Effects of variable viscosity and thermal conductivity on Unsteady MHD flow of non-Newtonian fluid over a stretching porous sheet

by

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The unsteady flow and heat transfer in an incompressible laminar, electrically conducting and non-Newtonian fluid over a non-isothermal stretching sheet with the variation in the viscosity and thermal conductivity in a porous medium by the influence of an external transverse magnetic field have been obtained and studied numerically. By using similarity analysis the governing differential equations are transformed into a set of non-linear coupled ordinary differential equations which are solved numerically. Numerical results were presented for velocity and temperature profiles for different parameters of the problem as power law parameter, unsteadiness parameter, radiation parameter, magnetic field parameter, porous medium parameter, temperature buoyancy parameter, Prandtl parameter, modified Eckert parameter, Joule heating parameter, heat source/sink parameter and others. A comparison with previously published work has been carried out and the results are found to be in good agreement. Also the effects of the pertinent parameters on the skin friction and the rate of heat transfer are obtained and discussed numerically and illustrated graphically.

Keywords: Unsteady flow, non-Newtonian fluid, MHD, porous medium.

Introduction

Many industrial fluids as the non-linear fluid rheology become of special interest and have practical applications. Hence the study of non-Newtonian fluid flow is important, different models have been proposed to explain the behaviour of non-Newtonian fluid. Among these, the power law, the differential type, and the rate type models gained importance. The knowledge of flow and heat mass transfer within a thin liquid film is crucial in understanding the coating process, designing of heat exchangers and chemical processing equipments. This interest stems from many engineering and geophysical applications such as geothermal reservoirs and other applications including wire and fibre coating, food stuff processing, reactor fluidization, transpiration cooling, thermal insulation, enhanced oil recovery, packed bed catalytic reactors, cooling of nuclear reactors and underground energy transport. The prime aim in almost every extrusion is to maintain the surface quality of the extricate. All coating processes demand a smooth glossy surface to meet the requirements for the best appearance and optimum service properties such as low friction, transparency and strength. Also, many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid, and in the process of drawing, these strips are stretched. This type of flow was firstly initiated by Sakiadis [1] for moving an inextensible sheet and later extended by Crane [2] to fluid flow over a linearly stretched sheet.

Thereafter, numerous investigations were done on the stretching sheet problem with linear stretching in different directions [3-8]. All the above studies restrict their analysis to Newtonian flows in the absence of the magnetic field. In recent years, it has been observed that a number of industrial fluids such as molten plastics, artificial fibers, polymeric liquids, blood, food stuff, and slurries exhibit non-Newtonian fluid behavior. It may also be pointed out that, many industrial processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. During this process, these strips are sometimes stretched. In all these cases, the properties of the final product depend to a great extent on the rate of cooling. By drawing such strips in an electrically conducting fluid subjected to a uniform magnetic field, the rate of cooling can be controlled and the desired

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characteristics of the final product can be obtained. Another important application of hydromagnetic flows to metallurgy lies in the purification of molten metal’s from non-metallic inclusion by the application of magnetic field. In view of these applications, Sarpakaya [9] was the first among others to study the magneto-hydrodynamic flow of non-Newtonian fluids. This work was later extended by many authors by considering the non-Newtonian visco-elastic flow, heat and mass transfer under different physical situations [10-16]. It is worth mentioning here that many inelastic non-Newtonian fluids, encountered in chemical engineering processes, are known to follow the empirical Ostwald–de Waele so called “power law model”. This model is described by a simple non-linear equation of state for inelastic fluids which includes linear Newtonian fluids as a special case.

The power law model provides an adequate representation of many non-Newtonian fluids over the most important range of shear stress. Although this model merely shows an empirical relationship between the stress and velocity gradients, it has been successfully applied to non-Newtonian fluids experimentally. The two constants in the model can be chosen with great ease for specific fluids and the model is found to be good in representing pseudo-plastic behavior. It is frequently used in oil engineering. A considerable amount of work has been done in this field by taking into account the heat and mass transfer. Schowalter [17] has introduced the concept of boundary layer theory of non-Newtonian fluids. Acrivos et al. [18] have investigated the steady laminar flow of non-Newtonian fluids over a plate. Lee and Ames [19] extended the above work to find the similarity solutions for non-Newtonian power law fluid. Andersson et al. [20] studied the boundary layer flow of an electrically conducting incompressible fluid obeying the power law model, in the presence of transverse magnetic field. Howell et al. [21] examined the momentum and heat transfer occurring in the laminar boundary layer on a continuously moving and stretching surface in a non-Newtonian power law fluid. However all these studies are restricted to the analysis of either flow characteristics or flow and heat transfer characteristics over an impervious stretching boundary. We know that the characteristic properties of the final product of the material depend to a great extent on the rate of cooling through the adjacent boundary. The rate of cooling associated with the heat transfer phenomena may be controlled by suction/blowing through the porous boundary in the presence of a constant transverse magnetic field. Hassanien et al. [22] presented a work on flow and heat transfer in power law fluid over a stretching porous surface with variable surface temperature. Very recently, Abel and Mahesha [23] considered the effects of buoyancy and variable thermal conductivity in a power law fluid past on a vertical stretching sheet in the presence of non-uniform heat source.

All the above investigators restrict their analyses to MHD flow and heat transfer over a stretching sheet. However the intricate flow and heat transfer problem with the effects of internal heat generation/absorption, viscous dissipation, work done by stress, and the thermal radiation is yet to be studied. This has applications to several industrial problems (say engineering processes involving nuclear power plants, gas turbines, and many others, see Vajravelu [24], Vajravelu and Nayfeh [25]). In all these studies, the thermo-physical properties of the ambient fluids were assumed to be constant. However it is well known that these properties may change with temperature, especially the thermal conductivity. Available literature on variable thermal conductivity [26–30] shows that this type of flow has not been investigated for power law fluids in the presence of suction/bowing and impermeability of the stretching sheet.

In view of these applications, we study the unsteady flow and heat transfer phenomena in a power law fluid over a porous stretching surface, in the presence of the influence of an external transverse magnetic field in a porous medium , taking into account the internal heat generation/absorption, viscous dissipation, work done by stress, variable thermal conductivities, variable viscosities, thermal radiation. This is a generalization of Andersson et al.’s [20] work to the case of power law fluid flow and heat transfer where the thermal conductivity is a function of temperature in the presence of transverse magnetic field. Recently, flow of non-Newtonian polymer solution was investigated by Savvas et al. [31] and it was shown that computer simulation is a powerful technique to predict the flow behavior. Because of the complexity and non-linearity of our problem, the resulting equations are solved numerically by the Keller–Box method. One of the important observations of the study is that suction reduces the horizontal velocity where as blowing increases the horizontal velocity for all values of power law index.
The results obtained herein, are compared with the solutions by Prasad et al. [32] which also studied heat transfer in the MHD flow of a power law fluid over a non-isothermal stretching sheet in absence of each of the unsteadiness parameter, i.e. $A=0$, the porous medium, i.e. $S=0$, the temperature buoyancy parameter, i.e. $G=0$, and variable viscosity=constant, to check the accuracy.

Hence, the objective of present paper is to study the above mentioned unsteady flow and heat transfer in an incompressible laminar, electrically conducting and non-Newtonian fluid over a non-isothermal stretching sheet with the variation in the viscosity and thermal conductivity in a porous medium by the influence of an external transverse magnetic field which have been obtained and studied numerically. Numerical results are presented for velocity and temperature profiles for different parameters of the problem as power law parameter, unsteadiness parameter, radiation parameter, magnetic field parameter, porous medium parameter, temperature buoyancy parameter, Prandtl parameter, modified Eckert parameter, Joule heating parameter and heat source/sink parameter, etc. In addition, the effects of the pertinent parameters on the skin friction and the rate of heat transfer are also discussed.

Mathematical analysis

Consider a viscous, unsteady two-dimensional flow of an incompressible, electrically conducting power law fluid in the presence of an external transverse magnetic field of strength $B(x,t)$ over a stretching sheet lying on the plane $y > 0$. The flow is confined to $y > 0$ (for details, see [20]). The thermo-physical properties of the sheet and the viscosity of the fluid vary as assumed to be $\mu(x,t) = \mu_0 b^{-3}x^{(1-n)/n}(1-\gamma t)^\gamma$ at which $\mu_0$ is the viscosity at temperature $T_w$, $b$ and $\gamma$ are positive constants with dimension reciprocal time. The fluid motion arises due to the stretching of the elastic sheet in a porous medium. The momentum and energy equations for a fluid with variable thermal conductivities and variable viscosities in the presence of internal heat generation/absorption, viscous dissipation and thermal radiation are studied. The $u$ and $v$ are the velocity of $x$ and $y$ components, $T$ is the temperature. Under these assumptions, the governing boundary layer equations of continuity, momentum and energy under Boussinesq approximations could be written as follows:

The equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

(1)

The equation of momentum:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial}{\partial y} \left( -\mu(x,t) \frac{\partial u}{\partial y} \right) - \frac{\sigma B^2 u}{\rho} + g \beta (T-T_w) - \frac{v}{K}.$$

(2)

The equation of energy:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \frac{\partial}{\partial y} \left( \alpha(T,t) \frac{\partial T}{\partial y} \right) - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q}{\rho c_p} (T-T_w) + \frac{1}{\rho c_p} \left( \mu(x,t) \frac{\partial u}{\partial y} \right)^{\alpha_1} + \frac{\sigma B^2 u}{\rho c_p}.$$

(3)

The initial and boundary conditions are:

$$t < 0: \quad u=v=T=0, \quad \text{for all point of} \quad s(x,y),$$

$$t \geq 0: \quad u(x,0) = U_w(x,t), \quad v(x,0) = v_w(x,t), \quad T(x,0) = T_w(x,t), \quad x \geq 0, \quad (4)$$

$$t \geq 0: \quad u(x,\infty) = 0, \quad T(x,\infty) = T_w, \quad (4)$$

Following Andersson et al. [33], the stretching velocity $U_w(x,t)$ is assumed to be $U_w(x,t) = bx/(1-\gamma t)$ and the velocity $v_w(x,t)$ across the stretching sheet is assumed to be $v_w(x,t) = (1-\gamma t)R/b$ in which $R$ is a suction velocity when $v_w < 0$ and $b$ is blowing velocity when $v_w > 0$. We have $b$ as the initial stretching rate and $b/(1-\gamma t)$ is increasing with time. In the
context of polymer extrusion, the material properties particularly the elasticity of the extruded sheet may vary with time even though the sheet is being pulled by a constant force.

With unsteady stretching, however, $\gamma^{-1}$ becomes the representative time scale of the resulting unsteady boundary layer problem. We assume that all of the surface temperature $T_w(x,t)$, the applied transverse magnetic field $B(x,t)$, the expansion coefficient of temperature $\beta(x,t)$, the volumetric heat generation/absorption rate $Q(x,t)$ and the porous medium $\mathcal{S}(x,t)$ are on a stretching sheet to vary with the distance $x$ along the sheet and time in the following forms:

$$B(x,t) = B_0(1-\gamma t)^{-3/2}, \quad \beta(x,t) = \beta_0(1-\gamma t)^{-1}, \quad Q(x,t) = Q_0(1-\gamma t)^{-1},$$

$$T_w(x,t) = T_w + A_1(x^2/l^2)(1-\gamma t)^{-1}, \quad \mathcal{S}(x,t) = \frac{\psi}{K} = S_0(1-\gamma t)^{-1} \quad (5)$$

Where $B_0$ is the uniform magnetic field acting normal to the plate, $g$ is the gravitational acceleration. $\beta_0$, $Q_0$ and $A_1$ are constants which depend on the properties of the fluid, $l$ is a characteristic length, $\rho, \mu, T_\infty$ and $c_p$ are the density, dynamic viscosity, the free stream temperature and the specific heat at constant pressure, respectively.

The subscripts $w$ and $\infty$ stand for the wall and free stream conditions, $\alpha(T,t)$ is the temperature-dependent thermal conductivity. We consider the temperature-dependent thermal conductivity in the following form Chiam [26]:

$$\alpha(T,t) = \alpha_\infty \left(1-\gamma t\right)^3 \left(1 + \varepsilon \frac{T - T_\infty}{\Delta T}\right) \quad (6)$$

where $\varepsilon$ is a small parameter, $\Delta T = T_w - T_\infty$. $T_w$ is the given temperature at the wall, and $\alpha_\infty$ is the thermal conductivity of the fluid far away from the sheet. The term containing $Q$ represents the temperature-dependent heat source when $Q > 0$ and heat sink when $Q < 0$; and it deals, respectively, with the situations of exothermic and endothermic chemical reactions. The third and fourth terms on the right-hand side of Eq. (3) represent, respectively, the viscous dissipation and the ohmic heating effect. The last term $q_r$ on the right-hand side of Eq. (3) represents the radiative heat flux. Using the Rosseland approximation (Sparrow and Cess [34] and El-Arabawy [35]), the radiative heat flux $q_r$ could be expressed by:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$$

(7)

Where the $\sigma^*$ represents the Stefan-Boltzman constant and $k^*$ is the Rosseland mean absorption coefficient.

Assuming that the temperature difference within the flow is sufficiently small such that $T^4$ could be approached as the linear function of temperature;

$$T^4 \equiv 4T_\infty^3 T - 3T_\infty^4 \quad (8)$$

The equation of continuity is satisfied if we choose a stream function $\psi(x,y)$ such that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. The governing partial differential equations (1)-(3) admit similarity solutions for obtaining the dimensionless stream function $f(\eta)$ and temperature $\theta(\eta)$. The relative parameters are introduced as:

$$\psi = \frac{x}{b}(1-\gamma t) f(\eta), \quad \eta = b^2 \left(1-\gamma t\right)^2 y, \quad T = T_\infty + T_0 b x (1-\gamma t)^{-1} \theta(\eta). \quad (9)$$

After introducing the similarity transformation, the equations (2)-(3) can be transformed into a set of following forms in terms with $f(\eta)$ and $\theta(\eta)$, expressed as:

$$n \left(-f^\prime \right)^{n-1} f^{\prime\prime\prime} + ff^{\prime\prime} - f^\prime 2 + A \left[f^\prime + 2\eta f^{\prime\prime}\right] - (M + S) f^\prime + G \theta = 0, \quad (10)$$

-4-
\[(k_0 + \alpha P, \alpha \theta^2) + \alpha \theta^2 - AP, \theta + f f' ) + \beta \theta + P, (E, f f) = 0, \quad (11)\]

With the appropriate boundary conditions:
\[\eta = 0; \quad f (0) = R, \quad f' (0) = 1, \quad \theta (0) = 1, \quad \eta \rightarrow \infty; \quad f' (\infty) = 0, \quad \theta (\infty) = 0. \quad (12)\]

Where the prime denotes a partial differentiation with respect to \(\eta\), \(k_0 = xA_1 / T_0 b l^2\) is constant parameter, \(R\) is introduced to represent the surface mass transfer which is positive for blowing, negative for suction, and shows that, the suction or blowing parameter \(R\) is used to control the strength and direction of the normal flow at the boundary and the dimensionless parameters are defined as:
\[A = \gamma / b \quad \text{(Unsteady parameter)}, \]
\[G = g\beta_0 T_0 / b \quad \text{(Temperature buoyancy parameter)}, \]
\[M = \frac{\sigma B_0^2}{\rho b} \quad \text{(Magnetic field parameter)}, \]
\[S = S_0 / b \quad \text{(Porous medium parameter)}, \]
\[\beta = A_1 Q_0 / b^3 \alpha_n T_0 l^2 \quad \text{(Heat source/sink parameter)}, \]
\[\delta = 16 \sigma^2 b^3 l^2 (T_w - T_\infty)^3 / 3 k^3 \rho c_p A_1^2 x^2 \quad \text{(Radiation parameter)}, \]
\[E = A_1^2 x^4 / l^4 b c_n T_0 (T_w - T_\infty)^2 \quad \text{(Modified Eckert parameter)}, \]
\[P_n = \rho c_p A_1 / \alpha_n T_0 b^3 l^2 \quad \text{(Prandtl number)}, \]
\[J = \frac{\sigma B_0^2 l^2 (T_w - T_\infty)}{T_0 b \rho c_p A_1 x^2} \quad \text{(Joule heating parameter)}.

**Skin-Friction Coefficient and Nusselt Number**

The parameters of engineering interest for the present problem are the local skin-friction coefficient and the local Nusselt number which indicate the physical wall shear stress and rate of heat transfer, respectively.

The equation defining the wall skin-friction is given by:
\[\tau_w = - \left( - \mu (x, t) \frac{\partial u}{\partial y} \right)_{y=0}^n, \]

The skin-friction coefficient is given by, Eq. (13):
\[C_f = - \tau_w / x u_0^n = - f'' (0)^n, \quad (13)\]

Now the heat flux \(q_w\) at the wall is given by:
\[q_w = - \alpha (T, t) \left( \frac{\partial T}{\partial y} \right)_{y=0}^n, \]

Hence the Nusselt number \(N_u\) is obtained as Eq. (14):
\[N_u = \frac{A_1 q_w}{\alpha_n T_0 \rho c_p A_1 x^2 (k_0 + \varepsilon)} = - \theta' (0), \quad (14)\]

**Numerical Computations**

The system of non-linear ordinary differential Eqs. (10)-(11) together with the boundary conditions (12) are locally similar and solved numerically by using the sixth order of Runge-Kutta
integration accompanied with the shooting iteration scheme. We have chosen a step size of \( \Delta \eta = 0.01 \) to satisfy the convergence criterion of \( 10^{-6} \) in all cases. The value of \( \eta_0 \) was found for each iteration loop by \( \eta_0 = \eta_0 + \Delta \eta \). The maximum value of \( \eta_0 \) for each group of parameters \( n, A, M, S, G, k_0, \delta, \varepsilon, P, \beta, E, R \) and \( J \) is determined when the value of the unknown boundary conditions at \( \eta = 0 \) is not changed to successful loop with error less than \( 10^{-6} \).

In order to verify the effects of the step size \( (\Delta \eta) \) we ran the code for our model with three different step sizes as \( \Delta \eta = 0.01, \Delta \eta = 0.004 \) and \( \Delta \eta = 0.001 \) and in each case we found excellent agreement among them. Fig. 1 shows the velocity profiles for different step sizes.

![Velocity profiles for different step sizes](image)

**Figure 1. Velocity profiles for different step sizes**

### Results and discussion

In order to gain physical insight the velocity, temperature and concentration profiles have been discussed by assigning numerical values to the parameter encountered in the problem in which the numerical results are tabulated and displayed with the graphical illustrations.

In order to verify the accuracy of our present method, we have compared our results with those of Prasad et al. [32] and Andersson et al. [20]. Table 1 shows the values of \( -f''(0) \) for various values of \( n \). The comparisons in all above cases are found to be excellent and agreed, also, the results are found to be similar to Prasad et al. [32] and Andersson et al. [20], so it is good.

Fig. 2a) and 2b), display the velocity and temperature profiles under the different power law parameter. The velocity increases, while the temperature profiles decrease with increasing power law parameter.

Fig. 3a) and 3b), display the velocity and temperature profiles under the different unsteadiness parameter. The velocity and the temperature profiles increase with increasing unsteadiness parameter. Fig. 4a) and 4b) , it is clear that the velocity decreases, while the temperature profiles increase with the increase of the magnetic parameter.

Fig. 5a) and 5b) , show the effects of porous medium parameter on the velocity and temperature profiles; also, we found that the velocity decreases, but the temperature profiles increase with the increase of porous medium parameters. Fig. 6a) and 6b) , show the effects of temperature buoyancy parameter on the velocity and temperature profiles; also, we found that the velocity and the temperature profiles increase in case \( n < 1.0 \), but decrease in case \( n > 1.0 \) with the increase of temperature buoyancy parameters. Fig. 7a) and 7b) , show the effects of surface mass transfer parameter on the velocity and temperature profiles; also, we found that the velocity and the temperature profiles decrease with the increase of surface mass transfer parameters. i.e. The thermal boundary layer becomes thicker for suction and thinner for blowing.

The effects of \( P_r \) on the velocity and the temperature profiles are shown in Fig. 8a) and 8b), respectively. It is observed that the velocity profile increases, while the temperature profile decreases
with the increase of Prandtl number. Physically, the thermal boundary layer thickness decreases with the increase of the values of Prandtl number. The effects Modified Eckert parameter on the velocity and the temperature profiles are shown in Fig. 9a) and 9b), respectively. It is observed that the velocity profile decreases, while the temperature profile increases with the increase of Modified Eckert parameter. This is in conformity with that fact that energy is stored in the fluid region as a consequence of dissipation due to viscosity and elastic deformation.

Fig. 10) shows the effects of Heat source/sink parameter on temperature profile; also, we found that the temperature profile increases with the increase of Heat source/sink parameters. The effects of Joule heating parameter on the velocity and the temperature profiles are shown in Fig. 11a) and 11b), respectively. It is observed that the velocity and the temperature profiles increase with the increase of Joule heating parameters. The effects of \( \delta \) on the velocity and the temperature profiles are shown in Fig. 12a) and 12b), respectively. It is observed that the velocity and the temperature increase with the increasing Radiation parameter.

### Table 1. Comparison of the values \(-f''(0)\) with Prasad et al.[32] and Andersson et al. [20]

<table>
<thead>
<tr>
<th>( n )</th>
<th>Prasad et al. [32]</th>
<th>Andersson et al. [20]</th>
<th>Present study</th>
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### Table 2. Numerical of the values Skin-friction coefficient \((C_f)\) and Nusselt number \((N_u)\) at the plate surface with \( n \)

<table>
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<th>( A )</th>
<th>( M )</th>
<th>( S )</th>
<th>( G )</th>
<th>( R )</th>
<th>( P_r )</th>
<th>( E_c )</th>
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Figure 2. Effect of power law parameter on (a) the velocity profile and (b) the temperature profile.

Figure 3. Effect of unsteadines parameter on (a) the velocity profile and (b) the temperature profile.

Figure 4. Effect of magnetic parameter on (a) the velocity profile and (b) the temperature profile.
Figure 5. Effect of porous medium parameter on (a) the velocity profile and (b) the temperature profile.

Figure 6. Effect of the temperature buoyancy parameter on (a) the velocity profile and (b) the temperature profile.

Figure 7. Effect of surface mass transfer parameter on (a) the velocity profile and (b) the temperature profile.
Figure 8. Effect of Prandtl number on (a) the velocity profile and (b) the temperature profile.

Figure 9. Effect of Modified Eckert parameter on (a) the velocity profile and (b) the temperature profile.

Figure 10. Effect of heat source/sink parameter on the temperature profile.
From Table 1, an increase for power law $n$ gives an increase in the value of the dimensionless quantities $-f''(0)$. In addition, by comparing our results with those of Prasad et al. [32] and Andersson et al. [20], the results are found to be similar, so it is good.

The numerical values of the Skin-friction and the Nusselt number are given in Table 2 and 3. It may be noted that with an increase in the power law parameter $n$, we observe that the Skin-friction coefficient increases in case of $n < 1$, while decreases in case of $n > 1$ and the Nusselt number increases. Also, with the increase in the unsteadiness parameter $A$, we observe that the Skin-friction coefficient increases in case of $n < 1$, while decreases in case of $n > 1$ and the Nusselt number decreases.

Both at $n < 1$ and $n > 1$, For an increase in each of the magnetic parameter $M$, porous medium parameter $S$ and Modified Eckert parameter $E_c$, we observe that the Skin-friction coefficient increases and the Nusselt number decreases. While, with an increase in the temperature buoyancy parameter $G$, we observe that the Skin-friction coefficient decreases and the Nusselt number increases.
Table 3: Numerical of the values Skin-friction coefficient \( (C_f) \) and Nusselt number \( (N_u) \) at the plate surface with \( A, M, S, G, R, P_r, E_c, \beta, J, \delta \) and \( n \)

<table>
<thead>
<tr>
<th>( A )</th>
<th>( M )</th>
<th>( S )</th>
<th>( G )</th>
<th>( R )</th>
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It may be noted that with an increase in the surface mass transfer parameter $R$ and Prandtl number $Pr$, we observe that the Skin-friction coefficient and the Nusselt number increase. While the Skin-friction coefficient and the Nusselt number decrease with increasing the Joule heating parameter $J$ and Radiation parameter $\delta$. With an increase in the Heat source/sink parameter $\beta$, we found that the value of the Nusselt number decreases. i.e. For positive $\beta$ we have a heat source in the boundary layer when $T_w < T_\infty$ and a heat sink when $T_w < T_\infty$. Physically these correspond, respectively, recombination and dissociation within the boundary layer. For the case of a cooled wall ($T_w < T_\infty$), there is heat transfer from the fluid to the wall even without heat source. The presence of a heat source $\beta > 0$ will further increase the heat flow to the wall. When $\beta$ is negative, this indicates a heat source for ($T_w > T_\infty$) and a heat sink ($T_w < T_\infty$). This corresponds to combustion and an endothermic chemical reaction. For the case of heated wall ($T_w > T_\infty$), the presence of a heat source creates a layer of hot fluid adjacent to the surface and therefore heat at the wall decreases. For the cooled wall case ($T_w < T_\infty$), the presence of a heat sink blankets the surface with a layer of cool fluid, and hence the heat flow at the surface decreases.

Conclusions

In this work the unsteady flow and heat transfer in an incompressible laminar, electrically conducting and non-Newtonian fluid over a non-isothermal stretching sheet with the variation in the viscosity and thermal conductivity in a porous medium by the influence of an external transverse magnetic field have been obtained and studied numerically. The governing equations were:

1. The effect of increasing values of the magnetic parameter and the porous medium parameter is to decrease the momentum boundary layer thickness and to increase the thermal boundary layer thickness.

2. The effect of increasing values of the variable viscosity parameter and the Temperature buoyancy parameter is to increase the momentum boundary layer as well as the thermal boundary layer thickness.

3. The effect of increasing values of the Prandtl number is to increase the momentum boundary layer thickness and to decrease the thermal boundary layer thickness. For an increase in the unsteadiness parameter, the Skin-friction coefficient increases in case of $n < 1$, while decreases in case of $n > 1$ and the Nusselt number decreases.

4. For an increase in each of the magnetic field parameter, the porous medium parameter and modified Eckert parameter, skin friction increases, but the rate of heat transfer decreases. While with an increase in the temperature buoyancy parameter, the Skin-friction coefficient decreases and the Nusselt number increases.

5. Of all the parameters considered, the variable viscosity parameter has the strong effect on the drag, heat transfer characteristics, the velocity and the temperature field in the boundary layer of a non-linearly stretching sheet.
Nomenclature

\( Q \) - internal heat generation/absorption 
\( (W/m^3) \)

\( q_w \) - local heat flux at the sheet \( (W/m^2) \)

\( q_r \) - radiative heat flux \( (W/m^2) \)

\( R \) - surface mass transfer

\( T \) - temperature distribution \( (K) \)

\( T_w \) - temperature at the sheet \( (K) \)

\( T_\infty \) - temperature of the fluid at infinity \( (K) \)

\( t \) - time \( (s) \)

\( u \) - velocity in the x-direction \( (m/s) \)

\( U \) - velocity of the sheet \( (m/s) \)

\( v \) - velocity in the y-direction \( (m/s) \)

\( x \) - horizontal distance \( (m) \)

\( y \) - vertical distance \( (m) \)

\( \varepsilon \) - small parameter

\( \theta \) - dimensionless temperature distribution

\( \sigma \) - electrical conductivity

\( \sigma^* \) - Stephan–Boltzmann constant

\( \Delta T = T_w - T_\infty \) - sheet temperature

\( \tau \) - shear stress \( (Pa) \)

Greek symbols

\( \varepsilon \) - small parameter

\( \theta \) - dimensionless temperature distribution

\( \sigma \) - electrical conductivity

\( \sigma^* \) - Stephan–Boltzmann constant

\( \Delta T = T_w - T_\infty \) - sheet temperature

\( \tau \) - shear stress \( (Pa) \)

Subscripts

\( w, \infty \) - conditions at the surface and in the free stream

References


[31] Savvas T.A., Markatos N.C., Papaspyrides C.D., On the flow of non-Newtonian