ESTIMATION OF THE LENGTH CONSTANT OF A LONG COOLING FIN BY AN ANCIENT CHINESE ALGORITHM

by

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Abstract

In this paper, an ancient Chinese algorithm is used to estimate the length constant of a long cooling fin, and an approximate solution formulation is obtained. The obtained results show that this method is a simple but promising method without any requirement for advanced calculus.

Key words: ancient Chinese mathematics, long cooling fin, approximate solution

Introduction

The Nine Chapters is the oldest and most influential work in the history of Chinese mathematics [1]. The Chapter 7 of the Nine Chapters is the Ying Buzu Shu, which is the oldest method for approximating real roots of an equation. Consider an algebraic equation:

\[ f(x) = 0 \]  

(1)

The basic idea of the Ying Buzu Shu is to give two trial-roots, \( x_1 \) and \( x_2 \), which lead to the remainders \( f(x_1) \) and \( f(x_2) \), respectively, and the approximate solution can be written in the following form:

\[ x_3 = x_2 \frac{f(x_1)}{f(x_2)} x_1 f(x_2) \]

(2)

The ancient Chinese mathematical algorithm was furthered extended to solve nonlinear differential equations by the famous Chinese mathematician, Dr. Ji-Huan He, see refs. [1-6], and many followers found Ji-Huan He’s idea extremely simple for engineering applications, see refs. [7-15]. In this paper, we will also follow Ji-Huan He’s idea to estimate the length constant of a long cooling fin.

Mathematical model for a long cooling fin

Figure 1 shows a cooling fin of thin rectangular section projecting from a hot plate held at a fixed temperature. The fin loses heat by convection to the surrounding air, and the governing equation, found by expressing the heat balance in a small length element, is [16]:

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with boundary conditions
\[ T(0) = T_0, \quad T(L) = 0 \]  

Here \( T \) is the excess temperature above that of the surrounding air at a distance \( x \) along the fin, \( K \) is the thermal conductivity, \( a \) is the height of the fin, \( b \) is its thickness (\( b \ll a \)) and \( h \) is a convective heat transfer coefficient. The index \( n \) may vary from 1 to 5/4 according to the conditions in the surrounding air.

The equation may be simplified by dividing throughout by \( Kab \). It may then be written as follows:
\[ \frac{d^2T}{dx^2} - cT^n = 0 \]  

with boundary conditions
\[ T(0) = T_0, \quad T(l) = 0 \]  

where \( c = 2h/Kb \).

**Estimation of the length constant of a long cooling fin**

Similar to He's frequency-amplitude formulation [2, 6], we choose two trial functions:
\[ T_1(x) = T_0 e^{-x} \]  
\[ T_2(x) = T_0 e^{-2x} \]  

Substituting eq. (5) and eq. (6) into, respectively, eq. (4), we obtain the following residuals:
\[ R_1(x) = T_0 e^{-x} - c(T_0 e^{-x})^n \]  
\[ R_2(x) = \lambda^2 T_0 e^{-2x} - c(T_0 e^{-2x})^n \]  

According to He's frequency-amplitude formulation (2, 6), we can approximately determine \( \lambda^2 \) in the following form:
\[ \lambda^2 = \frac{\lambda^2 R_2(0) - \lambda^2 R_1(0)}{R_2(0) - R_1(0)} = \frac{\lambda^2 T_0 - cT_0^n}{\lambda^2 T_0 - cT_0^n} = cT_0^{n-1} \]  

So the mathematical form of the temperature along a long cooling fin is:
\[ T(x) = T_0 e^{-x^{1/n}} \]  

In order to verify the correctness of the obtained frequency, we consider two special cases.
Case 1

If \( c = 1, \ n = 5/4, \ T_0 = 5 \), eqs. (3) and (4) reduce to:

\[
\frac{d^2T}{dx^2} - T^{5/4} = 0
\]

with boundary conditions

\[
T(0) = 5, \quad T(l) = 0
\]

Then we can obtain the length constant of a long cooling fin as follows:

\[
\lambda = 5^{1/8}
\]

Therefore, we can get the mathematical form of the temperature along a long cooling fin:

\[
T(x) = T_0 e^{-1.22284454499385 x}
\]

Comparison of the approximate solution, eq. (14), with that in ref. [2] is illustrated in fig. 2 showing a good agreement.

Case 2

If \( c = 2, \ n = 5/4, \ T_0 = 1.5 \), according to eqs. (3) and (4), we can obtain the following model:

\[
\frac{d^2T}{dx^2} - 2T^{5/4} = 0
\]

with boundary conditions

\[
T(0) = 1.5, \quad T(l) = 0
\]

Its approximate length constant and its approximate solution are, respectively, as follows:

\[
\lambda = \sqrt{2} \cdot 1.5^{1/4}
\]

and

\[
T(x) = T_0 e^{-1.48773782616449 x}
\]

which agrees well with the approximate solution obtained in ref. [16] as shown in fig. 3.

Conclusion

In this paper, we introduce the solution procedure using the basic concept of the ancient Chinese algorithm, and apply the method to obtain the length constant of a long cooling fin. The obtained solutions are in good agreement with ones

**Figure 2** Comparison of the approximate solution obtained in this paper with the approximate solution obtained in ref. [2]

**Figure 3** Comparison of the approximate solution obtained in this paper with the approximate solution obtained in ref. [16]
in ref. [16]. The results show that the solution procedure of the ancient Chinese algorithm is of deceptive simplicity and the method might find wide applications.

References


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