i)} \) is:

\[ \Delta T^{(i)} = \left( \hat{K}^{(i-1)} \right)^{-1} t+\Delta \hat{Q}^{(i-1)} \]  

where

\[ \hat{K}^{(i-1)} = K^{(i-1)} + \frac{1}{\Delta t} C^{(i-1)} \]  

and

\[ t+\Delta \hat{Q}^{(i-1)} = t+\Delta T^{(i-1)} - K^{(i-1)} t+\Delta T^{(i-1)} - \frac{1}{\Delta t} C^{(i-1)} \left( t+\Delta T^{(i-1)} - t+T \right) \]  

Iterative procedure is continued until temperature increment at nodes is not sufficiently low, what can be expressed in the following form:

\[ \| \Delta T^{(i)} \| \leq \varepsilon_a \]  

or
\[
\frac{\|\Delta T^{(0)}\|}{\|\Delta T^{(1)}\|} \leq \varepsilon_r
\]

where \(\varepsilon_a\) and \(\varepsilon_r\) are selected absolute and relative tolerances and \(\|\Delta T\|\) is temperature increment norm.

**Analysis of tank for transportation of liquid aluminum**

Tanks designed for liquid aluminum transportation must satisfy certain requirements regarding temperature of loading and unloading during transport. Liquid aluminum temperature during loading of the tank is 900 °C. Requested condition is to prevent liquid aluminum to cool down below 750 °C after 8 h, as well as to keep the temperature of the outer layer made of S235JRG2 steel below 100 °C.

Observed tank has multilayered insulation wall. Problem is modeled with 2D axisymmetrical four-node finite element, since the tank is axisymmetrical structure. One half of the tank model is shown in fig. 4.a. It should be noted that the tank is not filled with aluminum up to the top. Detailed view of the multilayered insulation wall is given in fig 4.b.

![Figure 4. a) Schematic representation of the tank model, b) Multilayered insulation wall](image)

Coefficients of conduction for used materials are temperature dependent, and they are presented in fig. 5. Materials are separately given in two diagrams since VATRAMIL® GM 22 and Insulfax Paper have very low conduction coefficients.
Figure 5. a) Coefficients of conduction of Steel S235JRG2 and Aluminum versus temperature, b) Coefficients of conduction of VATRAMIL® GM 22 and Insulfrax Paper versus temperature

Initial conditions for problem modeling are set up as follows: aluminum temperature at finite elements nodes 900 °C and initial temperature of the insulation wall 20 °C. Convection between the outer layer of the insulation wall and the environment is taken in account, with adopted convection coefficient 10 Wm⁻²K and environment temperature 20 °C.

Results and discussion

Solved problem is nonlinear since material characteristics are temperature dependent. The solution is obtained in 48 time steps with time increment of 600 s. The problem of nonlinear transient heat conduction is solved for three cases according to different layer thicknesses of VATRAMIL® GM 22 and Concrete as shown in Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Material thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td>Concrete</td>
<td>80</td>
</tr>
<tr>
<td>Insulfrax paper</td>
<td>5</td>
</tr>
<tr>
<td>VATRAMIL® GM 22</td>
<td>55</td>
</tr>
<tr>
<td>Steel S235JRG2</td>
<td>8</td>
</tr>
</tbody>
</table>

Calculation in software PAK-T [9] was performed to obtain the optimized solution for wall thickness in order to satisfy prescribed conditions. Results obtained in Case 1 and Case 2 did not completely satisfy prescribed conditions, since the liquid aluminum temperature is below 750 °C after 8 hours, what can be clearly seen in fig. 6.a. Prescribed condition for temperature on the outer layer of the insulation wall (below 100 °C), was satisfied in all three analyzed cases, what can be seen in fig 6.b.
Temperature field for all three analyzed cases of the material layer thickness, after finished calculations, is presented in fig. 7.

Temperature of the liquid aluminum in Case 3, is above 750 °C after 8 hours, and the temperature of the outer part of the insulation wall is below 100 °C.
Conclusions

This paper deals with possible optimization of real construction design of tank for liquid aluminum transport, by application of software package PAK-T. Complete software package is based on heat conduction theory, starting with balance equation and considering initial and boundary conditions. Forming of FE matrices and procedure for solving energy balance equations by implicit iterative methods were presented. The main objective was to found optimal solution for material layer thickness at the tank insulation wall. Obtained FE results shows that only case of analyzed material layer thicknesses used for insulation wall (70 mm Concrete, 5 mm Insulfrax paper, 65 mm VATRAMIL® GM 22 and 8 mm S235JRG2 steel) can be used for liquid aluminum transport.

Nomenclature

\[ c \] - specific heat, [Jkg\(^{-1}\)K\(^{-1}\)]
\[ h \] - coefficient of convection, [Wm\(^{-2}\)K\(^{-1}\)]
\[ h_0 \] - coefficient of radiation, [Wm\(^{-2}\)K\(^{-1}\)]
\[ h_1 \] - interpolation function, [-]
\[ k \] - coefficient of heat conduction, [Wm\(^{-1}\)K\(^{-1}\)]
\[ n \] - surface normal, [-]
\[ q \] - heat energy, [J]
\[ Q \] - heat source intensity, [Wm\(^{-3}\)]
\[ q_h \] - convection flux, [Wm\(^{-2}\)]
\[ q_g \] - given flux, [Wm\(^{-2}\)]
\[ q_r \] - radiation flux, [Wm\(^{-2}\)]
\[ q_x \] - heat flux component, [Wm\(^{-2}\)]
\[ S \] - surface, [m\(^2\)]
\[ t \] - time, [s]
\[ T \] - temperature, [K]
\[ T_0 \] - surface temperature, [K]
\[ T_e \] - environmental temperature, [K]
\[ T_r \] - temperature of radiation source, [K]
\[ \Delta T \] - temperature increment, [K]
\[ U \] - internal energy, [J]
\[ V \] - volume, [m\(^3\)]

Greek letters

\[ \varepsilon_a \] - absolute tolerance, [-]
\[ \varepsilon_r \] - relative tolerance, [-]
\[ \rho \] - material density, [kgm\(^{-3}\)]

Subscripts

\[ x, y, z \] - Cartesian co-ordinates
\[ \xi, \eta, \zeta \] - natural co-ordinates
\[ N \] - number of nodes per element

Superscripts

\[ h \] - convection
\[ k \] - conduction
\[ r \] - radiation
\[ s \] - surface
\[ t \] - time
\[ \Delta t \] - time increment

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