A NEW CURVE FOR TEMPERATURE-TIME RELATIONSHIP
IN COMPARTMENT FIRE

by

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An idealized temperature curve of compartment fire has three, distinct phases: growth phase, steady-burning (or fully developed) phase, and decay phase. Standard temperature-time curves are not suitable for describing the fire phenomena because it does not take into account fire load nor ventilation conditions, and fire according to these curves never decays. The temperature curve of compartment fire, especially the growth phase, may be treated like the pulse phenomena. This means that it is possible to approximate the fire development with some suitable function that satisfactorily describes the pulse phenomena. The shape of the time-temperature curve for fire with flashover has characteristic peak before the decay phase, or slow decreases before the decay phase – in absence of flashover. In this paper we propose the definition of the time-temperature curve by means of a unique function in which the quantities of fuel and ventilation conditions are defined with parameters. This function is very convenient for approximation of the development of compartment fire with flashover, for smouldering combustion which has fire curve without characteristic peak, this function can be used only for approximation of growth period of fire.

Key words: fire curve, fire development, temperature curve, approximation

Introduction

Generalization of the unique temperature-time fire curve is very complex task because many parameters such as ignitability and combustibility of the material in a compartment, rate of heat release, flame spread, etc., are to be taken into account. Further, shape of the temperature-time curve depends on parameters such as open doors or windows, vent flows and similar details.

An idealized temperature curve of compartment fire has three, distinct phases: growth phase – development phase from ignition to flashover, steady-burning (or fully developed) phase, and decay phase. There is a rapid transition stage called flashover between the pre-flashover and fully developed fire. This is shown in fig. 1 which illustrates the whole process in terms of heat released against time. Flashover is defined as the “relatively rapid transition between the primary fire which is essentially localized around the item first ignited, and the general conflagration when all surfaces within the compartment are burning.” If there

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is insufficient fuel, ventilation or propensity for fire spread, then a compartment fire may not achieve a rate of heat release sufficient for flashover to occur. The fire will remain small around the items first ignited (fig. 1, on the left side, the curve bellow).

In the most cases the front edge of a fire curve may be considered to be a reliable confirmation about fire growth. The growth phase has primary importance for obtaining a realistic prediction of detector and sprinkler activation, time to start evacuation, and time to initial exposure of occupants. Moreover, with suitable analytical function, which describes the growth phase, it is possible very fast — immediately after ignition, to predict the temperature development, its maximal value, and the time interval needed to achieve this value [1, 2].

Figure 1. Temperature-time phases of a well-ventilated compartment fire (idealized fire curve on the left; curve on the right is typical temperature history of gas contents of compartment in fire according to Harmathy, A New Look at Compartment Fires, Part II, p. 328 [3]).

There are many ways in which the fire can develop after ignition, the fire growth has smouldering period with short duration in the most cases, but from mathematical point of view it is very interesting approximation of fires with rapid development — with characteristic peek. The shape of fire curve depends on many parameters; however, fuel and compartment geometry, as well as ventilation are the main factors that determine the growth phase and the shape of fire curve. No single temperature-time curve can represent all fires, but the most fire curves are similar in their growth phases, independently of fuel type [4-7].

Time-temperature curves in engineering applications

Fire resistance testing of construction was formalized over 80 years ago although testing had been going on prior to that in an unplanned and informal manner. The earliest recorded tests were in UK, Germany, and USA. The heating conditions in the furnace are described by a standard temperature-time curve.

The British Standard (BS) temperature-time curve is given by (1), first published in 1932. The temperature of the furnace is programmed by controlling the rate of supply of fuel. Traditionally fire resistance design in the UK has assumed fire exposure to equal the BS fire curve:

$$ T = T_0 + 345\log(0.133t + 1) $$

where $t$ [s] – the time, $T$ – the temperature in the compartment, $T_0$ – the temperature in the compartment at the start of the fire – usually 20 °C.
The first ASTM standard for fire resistance testing, C19 (now E119), was published in 1918. The standard fire curve is prescribed by a series of points rather than an equation but is almost identical to the BS curve. The ASTM E119 standard temperature curve was not intended to be representative of a real fire scenario, but instead it is an envelope that represents maximal values of temperature during fire that may occur in buildings. Curves similar to that of ASTM E119 are used in National Fire Protection Association, Underwriters Laboratories, and International Organization for Standardization (ISO) standards [8].

Generally, there are three different approaches to the temperature-time curve and they incorporate various factors. Considering, in a simplistic form, different fuel types and ventilation conditions, BS EN1991-1-2: 2002 and PD7974-1: 2003 provides nominal temperature-time curves (nominal fire models) given by eq. (2), eq. (3), and eq. (4). In all models, variables have meanings as: \( t \) [min.], \(-t\) time, \( T\) – temperature in the compartment, and \( T_0 \) – temperature at the start of the fire.

1. Standard time-temperature relationship according to ISO 834 (for representing a fully developed compartment fire):

\[
T = \begin{cases} 
T_0 + 504t^{0.141} & t < 10 \text{ minutes} \\
T_0 + 345\log(8t + 1) & t \geq 10 \text{ minutes}
\end{cases}
\] (2)

This model can be used in many engineering applications; the use of this model depends on purpose of calculations and should be used when no extra information of the fire is available. The main characteristics of this model are: the fire is assumed to be active within the whole compartment (even if the compartment is huge), independent of the actual size of this compartment; the fire never decays, not even after all combustible materials have been exhausted; it does not depend on the compartment’s fire load nor on ventilation conditions [9-12].

2. External temperature-time curve (for the outside of external walls, which can be exposed to fire from different parts of the facade):

\[
T = T_0 + 660[1 - 0.686\exp(-0.32t) - 0.313\exp(-3.8t)]
\] (3)

This model is applicable to the external face of separating walls which are exposed to smoke coming from a fire developing within the compartment that is internal to the walls. Such a fire is characterized by less elevated temperatures and should therefore not be used for structural elements that are exterior to the compartment but that can still be exposed to higher temperatures (e.g. through openings). In this case, a different model should be used.

3. Hydrocarbon temperature-time curve (for representing a fire with hydrocarbon or liquid fuel):

\[
T = T_0 + 1080[1 - 0.325\exp(-0.167t) - 0.675\exp(-2.5t)]
\] (4)

This model is applicable to fire hazards which are caused by the ignition of hydrocarbons and is characterized by significantly elevated temperatures.

Both the BS temperature time curve and the ASTM curve are shown in fig. 2, on the left side, and ISO 834 temperature-time curve on the right side.

In addition to described models, parametric fires are also described by a temperature-time curve, which however depends on a considerable number of environmental parameters and therefore provides a more realistic approach to how a fire hazard develops and evolves. A parametric fire curve takes into account the compartment’s ventilation conditions and thermal properties of its bounding walls. Parametric fire curves furthermore, consider the
fire’s decay phase, thus allowing for a temperature decrease once the fire load has been exhausted. As temperatures are assumed to be uniformly distributed within the compartment, those fire models should in principle only be applied to compartments of a moderate size. Parametric fires (basic fire model) are valid for fire compartments up to 500 m² of floor area, with maximum height of 4 m and without openings in the roof. It should be noted that some recent papers [13] point that the uniform temperature assumption is not quite correct.

Figure 2. Standard temperature-time curves

Typical parametric fire curve in accordance with BS EN1991-1-2 [14] is shown on fig. 3, on the left side. A complete fire curve comprises a heating phase represented by an exponential curve that lasts maximal value of temperature $T_{\text{max}}$, followed by a linearly decreasing cooling phase until a residual temperature (usually the ambient temperature). The maximal temperature $T_{\text{max}}$ and fire duration $t_{\text{max}}$ are two primary factors affecting the behavior of a structure in fire. Consequently, they are adopted as the governing parameters in the design formulae for the parametric fires.

On the right side in fig. 3, the parametric curves for compartment with area of 300 m² and fire load of 800 MJ/m², are shown. (The fire load 40 kg/m² timber cribs is equivalent to 720 MJ/m², [15], pp. 307). Shape of those curves depends on opening factor (0.04 m$^{1/2}$, 0.08 m$^{1/2}$, etc.).

Figure 3. Parametric fire curves
It is obvious that the relationship between the standard fire test and “real fires” is the subject of questions and controversy. At the beginning of century, 1928, Ingberg [16] proposed a solution to the problem of standard fire curves not representing real fires. He suggested that fire severity could be related to the fire load of a room and expressed as an area under the temperature-time curve. The severity of two fires was equal if the area under the temperature-time curves were equal (above a base line of 300 °C). Thus, any fire temperature-time history could be compared to the standard curve. In fig. 4, on the left side, the numbers identified with each curve indicate the fire load in kg/m² (60, 30 or 15), and the ventilation area (½ or ¼) as a proportional fraction of one wall. For example, number 60(1/2) denotes 60 kg/m² fire load and 50% open wall.

On the other side, Magnusson et al. [17] made the same assumptions when setting up a computer model for solving the problem as follows. The energy release rate is ventilation-controlled during the fully developed stage, but based on data from full-scale experiments during the growth and decay stages, combustion is complete and takes place entirely within the confines of the enclosure. The temperature is uniform within the enclosure at all times, a single surface heat transfer coefficient is used for the entire inner surface of the enclosure and the heat flow to and through the enclosure boundaries is one-dimensional (i.e., effects of corners and edges are ignored) and the boundaries are assumed to be “infinite slabs”. The result of this approach is known as “Swedish curves” (fig. 4, on the right side [9, 17]).

The compartments used in the both tests were small by modern standards, but the results are indicative of the influence of fire load and ventilation on the temperature-time environment generated within fire compartments. In the other words, fire load and ventilation should be basic parameters in any approximation of temperature-time fire curve in a compartment.

Lie et al. [6] developed temperature curves based on results of the analysis of Kawagoe et al. [18] for ventilation-controlled fires. The expression is:
\[ T = \frac{0.1}{250(10F)^{0.7}} \exp(-F^{2}t)[3(1-\exp(-0.6t)) - (1-\exp(-3t)) + 4(1-\exp(-12t))] + C\left(\frac{600}{F}\right)^{0.5} \] (5)

where \( T \) is the temperature, \( t \) [h] – the time, \( C \) – the constant which depends on the boundary material properties (\( C = 0 \) for heavy materials – \( \rho \geq 1600 \text{ kg/m}^3 \), \( C = 1 \) for light materials – \( \rho < 1600 \text{ kg/m}^3 \)), and \( F \) – opening factor [m\(^{1/2}\)].

This expression is valid for \( t \leq 1 + 0.008F \) and \( 0.01 \leq F < 0.15 \). Value of \( t = 1 + 0.008F \) should be used for \( t + 0.008F \), and if \( F > 0.15 \) then \( F = 0.15 \) should be used. During decay period temperature is:

\[ T = -600\left(\frac{t}{\tau} - 1\right) + T_{\tau} \] (6)

where \( T \) is the temperature, \( t \) [h] – the time, \( \tau \) [h] – the time at which decay period begins, \( T_{\tau} \) – the temperature at time \( t = \tau \) and \( t > \tau \) as well as \( T \geq 20 \degree C \).

The temperature of fire gases in compartment depends on relationship between fire load and ventilation. Highest fire temperature often occur within ventilation controlled regime, but heat penetration into compartment boundaries is always most intense at the point of transition between ventilation-controlled and fuel controlled-regime with air flow factor = 1 (dimensionless), (fig. 5) [19, 20].

According to Harmathy [3], the rate of decrease of temperature for fires with relative short duration is in order of 15-20 \degree C and for longer duration of fully developed period of fire, the rate of decrease of temperature is lower. This and other approaches have been compared in the engineering guide SFPE by Lie [21, 22], (fig. 6).

From fig. 6 is obvious that the fire load determines the duration of fire, and opening factor determines both the duration and the intensity of the fire. Characteristic temperature curves for various fire loads are shown on the left side of fig. 6 (opening factor \( F = 0.05 \text{ m}^{1/2} \) – heavy bounding materials), and influence of opening factor on fire temperature course, on the right side of fig. 6.

Some recent models, for example fire curve known as “BDF curve” proposed by Barnett [23-25], use three factors for temperature-time relationship: maximal gas temperature, the time at which this temperature occurs and constant which defines shape for the curve. The basic equation that produces a BDF curve is:
where $T$ is the temperature, $T_a$ – the ambient temperature, $T_m$ – maximum value of temperature generated above $T_a$, $z = (\log t - \log t_m)^2/s_c$ where $t$ [min.] is time of ignition and $t_m$ [min.] is the time at which $T_m$ occurs, and $s_c$ – the dimensionless number which defines shape for time-temperature curve. In fig. 7, BDF temperature-time curves using different values for factors $T_m$ and $t_m$ are shown (at the top, and at the bottom of fig. 7, respectively).
From previous considerations it can be seen that would be very reasonable to find convenient analytical function that characterizes temperatures during the fire with minimum numbers of parameters. That function should contain a parameter which is obtained on the basis of relationship between fire load and ventilation, and this parameter should be represented by factor which defines shape of temperature-time curve.

From the mathematical point of view, it is obvious that the growth phase of fire may be treated like pulse phenomena. This means that it is possible to approximate the fire growth phase with Heaviside function, with a linear combination of exponential functions or with some suitable function that satisfactorily describes the pulse phenomena.

The steady burning phase may originate with or without flashover. At the post-flashover level the steady burning phase is fuel or ventilation-controlled. From this reason, the shape of the temperature-time curve will have characteristic peak before the decay phase – in a point of flashover, or slow decrease before the decay phase – in absence of flashover.

The decay phase is characterized with a fast decrease because a burning rate declines as the fuel is exhausted. Harmathy [3] indicates the decay period begins at a temperature of 0.8 $T_{\text{max}}$ after the peak of fire curve (fig. 1, on the right side). In engineering applications and in absence of experimental data, this phase may be considered as the inverse of the growth phase. It is often assumed that the decay phase begins when 20% of fuel is left. With this assumption, the inverse shape of the decay phase from the growth phase is technically reasonable.

**The new function for describing time-temperature relationship**

During our research, in the absence of experimental results, we used fire test data found on available literature, as well as previously described curves obtained from temperature-time correlations related to compartment fires. The basic idea presented in this paper is to define the family of curves that satisfactorily and accurately describe the phases of fire. With this approach, each of the curves differs only in one parameter – according to fuel type and ventilation. Two factors in the new function are related to maximal temperature and time elapsed to reach this temperature, while the third factor determines the shape of the curve [26, 27].

For approximation of fire phases, we propose, as very simple and accurate, function as follows:

$$y = a \left[ \frac{x}{b} \exp \left( 1 - \frac{x}{b} \right) \right]^c$$  \hspace{1cm} (8)

where $y$ is the temperature, $x$ [min. – the time, $a$ – the maximal temperature in steady burning phase, depends on fuel type and ventilation, $b$ [min.] – the time instance for maximal temperature, $c$ – the correction parameter (dimensionless) that depends on other conditions (compartment geometry, condition related to ignition, flame spread, etc.)

The parameter $c$ that defines fuel type and ventilation is, in fact, the rate of rise of the given function. This approach is similar to the approach for the construction of BDF curves, in the sense that the proposed function is described by three parameters: the maximal temperature, the time reaching the maximal temperature and the factor that determines the shape of the curve. Naturally, the full justification for usage of this function can be proved only by experimental researches that will assign full physical meaning for each parameters in eq. (8), especially for parameter $c$, whose value will depend on fire load and ventilation.
In fig. 8, the curves in accordance to the function defined with fig. (8) for fixed values of $a$ and $b$, and for values $c > 1$, $c = 1$, and $c < 1$ are shown.

In order to prove the proper choice of function (8) for temperature-time relationship, we must know the maximal value of temperature raised during the steady burning phase and the time instance for that value. In this way, it is necessary to find only a parameter $c$. On the other hand, it is obvious that the shape of curves shown in fig. 8 can be used to describe a time-temperature process for the most fires, especially flaming ones (with characteristic peak). For smouldering fires, this function can be used only for the growth phase of fire-between ignition and flashover. The function defined in eq. (8) was chosen after through analysis of fire data and fire curves presented in literature by various authors (for example, tables from [28] and curves from [29-34], etc.)

**Using the new function**

For numerical experiments and for illustration of this approach, some curves from figs. 3-7, was chosen. The aim was to compare shapes of the various experimental curves with those obtained by approximation. The simulation software is being developed using programming package Microsoft Fortran PowerStation 4.0.

In the beginning of the numerical experiment, usage of new function given with eq. (8) will be illustrate on parametric fire curves for various opening factors from fig. 3, and on curves from fig. 4, in the two ways:

1. In the first step, assuming that power $c = 1$, and by settings the values of coefficients $a$ and $b$ to pair $(t_{\text{max}}, T_{\text{max}})$ for each curve from figs. 3 and 4 (on the left side), we obtain a family of curves based on eq. (8), as shown on fig. 9. Those curves was chosen because of different approaches, and consequently, different shapes.
At the first sight, we may yield some conclusions about approximation with simplified function (power \( c = 1 \)). Regarding to approximation of parametric fire curves it is obvious that the first edge and the slope of the curves are not appropriate. In order to obtain more appropriate approximation curve, we must calculate the parameter \( c \) which defines rate of rise of given function. The decay phase described by parametric fire curves is linear, and consequently, it can not be described with proposed function.

Temperature-time curves based on fire load are much more convenient for approximation with proposed function. For curves from fig. 9, there is no need to calculate parameter \( c \) of given function, especially for cases of fire loads less then 60 kg/m\(^2\), for example, 30(1/4), 30(1/2), etc. In other words, this means that we need in the first step, to establish relationship between parameters \( a \) and \( b \) (coefficients of function) with fire load and opening factors during the fire.

With this approach, usage of proposed function gets the full meaning, because of describing fire development by unique function which shape depends on combustible material and conditions of ventilation. This thinking is completely in line with “fire triangle” logic, a simple combustion model which contains necessary ingredients for most fires as follows: fuel (fire load), oxygen (conditions of ventilation-oxidizing), and heat (source of ignition).

Therefore, the numerical experiment was continued in a way that approximates only that experimental fire curves which are obtained on the data of fire load and ventilation. Better match experimental fire curves and proposed function can be achieved by adjusting the parameter \( c \).

(2) In the second step, if we know the maximal value of temperature in fire and the corresponding time co-ordinate (the parameters \( a \) and \( b \)), the parameter \( c \) can be calculated by adjusting the function to an accurate value in any point with co-ordinates \((t_1, T_1)\) at each experimental fire curve. Adjusting of function (8) in any arbitrary point \((t_1, T_1)\) gives:

\[
c = \frac{\ln \frac{T}{a}}{\ln \frac{x}{b} + 1 - \frac{x}{b}} = \frac{\ln \frac{T_1}{T_{\text{max}}}}{\ln \frac{t_1}{t_{\text{max}}} + 1 - \frac{t_1}{t_{\text{max}}}}.
\]  

(9)

![Figure 10. Approximations with proposed function (curves from fig. 5 and fig. 6)](image-url)
The first example of approximation (fig. 9) shows that proposed function is not quite suitable for use as a parametric curve, but as real temperature-time curve. For instance, two approximations with proposed function are presented in fig. 10. In the first approximation (fig. 10, on the left side), for each of three curves value of parameter \( c \) is chosen to be equal to the value of the air flow factor for which the corresponding function has a maximum. In the second approximation (fig. 10, on the right side), parameter \( c \) is adjusted as previously described.

Due to the reasons described, when creating a function, the authors wanted the shape of a new function to be similar to the shape of a function from fig. 1, and to correspond to the curve which describes the development of a real fire. Therefore, in a numerical experiment to follow, the selected curves correspond to the real scenario of a fire development in a compartment.

The approach suggested in this paper is the most similar to the approach used for creating BDF curves. The starting point of both approaches is following: two parameters are defined by maximal temperature and the time for reaching this temperature, and the third parameter defines shape of temperature-time curve. The similarity and the difference between the two approaches are obvious, as if we look at fig. 11. An obvious similarity is in the growth phase and phase of fully developed fire, whereas in the decay phase there is a significant difference between two curves. The tab. 1 shows values \( a \), \( b \), and \( c \) of proposed function for approximation of BDF curves.

In accordance with previously set hypothesis that the shape of the proposed function in a satisfactory way describes the fire development in a compartment for known values \( a \) and \( b \), numerical simulation is continued on the way of setting parameter \( c \).

The results of numerical experiments which carried out in the second step, for some experimental fire curves from fig. 4, are shown in fig. 12. These curves were chosen because they well illustrate how fuel load and opening factor affect on shape of temperature-time relationships in fire.

Value for parameter \( c \), for each experimental fire curve, was obtained by adjusting the proposed
function in a saddle point using eq. (8). Results of calculation for the parameter \( c \), and values of \( a \) and \( b \), according to co-ordinates \((t_{\text{max}}, T_{\text{max}})\) for each curve, are shown in tab. 2.

![Figure 12. Fire curves and approximations obtained by adjusting parameter \( c \)](image)

<table>
<thead>
<tr>
<th>Curve</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60(1/4)</td>
<td>30</td>
<td>1180</td>
<td>0.70</td>
</tr>
<tr>
<td>60(1/2)</td>
<td>19</td>
<td>1075</td>
<td>0.94</td>
</tr>
<tr>
<td>30(1/4)</td>
<td>13</td>
<td>1062</td>
<td>1.18</td>
</tr>
</tbody>
</table>

After adjusting the parameter \( c \), as the correction parameter for conditions which are specific for any compartment, the curves shown on fig. 12 were obtained. This process oriented on finding the best approximation of time-temperature relationship in fire, can be continued in terms of looking at relationship between parameter \( c \), and parameters \( a \) and \( b \) of proposed function. For this purpose, from mathematical point of view in further investigations the least squares method can be applied, or some similar method for obtaining polynomial approximation. However, mentioned process requires establishment of a reliable basis for assessing or calculation the parameters \( a \) and \( b \). After that, the calculation of the parameter \( c \) has sense. The numerical results obtained in the second step, for curves from fig. 12 are shown in tab. 3.

### Conclusions

The temperature development in fire can be divided into growth period, a fully developed period and a decay period. Growth period of time-temperature curves has the shape similar to pulse phenomena and are very convenient for approximation with function proposed in this paper. For smouldering fires (without characteristic peak) this function can be used only for approximation of smaller or larger part in the development phase of fire. In any way, this function provides significantly better results related to the already known standard time-temperature curves.

With the aim to create a new function, the authors used available research results from laboratories worldwide, and made efforts to create a new function, which will authentically describe development of compartment fire from the mathematical point of view. In order to confirm the accuracy of a function suggested in this paper, it is necessary to carry out experiments for the compartment fires with diverse fire load and ventilation conditions.
Nomenclature

- $a$ – y co-ordinate for maximal value of temperature in function $T(t)$, [°C]
- $b$ – x co-ordinate for time instance for maximal value of temperature in function $T(t)$, [min.]
- $C$ – constant (0/1), [-]
- $c$ – correction parameter that defines shape of light or proposed function, [-]
- $F$ – opening factor, $[m^2]$]
- $s_e$ – number which defines shape for time-temperature curve, [-]
- $T$ – temperature, [°C]
- $T_a$ – ambient temperature, [°C]
- $T_m$ – maximal value of temperature, [°C]
- $T_m'$ – maximal value of temperature in function which are approximated, [°C]
- $T_e$ – temperature at time $t = \tau$ and $t > \tau$, [°C]
- $T_0$ – temperature at the start of the fire, [°C]

Greek symbols

- $\rho$ – material density, $[kgm^{-3}]$
- $\tau$ – time at which decay period begins [h],

Acronyms

- BS – British Standard
- ISO – International Organization for Standards

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