MATHEMATICAL MODELING OF A MULTI-STREAM BRAZED ALUMINUM PLATE FIN HEAT EXCHANGER

by

Ahmed A. KOHIL a, Hassan A. FARAG a, and Mona E. OSSMAN b*

a Chem. Eng. Dept., Faculty of Engineering, Alexandria University, Alexandria, Egypt
b Mubarak City for Scientific Research and Technological Application, Alexandria, Egypt

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The need for small size and lightweight heat exchangers in many applications has resulted in the development of many heat transfer surfaces. This type of heat exchanger is much more compact than can be practically realized with circular tubes. In this work a steady-state mathematical model that representing one of the plate fin heat exchangers enclosed in cold box of an ethylene plant has been developed. This model could evaluate the performance of the heat exchanger by predicting the outlet temperatures of the hot and cold streams when the inlet conditions are known. The model has been validated by comparing the results with actual operating values and the results showed good agreement with the actual data. Sensitivity analysis was applied on the model to illustrate the main parameters that have the greatest influence on the model calculated results. The sensitivity analysis showed that the hot stream outlet temperature is more sensitive to cold streams inlet temperature and less sensitive to hot stream inlet temperature and thermal resistance (fouling), while the cold stream outlet temperature is more sensitive to cold streams inlet flow rate and less sensitive to fouling.

Key words: fouling, heat exchanger, modeling, simulation

Introduction

Studies on enhanced heat transfer have been reported for more than 100 years now. In recent years, due to the increasing demand by industries for heat exchangers that are more efficient, compact and less expensive, heat transfer enhancement has gained serious momentum [1]. Brazed aluminum plate fin heat exchangers have been successfully used in a variety of applications. The major applications have been in the cryogenic separation, Air Separation Unit (ASU); natural gas processing (NGP) and liquefaction of natural gas (LNG); the production of petrochemicals and treatment of off gases; large refrigeration systems [2]. This type of heat

* Corresponding author; e-mail: ai4728@wayne.edu, mhr1410@hotmail.com
exchangers is capable of handling a wide variety of fluids in many different types of applications. In general, fluids should be clean, dry, and non-corrosive to aluminum, trace impurities of \( \text{H}_2\text{S}, \text{NH}_3, \text{CO}_2, \text{SO}_2, \text{NO}_2, \text{CO}, \text{Cl} \), and other acid-forming gases do not create corrosion problems in streams with water dew point temperatures lower than the cold-end temperature of the brazed aluminum plate-fin heat exchanger [3]. The design and simulation of multi stream plate fin heat exchangers are markedly different from those of two-fluid exchangers. Features like bypass heat transfer [4] or crossover in temperature [5], common in multi stream heat exchangers, have no equivalent in two-stream units. In the simplest form a multi stream handles three different streams of fluids. Sorlie [6] developed a design theory for three-fluid heat exchangers of the concentric-tube and plate fin types, in which the intermediate and cold streams were thermally insulated. He derived closed form solutions for the temperatures of all the streams by solving a set of three first order linear ordinary differential equations and defined an expression for the overall effectiveness. Some of the theoretical results were compared with experiments and excellent agreement was obtained. Aulds et al. [7] extended the work of Sorlie by analyzing the case, in which all three streams were in thermal communication, which is relevant to many three-fluid heat exchangers used in cryogenic systems. Ghosh [8] developed a new algorithm for the analysis of multi stream heat exchangers. The numerical technique involves partitioning of the exchanger in both axial and normal directions. Conservation equations written for each segment are solved using an iterative procedure. The algorithm has been tested against published results and good agreement has been observed. Georgiadis [9] developed a mathematical modeling and simulation of complex plate heat exchanger arrangements under milk fouling, using detailed dynamic models. A complex fouling model based on a reaction mass transfer scheme is coupled with a general thermal dynamic model of plate heat exchangers. All the important factors affecting milk heat treatment are formally quantified. The final model comprises a set of partial differential, integral, and algebraic equations. Reibero [10] developed an algorithm for the steady-state simulation of a plate heat exchanger. In this algorithm he took into account a general unit with \( n \) flow channels, in which the hot and cold streams may flow co- or counter currently. The algorithm was successfully utilized to simulate the steady-state operation of an industrial plate heat exchanger used for pasteurizing milk. Gut [11] developed a mathematical model in algorithmic form for the steady-state simulation of gasket plate heat exchangers with generalized configurations. The configuration is defined by the number of channels, number of passes at each side, fluid locations, feed connection locations, and type of channel-flow. The main purposes of this model are to study the configuration influence on the exchanger performance and to further develop a method for configuration optimization. The main simulation results are: temperature profiles in all channels, thermal effectiveness, distribution of the overall heat transfer coefficient and pressure drops. Moreover, the assumption of constant overall heat transfer coefficient is analyzed. Tovazhyansky [12] developed numerical simulation of multi component mixtures condensation in plate condensers. A numerical simulation using semi-empirical equations of heat and mass transfer performance along the surface of plate condensers was carried out for different multi component mixtures with non condensable components. The plates with cross-corrugated patterns for plate condensers were used. The simulation was done for four different types of corrugated plates of industrially manufactured plate heat exchangers. The complexity of compact exchanger design equations results from the exchanger’s unique ability to transfer heat between multiple process streams [13] and the wide array of possible flow configurations. These complexities make hand calculations tedious and simple correlations inapplicable. However, computer programs and process simulators allow engineers to more easily
evaluate complex brazed aluminum exchangers. Pinguad et al. [14] and Luo et al. [15] have also carried out steady-state and dynamic simulation of plate fin heat exchangers. Luo et al. [16] have developed an analytical model of a multi stream exchanger with constant physical properties. In a separate paper [17], the authors have proposed a more generalized analytical solution for predicting the thermal performance of multi stream heat exchangers and their networks. This model is also applicable to other types of one-dimensional heat exchangers such as shell and tube and plate heat exchangers. The objective of the present work is to develop a steady-state mathematical model for one of the series of plate fin heat exchangers used in an ethylene plant located in Alexandria, Egypt. This model could evaluate the performance of the heat exchanger by predicting the outlet temperatures of the cold and hot streams when the inlet conditions are known.

The mathematical model

This study will focus on one exchanger of the cold box of Sidi Kerir Petrochemicals Company, Alexandria, Egypt. The plate fin heat exchanger contains the following streams (fig. 1):

Cold streams: Normal temperature [°C]
- methane off gas (gas), -62 °C
- low pressure hydrogen (gas), -62 °C
- high pressure hydrogen (gas), -62 °C
- de-ethanizer bottom feed (liq.), -57 °C
- de-ethanizer top feed (liq.), -57 °C
- recycle ethane (liq.), -42 °C

Hot stream:
- ethylene refrigerant (liq.) -14 °C

Arranged in the order shown in fig. 1.

Assumptions

(1) Steady-state operation of the heat exchanger and neglect influence of inlet/outlet expansion/contraction effect.
(2) Multi-component streams are assumed to be two-component streams (one main cold stream and one main hot stream).
(3) Zero heat losses to the surroundings.
(4) No heat transfer through inlet and outer headers and distributors.
(5) Thermal resistance in the cold side may increase in the de-ethanizer feed streams only because they may contain heavier-than-design components during the start up period conditions, which may cause freezing to take place on the heat transfer surfaces. (Normally during startup this is prevented from reaching the exchanger, but sometimes it may reach by mistake). The other streams are either light gas components from the overheads of flash drums and distillation columns mainly consisting of hydrogen, methane, and ethane, or a light component liquid like ethylene.

Figure 1. Schematic of cold and hot streams for the exchanger
The steps of the calculations are performed in the following sequence to fulfill the steady-state heat balance equation:

$$Q_c = -Q_h = Q_{UA}$$  \hspace{1cm} (1)

where $Q_c$ is the total heat energy gained by the cold streams, $Q_h$ – the total heat energy given away by the hot stream, and $Q_{UA}$ – the heat transferred through the heat exchanger based on the overall heat transfer coefficient.

Heat gained by cold streams $Q_c$ can be obtained from:

$$Q_c = \sum_{i=1}^{6} \left( m_{ci} \cdot C_{pc} \cdot \Delta T \cdot \frac{T_{c,i}^{\text{out}}}{T_{c,i}^{\text{in}}} \right)$$  \hspace{1cm} (2)

Heat gave away by hot streams $Q_h$ can be obtained from:

$$Q_h = m_h \cdot C_{ph} \cdot \Delta T \cdot \frac{T_{h,i}^{\text{out}}}{T_{h,i}^{\text{in}}}$$  \hspace{1cm} (3)

Heat energy based on the overall heat transfer coefficient $Q_{UA}$ can be obtained from:

$$Q_{UA} = UA \cdot (dTLM)^*$$  \hspace{1cm} (4)

$$(dTLM)^* = \frac{\sum_{i=1}^{6} m_{ci} \cdot (dTLM)_i}{m_c}$$  \hspace{1cm} (5)

where $dTLM$ [K] is the logarithmic mean temperature difference, $T_{hi}$ [K] – the inlet temperature of hot fluid, $T_{ho}$ [K] – the outlet temperature of hot fluid, $T_{ci}$ [K] – the inlet temperature of cold fluid, and $T_{co}$ [K] – the outlet temperature of cold fluid.

An approach has been suggested to calculate an average value for the logarithmic mean temperature difference $(dTLM)^*$ as follows:

$$\frac{1}{n} \sum_{i=1}^{n} (dTLM)_i = \frac{1}{n} \sum_{i=1}^{\frac{n}{2}} \left( m_{ci} \cdot (dTLM)_i \right)$$

The heat transfer surface area [2] for the finned passages $A$ (fig. 3), which consists of the primary and secondary transfer surfaces can be obtained from:

$$A = 2N_p (LW) (1 - nd) + 2n(H - d)$$  \hspace{1cm} (7)

where per unit area of each parting sheet:
the primary surface is given by \( 2(1 - nd) \)
the secondary surface is given by \( 2n(H - d) \)
and \( n \) [\( \text{m}^{-1} \)] is the fin density; \( d \) [\( \text{m} \)] – the fin thickness, \( H \) [\( \text{m} \)] – the fin height, \( N_p \) – the number of passages per core, \( L \) [\( \text{m} \)] – the passage length, and \( W \) [\( \text{m} \)] – the passage width.

The overall heat transfer coefficient \( (U) \) \(^2\) can be obtained from:

\[
\frac{1}{UA} = \frac{1}{\sum(h_iA)_hi} + \frac{1}{\sum(h_iA)_ci}
\]

(8)

where \( h_i \) [\( \text{kJh}^{-1}\text{m}^{-1}\text{K}^{-1} \)] is the effective heat transfer coefficient of a stream, \( A \) [\( \text{m}^2 \)] – the overall heat transfer surface and subscripts \( hi \), and \( ci \): hot or cold stream \( i \).

\[
\frac{1}{h_o} = \frac{1}{h} + r
\]

(9)

where \( h \) [\( \text{kJh}^{-1}\text{m}^{-1}\text{K}^{-1} \)] is the heat transfer coefficient of a stream, and \( r \) [\( \text{hm}^2\text{KkJ}^{-1} \)] – the thermal resistance.

Heat transfer coefficient of streams \( (h) \) \(^3\) can be obtained from:

\[
h = \frac{jG_mC_p}{3\sqrt{Pr}}
\]

(10)

where \( j \) is the Colburn factor for a finned passage, \( G_m \) [\( \text{kgm}^{-1}\text{h}^{-1} \)] – the mass flux of a stream, \( C_p \) [\( \text{kJkg}^{-1}\text{K}^{-1} \)] – the specific heat capacity of a stream at constant pressure, and \( Pr \) – the Prandtl number of a stream.

Colburn factor \( (j) \) (or \( \text{StPr}^{2/3} = \Phi(\text{Re}) \)) can be obtained from the equations developed from interpolation/extrapolation of the empirical data tables \(^3\) relating the \( j \) factor to \( \text{Re} \) (fig. 4) to match the exchanger dimensions and conditions. These equations have the formula:

\[
j = a\text{Re}^{-b}
\]

(11)

Figure 4. Colburn factor \( (j) \) as a function of \( \text{Re} \) number
Reynolds number [18] can be obtained from:

\[ \text{Re} = \frac{4R_h G_m}{\mu} \]  

(12)

where \( R_h [\text{m}] \) is the hydraulic radius \((A_c L/A)\), and \( \mu [\text{kgm}^{-1}\text{h}^{-1}] \) is the viscosity of fluid.

Mass flux of a stream \( G_m \), also known as the mass velocity, is obtained from [2]:

\[ G_m = \frac{m}{A_c} \]  

(13)

where \( m [\text{kgh}^{-1}] \) is the mass flow rate.

The free flow area \((A_c) [2]\) can be obtained from:

\[ A_c = (H - t)(p - t) \text{ (No. of fins)} \]  

(14)

Prandtl number for the streams is obtained from:

\[ \text{Pr} = \frac{C_p \mu}{k} \]  

(15)

and for simplifying the calculations, the physical properties \((C_p, \mu, \text{and } k, \text{where } k [\text{kJh}^{-1}\text{m}^{-1}\text{K}^{-1}] \) is the thermal conductivity) were evaluated at the average temperature, or \( T_m = (T_{\text{in}} + T_{\text{out}})/2 \).

**Validation of the model**

The proposed model was validated according to fig. 5, to check whether the model reproduces system behavior within acceptable bounds. The model is coded using MATLAB. The validation done by comparing predicted model output to actual measured output. The predicted model output generated from the developed model will be compared with actual running data of the exchanger after one year of being in service. By looking at tab. 1, which contains summary of the model results and validation of the model, it was found that the validation showed that the model results showed good agreement with the actual measured output form the heat exchanger.

The developed model has been used to simulate the effect of different parameters on the outlet temperatures of the cold and hot streams in the steady state conditions. The results of these simulations are represented as follows.

<table>
<thead>
<tr>
<th>Plant load [%]</th>
<th>Actual value (^\circ\text{C})</th>
<th>Calculated value (^\circ\text{C})</th>
<th>Error in calculation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>(T_{c \text{ out}}) = -31.3</td>
<td>(T_{c \text{ out}}) = -30.6</td>
<td>-2.29</td>
</tr>
<tr>
<td></td>
<td>(T_{h \text{ out}}) = -51.5</td>
<td>(T_{h \text{ out}}) = -52.2</td>
<td>1.34</td>
</tr>
<tr>
<td>83</td>
<td>(T_{c \text{ out}}) = -30.6</td>
<td>(T_{c \text{ out}}) = -31.7</td>
<td>3.47</td>
</tr>
<tr>
<td></td>
<td>(T_{h \text{ out}}) = -50.8</td>
<td>(T_{h \text{ out}}) = -53.0</td>
<td>4.15</td>
</tr>
<tr>
<td>78</td>
<td>(T_{c \text{ out}}) = -33.13</td>
<td>(T_{c \text{ out}}) = -30.61</td>
<td>-8.23</td>
</tr>
<tr>
<td></td>
<td>(T_{h \text{ out}}) = -45.08</td>
<td>(T_{h \text{ out}}) = -51.7</td>
<td>12.8</td>
</tr>
</tbody>
</table>
Effect of inlet flow rate of cold and hot streams

It is noticed that increasing the cold streams inlet flow rates, with all other parameters kept constant will result in lower cold streams outlet temperatures $T_{c\text{ out}}$, (fig. 6). This is because increasing the flow rate of the cold streams entering the heat exchanger will decrease the residence time, therefore the outlet temperature of the cold streams, $T_{c\text{ out}}$ will be lower. Also increasing the cold streams inlet flow rates will lower the hot stream outlet temperature $T_{h\text{ out}}$.

This can be explained by the simple eq. (16):

$$mC_p\Delta T_c = mC_p\Delta T_h$$

(16)

To keep this equation balance then the $\Delta T_h$ must increase and since the simulation is done by changing one process variable, keeping all other variables constant, therefore $T_{h\text{ out}}$ must decrease for the above equation to apply. This is also can explain the effect of increasing the hot stream flow rate which represented in fig. 7.

The figure shows that increasing the hot streams inlet flow rate, with all other parameters kept constant will result in higher cold streams outlet temperatures $T_{c\text{ out}}$ and higher hot stream outlet temperature $T_{h\text{ out}}$ which agrees with the basic principles of heat transfer [19].

Effect of cold stream inlet temperature

Increasing the cold streams inlet temperature, with all other parameters kept constant will result in higher cold streams outlet temperatures $T_{c\text{ out}}$, and higher hot stream outlet temperature $T_{h\text{ out}}$, (figs. 6 and 8).
Effect of hot stream inlet temperature

By decreasing the cold streams inlet temperature, the model predicts that the outlet temperature of the cold streams will decrease as well as the outlet temperatures of hot stream keeping all other variables constant. These computed results are given in fig. 9. The obtained results show that they are all in agreement with the basic principles of heat transfer within any heat exchanger.

Figure 9 shows that increasing the hot streams inlet temperature, with all other parameters kept constant will result in higher cold streams outlet temperatures $T_{c_{out}}$ while results in almost no change in the hot stream outlet temperature $T_{h_{out}}$ and this results showed good agreement with the heat transfer principles [19].

Effect of thermal resistance (fouling)

Figure 10 shows that if the cold streams side thermal resistance (fouling on the two liquid de-ethanizer feed streams side) is increased this will result in a slightly lower cold streams outlet temperatures $T_{c_{out}}$, and a slightly higher hot stream outlet temperature $T_{h_{out}}$, while fig. 11 shows that if the hot stream side thermal resistance (fouling) is increased this will result in a slightly lower cold streams outlet temperatures $T_{c_{out}}$, and a slightly higher hot stream outlet temperature $T_{h_{out}}$.

Sensitivity analysis of the model

A sensitivity analysis was performed on the model to identify to what extent the model is sensitive to the variation in some of the parameters, and which of these parameters has a higher impact on the results calculated by the model.
Figure 12 represents that the cold streams outlet temperature $T_{c\text{ out}}$ is more sensitive to cold streams inlet flow rate $F_{c\text{ in}}$, and hot stream inlet flow rate $F_{h\text{ in}}$. While it is less sensitive to the thermal resistance on the cold side $r_c$ and the thermal resistance on the hot side $r_h$.

Figure 13 shows that the hot stream outlet temperature $T_{h\text{ out}}$ is more sensitive to cold streams inlet temperature $T_{c\text{ in}}$. While it is less sensitive to the hot stream inlet temperature $T_{h\text{ in}}$ and the thermal resistance on the cold and hot sides $r_c$ and $r_h$. 

![Figure 12. Sensitivity analysis of cold streams outlet temperature $T_{c\text{ out}}$.](image)
Conclusions

A mathematical model developed for the plate fin heat exchanger which contains 6 cold streams and one hot stream has been tested against actual operating conditions of the plant that has been running for more than 6 years, taking into consideration any expected effect for present or future fouling on the heat transfer surfaces. Multi-component streams were assumed to be two component streams for simplification; excluding components with minor percentages. The model showed very close results to the actual values for the cold and hot streams outlet temperatures. The effect of thermal resistance (fouling) which was susceptible after more than 6 years of operation and due to abnormal conditions during startups in the two liquid cold streams side was tested and found to have a slight effect on the rate of heat transfer and the streams outlet temperatures for both cold and hot sides. The other test which was done for the effect of the thermal resistance (fouling) in the hot stream passages showed also a slight effect of this resistance on the rate of heat transfer and streams outlet temperatures. The sensitivity analysis showed that the hot stream outlet temperature $T_{h_{out}}$ is more sensitive to cold streams inlet temperatures $T_{c_{in}}$ and less sensitive to hot stream inlet temperature $T_{h_{in}}$ and thermal resistance (fouling), while the cold stream outlet temperature $T_{c_{out}}$ is more sensitive to cold streams inlet flow rate $F_{c}$ and less sensitive to fouling. The model by predicting the streams outlet temperatures is a useful tool for monitoring the exchanger performance during normal operation.

Nomenclature

\begin{itemize}
  \item $A$ – effective heat transfer surface of a passage or layers of a stream, [m$^2$]
  \item $A_{c}$ – exchanger minimum free-flow area, [m$^2$]
  \item $A_{d}$ – designed (or estimated) overall effective heat transfer surface, [m$^2$]
  \item $A_{r}$ – required overall effective heat transfer surface, [m$^2$]
\end{itemize}
References


