Finite Time Thermodynamic Modeling and Analysis for an Irreversible Atkinson Cycle

by

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Performance of an air-standard Atkinson cycle is analyzed by using finite-time thermodynamics. The irreversible cycle model which is more close to practice is founded. In this model, the non-linear relation between the specific heats of working fluid and its temperature, the friction loss computed according to the mean velocity of the piston, the internal irreversibility described by using the compression and expansion efficiencies, and heat transfer loss are considered. The relations between the power output and the compression ratio, between the thermal efficiency and the compression ratio, as well as the optimal relation between power output and the efficiency of the cycle are derived by detailed numerical examples. Moreover, the effects of internal irreversibility, heat transfer loss and friction loss on the cycle performance are analyzed. The results obtained in this paper may provide guidelines for the design of practical internal combustion engines.

Key words: finite-time thermodynamics, Atkinson cycles, heat resistance, friction, internal irreversibility, performance optimization

Introduction

Finite time thermodynamics can answer some global questions which classical thermodynamics does not try to answer and conventional irreversible thermodynamics can not answer because of its micro, differential viewpoint. Examples of such questions are: (1) What is the least energy required by a given machine to produce a given work in a given time? (2) What is the most work that can be produced by a given machine in given time, utilizing a given energy? (3) What is the most efficient way to run a given thermodynamic process (optimal path) in finite time? (4) What is the optimal time-dependent (on and off) process? (5) What is the optimal distribution between heat exchanger heat transfer surface areas or heat conductances corresponding to the optimal performance of the thermodynamic devices for the fixed total heat exchanger heat transfer surface area or total heat conductance? (6) What are the quantitative and qualitative features of the effects of heat resistance, internal irreversibility and heat leakage on the performance of real thermodynamic processes and devices? A series of achievements have been made since finite-time thermodynamics was used to analyze and optimize performance of real heat engines [1-10]. Chen et al. [11] studied the efficiency of an Atkinson cycle at maxi-
mum power density without any loss. Qin et al. [12] and Ge et al. [13] derived the performance characteristics of Atkinson cycle with heat transfer loss [12] and with heat transfer and friction-like term losses [13], respectively. Ge et al. [14, 15] considered the effect of variable specific heats on the cycle process and studied the performance characteristic of endoreversible and irreversible Atkinson cycles when variable specific heats of working fluid are linear functions of its temperature and the maximum temperature of the cycle is not fixed. Wang et al. [16] analyzed and compared the performance of an Atkinson cycle coupled to variable-temperature heat reservoirs under maximum power and maximum power density conditions. Zhao et al. [17] analyzed the performance and optimized the parametric criteria of an irreversible Atkinson heat engine. Hou et al. [18] compared the performance of air standard Atkinson and Otto cycles with heat transfer considerations. Lin et al. [19] analyzed the influence of heat loss, as characterized by a percentage of fuel’s energy, friction, and variable specific heats of working fluid on the performance of an air-standard Atkinson cycle when variable specific heats of working fluid are linear functions of its temperature and the maximum temperature of the cycle is not fixed. Al-Sarkhi et al. [20] outlined the effect of maximum power density on the performance of the Atkinson cycle efficiency when the variable specific heats of working fluid are linear functions of its temperature. Abu-Nada et al. [21] and Al-Sarkhi et al. [22] advanced a non-linear relation between the specific heats of working fluid and its temperature and compared the performance of the cycle with constant and variable specific heats. Parlak et al. [23] defined the internal irreversibility by using entropy production, and analyzed the effect of the internal irreversibility on the performance of irreversible reciprocating heat engine cycles. Zhao et al. [24-26] defined the internal irreversibility by using compression and expansion efficiencies and analyzed the performance of diesel, Otto, dual, and Miller cycles when the maximum temperature of the cycle is fixed and the efficiency has a new definition. Zhao et al. [27, 28] used the model of specific heats advanced in refs. [14, 15], the internal irreversibility defined in [24-26], and studied the optimum performance of Otto and diesel cycles when the maximum temperature of the cycles is fixed. Ge et al. [29-30] adopted the specific heat model advanced in refs. [21, 22], the internal irreversibility defined in refs. [24-28], and the friction loss defined in ref. [31], and studied the performance of an irreversible Otto, diesel, and dual cycles when heat transfer, friction, and internal irreversibility losses are considered. This paper will adopt the specific heats model advanced in refs. [21, 22, 29, 30], the internal irreversibility and efficiency defined in refs. [24-30] and the friction loss defined in refs. [29-31], and study the performance of an irreversible Atkinson cycle when heat transfer, friction, and internal irreversibility losses and non-linear variable specific heats of the working fluid are considered.

**Cycle model and analysis**

An air standard Atkinson cycle model is shown in fig. 1. Process 1 → 2S is a reversible adiabatic compression, while process 1 → 2 is an irreversible adiabatic process that takes into account the internal irreversibility in the real compression process. The heat addition is an isochoric process 2 → 3. Process 3 → 4S is a reversible adiabatic expansion, while 3 → 4 is an irreversible adiabatic process that takes into account the internal irreversibility in the real expansion process. The heat rejection is an isobaric process 4 → 1.
In most cycle model, the working fluid is assumed to behave as an ideal gas with constant specific heats. But this assumption can be valid only for small temperature difference. For the large temperature difference encountered in practical cycle, this assumption cannot be applied. According to ref. [21], for the temperature range of 200-1000 K, the specific heat capacity with constant pressure can be written as:

$$C_p = (3.56839 - 6.788729 \times 10^{-4} T + 1.5537 \times 10^{-6} T^2 - 3.29937 \times 10^{-12} T^3 - 466.395 \times 10^{-15} T^4)R_g$$  \hspace{1cm} (1)

For the temperature range of 1000-6000 K, the equation is written as:

$$C_p = (3.08793 + 12.4597 \times 10^{-4} T - 0.42372 \times 10^{-6} T^2 + \frac{67.4775 \times 10^{-12} T^3 - 3.97077 \times 10^{-15} T^4}{R_g}$$  \hspace{1cm} (2)

Equations (1) and (2) can be applied to a temperature range of 200-6000 K which is too wide for the temperature range (300-3500 K) of practical engine. So a single equation was used to describe the specific heat model which is based on the assumption that air is an ideal gas mixture containing 78.1% nitrogen, 20.95% oxygen, 0.92% argon, and 0.03% carbon dioxide.

$$C_p = 2.506 \times 10^{-11} T^2 + 1.454 \times 10^{-7} T^{1.5} - 4246 \times 10^{-7} T + 3162 \times 10^{-5} T^{0.5} + 13030 - 1512 \times 10^4 T^{1.5} + 3063 \times 10^5 T^{-2} - 2212 \times 10^7 T^{-3}$$  \hspace{1cm} (3)

According to the relation between specific heat with constant pressure and specific heat with constant volume:

$$C_v = C_p - R_g$$  \hspace{1cm} (4)

the specific heat with constant volume can be written as:

$$C_v = C_p - R_g = 2.506 \times 10^{-11} T^2 + 1.454 \times 10^{-7} T^{1.5} - 4246 \times 10^{-7} T + 3162 \times 10^{-5} T^{0.5} + 13033 - 1512 \times 10^4 T^{1.5} + 3063 \times 10^5 T^{-2} - 2212 \times 10^7 T^{-3}$$  \hspace{1cm} (5)

where $R_g = 0.287$ kJ/kgK is the gas constant of the working fluid. The unit of $C_v$ and $C_p$ is [kJ/kg·K].

The heat added to the working fluid during process $2 \rightarrow 3$ is:

$$Q_{in} = \int_{T_1}^{T_2} C_p \, dT = \int_{T_1}^{T_2} (2.506 \times 10^{-11} T^2 + 1.454 \times 10^{-7} T^{1.5} - 4246 \times 10^{-7} T + 3162 \times 10^{-5} T^{0.5} + 13033 - 1512 \times 10^4 T^{1.5} + 3063 \times 10^5 T^{-2} - 2212 \times 10^7 T^{-3}) \, dT = M[8353 \times 10^{-12} T^3 + 5816 \times 10^{-8} T^{2.5} - 2123 \times 10^{-7} T^2 + 2108 \times 10^{-5} T^{1.5} + 10433T + 3024 \times 10^4 T^{-0.5} - 3063 \times 10^5 T^{-1} + 1106 \times 10^7 T^{-2}]_{T_1}^{T_2}$$  \hspace{1cm} (6)

The heat rejected by the working fluid during process is:

$$Q_{out} = \int_{T_1}^{T_2} C_p \, dT = \int_{T_1}^{T_2} (2.506 \times 10^{-11} T^2 + 1.454 \times 10^{-7} T^{1.5} - 4246 \times 10^{-7} - 4246 \times 10^{-7} T + 3162 \times 10^{-5} T^{0.5} + 13033 - 1512 \times 10^4 T^{1.5} + 3063 \times 10^5 T^{-2} - 2212 \times 10^7 T^{-3}) \, dT = M[8353 \times 10^{-12} T^3 + 5816 \times 10^{-8} T^{2.5} - 2123 \times 10^{-7} T^2 + 2108 \times 10^{-5} T^{1.5} + 13033T + 3024 \times 10^4 T^{-0.5} - 3063 \times 10^5 T^{-1} + 1106 \times 10^7 T^{-2}]_{T_1}^{T_2}$$  \hspace{1cm} (7)
where \( M \) is the mass flow rate of the working fluid, \( T_1, T_2, T_3, \) and \( T_4 \) [K] are the temperatures at states 1, 2, 3, and 4.

For the two adiabatic processes 1 \( \rightarrow \) 2 and 3 \( \rightarrow \) 4, the compression and expansion efficiencies can be defined as [24-30]:

\[
\eta_c = \frac{T_{2S} - T_1}{T_2 - T_1} \tag{8}
\]

\[
\eta_e = \frac{T_{4S} - T_3}{T_4 - T_3} \tag{9}
\]

These two efficiencies can be used to describe the internal irreversibility of the processes.

Since \( C_p \) and \( C_v \) are dependent on temperature, adiabatic exponent \( k = C_p/C_v \) will vary with temperature as well. Therefore, the equation often used in reversible adiabatic process with constant \( k \) can not be used in reversible adiabatic process with variable \( k \). However, according to refs. [14, 15, 29-40], a suitable engineering approximation for reversible adiabatic process with variable \( k \) can be made, i.e. this process can be broken up into a large number of infinitesimally small processes and for each of these processes, adiabatic exponent \( k \) can be regarded as a constant. For example, for any reversible adiabatic process between states \( i \) and \( j \) can be regarded as consisting of numerous infinitesimally small processes with constant \( k \). For any of these processes, when an infinitesimally small change in temperature \( dT \), and volume \( dV \) of the working fluid takes place, the equation for reversible adiabatic process with variable \( k \) can be written as follows.

\[
T \frac{V^{k-1}}{V^k} = (T + dT)(V + dV)^{k-1} \tag{10}
\]

For an isochoric heat addition process \( i \rightarrow j \), the heat added is

\[
Q_m = C_v(T_j - T_i) = T \Delta S_{v_{eq}} = T C_v \ln(T_j/T_i) \]

So one has

\[
\bar{T} = (T_j - T_i)/\ln(T_j/T_i)
\]

where \( \bar{T} \) is the equivalent temperature of heat absorption process. When \( C_v \) is the function of temperature, the \( C_v(\bar{T}) \) can be regarded as mean specific heat with constant volume.

From eq. (10), one gets

\[
C_v \ln \frac{T_j}{T_i} R_g \ln \frac{V_i}{V_j} \tag{11}
\]

where the temperature in the equation of \( C_v \) is

\[
T = (T_j - T_i)/\ln(T_j/T_i)
\]

The compression ratio is defined as:

\[
\gamma = \frac{V_1}{V_2} \tag{12}
\]

Therefore, equations for reversible adiabatic processes 1 \( \rightarrow \) 2S and 3 \( \rightarrow \) 4S are:

\[
C_v \ln \frac{T_{2S}}{T_1} = R_g \ln \gamma \tag{13}
\]

\[
C_v \ln \frac{T_{4S}}{T_3} - R_g \ln \frac{T_1}{T_{4S}} = -R_g \ln \gamma \tag{14}
\]

For an ideal Atkinson cycle model, there are no heat transfer losses. However, for a real Atkinson cycle, heat transfer irreversibility between working fluid and the cylinder wall is
not negligible. One can assume that the heat transfer loss through the cylinder wall (i.e. the heat leakage loss) is proportional to average temperature of both the working fluid and the cylinder wall and that the wall temperature is constant, \( T_0 \) [K]. If the released heat by combustion per second is \( A_1 \) [kW] the heat leakage coefficient of the cylinder wall is \( B_1 \) [kJkg\(^{-1}\)K\(^{-1}\)] which has considered the heat transfer coefficient and the heat exchange surface, one has the heat added to the working fluid per second by combustion in the following linear relation [12-15]:

\[
Q_{in} = A_1 - MB_1 \frac{T_2 + T_3}{2 - T_0}
\]  

(15)

From eq. (15), one can see that \( Q_{in} \) contained two parts, the first part is \( A_1 \), the released heat by combustion per second, and the second part is the heat leak loss per second, it can be written as:

\[
Q_{\text{leak}} = MB(T_2 + T_3 - 2T_0)
\]  

(16)

where \( B = B_1/2 \).

Taking into account the friction loss of the piston as recommended by Chen et al. [31] for the Dual cycle and assuming a dissipation term represented by a friction force which in a liner function of the velocity gives

\[
f_\mu = \mu v = \mu \frac{dx}{dt}
\]  

(17)

where \( \mu \) [Nsm\(^{-1}\)] is a coefficient of friction which takes into account the global losses and \( x \) is the piston displacement. Then, the lost power is:

\[
P_\mu = \frac{dW_\mu}{dt} = \mu \frac{dx}{dt} \frac{dx}{dt} = \mu v^2
\]  

(18)

If one specifies the engine is a four stroke cycle engine, the total distance the piston travels per cycle is:

\[4L = 4(x_1 - x_2)\]  

(19)

For a four stroke cycle engine, running at \( N \) cycles per second, the mean velocity of the piston is:

\[\bar{v} = 4LN\]  

(20)

where \( x_1 \) and \( x_2 \) [m] are the piston position at maximum and minimum volume and \( L \) [m] is the distance that the piston travels per stroke, respectively.

Thus, the power output is:

\[
P_{at} = Q_{in} - Q_{out} - P_\mu =
\]

\[
= M[8.353 \cdot 10^{-12}(T_3^3 + T_4^3 - T_2^3 - T_4^3) + 5816 \cdot 10^{-8}(T_3^{2.5} + T_4^{2.5} - T_2^{2.5} - T_4^{2.5}) -
-2.123 \cdot 10^{-7}(T_3^2 + T_4^2 - T_2^2 - T_4^2) + 2108 \cdot 10^{-5}(T_3^{1.5} + T_4^{1.5} - T_2^{1.5} - T_4^{1.5}) +
+1.0433(T_3 - T_2) - 1.3333(T_4 - T_1) + 3.024 \cdot 10^4(T_3^{-0.5} + T_4^{-0.5} - T_2^{-0.5} - T_4^{-0.5}) -
-3.063 \cdot 10^3(T_3^{-1} + T_4^{-1} - T_2^{-1} - T_4^{-1}) + 4.1666 \cdot 10^3(T_2^{-1} + T_4^{-1} - T_2^{-1} - T_4^{-1})] - \mu \bar{v}^2
\]  

(21)

The efficiency of the cycle is:
\[ \eta_{\text{ut}} = \frac{P_{\text{ut}}}{Q_{\text{in}} + Q_{\text{leak}}} = \]

\[ M[8.353 \cdot 10^{-12}(T_3^3 + T_1^3 - T_2^3 - T_4^3) + 5.816 \cdot 10^{-8}(T_1^{2.5} + T_2^{2.5} - T_3^{2.5} - T_4^{2.5}) - \]

\[ -2.123 \cdot 10^{-7}(T_3^2 + T_1^2 - T_2^2 - T_4^2) + 2.108 \cdot 10^{-5}(T_3^{1.5} + T_1^{1.5} - T_2^{1.5} - T_4^{1.5}) + \]

\[ + 1.043(T_3 - T_2) - 1.3303(T_3 - T_1) + 3.024 \cdot 10^4(T_3^{-0.5} + T_1^{-0.5} - T_2^{-0.5} - T_4^{-0.5}) - \]

\[ -3.063 \cdot 10^5(T_3^{-1} + T_1^{-1} + T_2^{-1} - T_4^{-1}) + 1.106 \cdot 10^7(T_3^{-2} + T_1^{-2} - T_2^{-2} - T_4^{-2})]\]

\[ \mu^2 \]

\[ M[8.353 \cdot 10^{-12}(T_3^3 - T_1^3) + 5.816 \cdot 10^{-8}(T_1^{2.5} - T_2^{2.5}) - 2.123 \cdot 10^{-7}(T_1^2 - T_2^2) + \]

\[ + 2.108 \cdot 10^{-5}(T_1^{1.5} - T_2^{1.5}) + 1.043(T_3 - T_2) + 3.024 \cdot 10^4(T_3^{-0.5} - T_2^{-0.5}) - \]

\[ -3.063 \cdot 10^5(T_3^{-1} - T_2^{-1}) + 1.106 \cdot 10^7(T_3^{-2} - T_2^{-2})] + MB(T_2 + T_3 - 2T_0) \]

When \( \gamma, T_1, T_3, \eta_s, \) and \( \eta_u \) are given, \( T_{25} \) can be obtained from eq. (13), then, substituting \( T_{25} \) into eq. (8) yields \( T_2, T_{35} \) can be obtained from eq. (14), and the last, \( T_4 \) can be solved out by substituting \( T_{35} \) into eq. (9). Substituting \( T_2 \) and \( T_4 \) into eqs. (21) and (22) yields the power and efficiency. Then, the relations between the power output and the compression ratio, between the thermal efficiency and the compression ratio, as well as the optimal relation between power output and the efficiency of the cycle can be obtained.

**Numerical examples and discussion**

According to ref. [29-31], the following parameters are used: \( T_1 = 350 \) K, \( T_3 = 2200 \) K, \( x_1 = 8 \cdot 10^{-2}, x_2 = 1 \cdot 10^{-2} \) m, \( N = 30, \) and \( M = 4.553 \cdot 10^3 \) kg/s. Figures 2-4 show the effects of the internal irreversibility, heat transfer loss, and friction loss on the performance of the cycle. One can see that when the above three irreversibilities are not included, the power output vs. compression ratio characteristic and the power output vs. efficiency characteristic are parabolic-like curves, while the efficiency will increase with the increase of the compression ratio. When more than one irreversibilities are included, the power output vs. compression ratio characteristic and the efficiency vs. compression ratio characteristic are parabolic like curves and the power output vs. efficiency curve is loop-shaped one.

According to eq. (21), the definitions of the power output, the heat transfer loss has no effect on the power output of the cycle. So fig. 2 only shows the effects of the internal irreversibility and friction loss on the power output of the cycle. Comparing curves 1 with 1’ and 2 with 2’, one can see that the power output increases with the decrease of internal irreversibility. Comparing curves 1 with 2 and 1’ with 2’, one can see that the power output decreases with the increase of friction loss.

Figure 3 shows the effects of the internal irreversibility, heat transfer loss and friction loss on the efficiency of the cycle. Curve 1 is the efficiency vs. compression ratio characteristic without irreversibility. Under this circumstance, the effi-
ciency increases with the increase of compression ratio. Other curves are efficiency vs. compression ratio characteristic with one or more irreversibilities and these curves are parabolic-like ones. Comparing curves 1 with 1’, 2 with 2’, 3 with 3’, and 4 with 4’, one can see that the efficiency increases with the decrease of internal irreversibility. Comparing curves 1 with 3, 2 with 4, 1’ with 3’, and 2’ with 4’, one can see that the efficiency decreases with the increase of heat transfer loss. Comparing curves 1 with 2, 3 with 4, 1’ with 2’, and 3’ with 4’, one can see that the efficiency decreases with the increase of friction loss.

Figure 4 shows the effects of the internal irreversibility, heat transfer loss, and friction loss on the power output vs. the efficiency characteristic. Curve 1 which is a parabolic-like curve is the power output vs. efficiency characteristic of the cycle without irreversibility, while else curves are loop-shaped ones with one or more irreversibilities. Comparing curves 1 with 1’, 2 with 2’, 3 with 3’, and 4 with 4’, one can see that the maximum power output and the efficiency at the maximum power output decrease with the increase of internal irreversibility. Comparing curves 1 with 3, 2 with 4, 1’ with 3’, and 2’ with 4’, one can see that the maximum power output is not influenced by heat transfer loss, while the efficiency at the maximum power output decreases with the increase of heat transfer loss. Comparing curves 1 with 2, 3 with 4, 1’ with 2’, and 3’ with 4’, one can see that both the maximum power output and the corresponding efficiency decrease with the increase of friction loss.

Conclusions

In this paper, an irreversible air standard Atkinson cycle model which is more close to practice is founded. In this model, the non-linear relation between the specific heats of working fluid and its temperature, the friction loss computed according to the mean velocity of the piston, the internal irreversibility described by using the compression and expansion efficiency, and heat transfer loss are presented. The performance characteristics of the cycle were obtained by detailed numerical examples. The effects of internal irreversibility, heat transfer loss and fric-
tion loss on the performance of the cycle were analyzed. The results obtained herein may provide guidelines for the design of practical internal combustion engines.

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Nomenclature

\begin{tabular}{ll}
A_1 & – heat released by combustion per second, [kW] \\
B & – constant related to heat transfer, [kJ kg^{-1} K^{-1}] \\
C_p & – specific heat with constant pressure, [kJ kg^{-1} K^{-1}] \\
C_v & – specific heat with constant volume, [kJ kg^{-1} K^{-1}] \\
k & – ratio of specific heats, [-] \\
L & – total distance of the piston traveling per cycle, [m] \\
M & – mass flow rate of the working fluid, [kgs^{-1}] \\
N & – number of the cycle operating in a second, [-] \\
P_{2,3} & – pressure at different states 2 and 3, [Pa] \\
P_{\mu} & – power output of the cycle, [kW] \\
Q_{in} & – heat added to the working fluid in a second, [kW] \\
Q_{out} & – heat rejected by the working fluid in a second, [kW] \\
R_e & – air constant of the working fluid, [kJ kg^{-1} K^{-1}] \\
T_{1,2,5,2S,5S} & – temperature at different states, [K] \\
V_{1,2} & – volume at different states 1 and 2, [m^3] \\
v & – velocity of the piston, [m s^{-1}] \\
x_1 & – the piston position at maximum volume, [m] \\
x_2 & – the piston position at minimum volume, [m] \\
\eta_c & – efficiency of the cycle, [-] \\
\eta_e & – expansion efficiency, [-] \\
\mu & – coefficient of friction, [N m^{-1}] \\
\gamma & – compression ratio, [-]
\end{tabular}

Greek letters

\begin{tabular}{ll}
\rho_a & – density of the air, [kg m^{-3}] \\
p & – density of the working fluid, [kg m^{-3}] \\
T & – absolute temperature, [K] \\
V & – volume, [m^3] \\
V_0 & – volume at atmospheric pressure, [m^3] \\
W & – work output of the cycle, [kW] \\
\xi & – flow coefficient, [-] \\
\beta & – volume expansion coefficient, [-] \\
\lambda & – thermal conductivity, [W m^{-1} K^{-1}] \\
\kappa & – thermal diffusivity, [m^2 s^{-1}] \\
\alpha & – thermal diffusivity, [m^2 s^{-1}] \\
\phi & – thermal ejection coefficient, [-]
\end{tabular}

References

