DETERMINATION OF THE DIFFUSION COEFFICIENT OF NEW INSULATORS COMPOSED OF VEGETABLE FIBERS

by

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The knowledge of the moisture transport of building materials is necessary for the performance of building structures. The control of moisture transport is essential to describe the moisture migration process through the building walls. The present work's aim is to determine through experiment the water diffusion coefficient of different insulators in unsteady-state based on the Fick's second law equation. This equation was solved analytically by the separation of variables method and by the change of variables method. The moisture diffusion coefficient for building material was experimentally predicted by using the weighing technique and different analytical methods. The results were compared with experimental data.

Key words: *diffusion coefficient, insulators, moisture transport, moisture content*

Introduction

The problem of moisture transport in buildings induced a great interest, since the understanding of moisture transport mechanisms allows the specialists to select the most appropriate material for the construction. Mass diffusivity is one of the most important parameters of moisture transfer through building walls. The transport of moisture through common building materials depends on the characteristics of the pores in these materials. Moisture transport in buildings directly leads to structural damage. In fact moisture condensation can result in microbial growth, decomposition and deterioration of materials in building envelops. The relative humidity of the air has a direct impact on the moisture content of building materials. Thus, the knowledge of the moisture transport phenomena through building materials is necessary to predict building structures behavior.

The mathematical formulation of moisture transfer in porous materials is often based on the diffusion equation described by Fick's law. The diffusion coefficient has been determined experimentally. Two methods based on steady and unsteady states were developed to determine the diffusion coefficient.

The steady-state method is based upon Fick's first law of diffusion. The unsteady-state experiments are based on Fick's second law of diffusion where the flux is proportional to the gradient of some chosen potential. The different properties determined by the two methods were reported by many authors. Problems involving moisture migration in porous building materials

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were treated by several studies. Zohoun *et al*. [1] measured the moisture diffusivity of "PVC-CHA" by the steady-state method, Comstock [2] calculated moisture diffusion coefficients in wood from adsorption, desorption, and steady-state data. Richards [3] considered a steady-flux method for measuring the distribution of 1-D moisture content prevalent in a material. With the aid of an inverse method, Olek *et al*. [4] evaluated the diffusion coefficient during water sorption in wood. The different mechanisms of moisture migration in building walls were investigated by Philip *et al*. [5] and Freitas *et al*. [6]. Agoua *et al*. [7] used a double climatic chamber to determine the diffusion coefficient of wood based on unsteady-state method. Wadso [8] studied water vapor transport and sorption in wood. Siau [9] studied water vapor transport processes in wood. All these previous works used the weighing technique in transient state, but the control of the relative humidity was treated in different ways by researchers. Hjort [10, 11] developed a 2-D program for calculating the moisture conditions in painted wooden structures. In order to find the moisture transport coefficient Crank [12] presented solutions of the Fick's second law for different types of initial and boundary conditions.

Several works have been elaborated to determine the diffusion coefficient under isothermal conditions *e. g*. ASTM E 96/E 96M-05 [13], the THALES underwater systems [14], and international standards [15, 16]. All of the above references used a weighing technique called the cup method. The vapor transfer rate through the sample was determined by weighing method. The weigh of each cup was then plotted as a function of time. The cup method owes its popularity to its simplicity and low cost. However, it needs a relatively long measurement time.

In this study, we have reported a method for measuring the mass diffusivity coefficient in porous media in unsteady-state. The objectives of this work are:

– to propose an insulating brick using vegetable fibers which is less expensive and having the advantage of preserving the environment,
– to build a reliable experimental device using weighing technique for mass diffusivity determination, and
– to extract the mass diffusion coefficient from the experimental curves using two classical methods.

Experimental device

To determine the diffusion coefficient we used the weighing method. The principle of this method is to weigh the mass desorbed or adsorbed by the sample at regular time intervals during the experience.

In the present work we have developed an experimental set-up presented in fig.1. This set-up is formed by three elements: a climatic chamber, a cryostat, and an electronic balance. The climatic chamber is highly insulated thanks to a thick layer of expanded polystyrene. The temperature is regulated by the cryostat and measured with platinum probes. The relative humidity is constant due to saturated soil solutions of potassium (KCl). The relative humidity is measured by an hygrometric probe. During the experiment, the sample is put on the

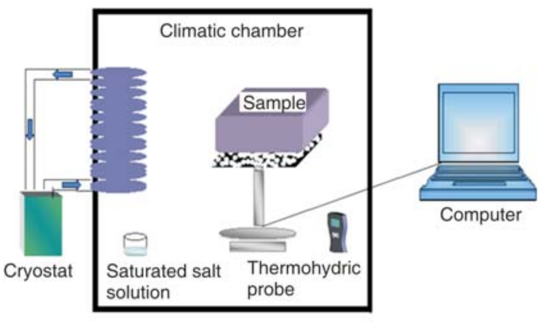

Figure 1. The experimental device

grill in order to ensure the 1-D moisture transfer. The sample and the grill are both weighed at regular time intervals using an electronic balance connected to computerized acquisition system.

Mathematical models

There are several analytical methods to predict the diffusion coefficient. Different researchers use an experimentation based on the follow-up of the water content during time of a sample balanced in a drying oven with controlled relative moisture. The steady-state regime of the water vapor flux through the sample during the experiment is identified when the time variation of the accumulated mass of the cup and the sample becomes linear. In this case, the water vapor flux is determined by evaluating the slope of the mass time variation curve. The water vapor diffusion coefficient is obtained according to the second Fick's law [11].

The present model assumes that the moisture migration by diffusion can be described by the Fick's second law. To obtain an appropriate analytical solution it is necessary to make some simplifying assumptions. We assume that moisture transfer is unidirectional, the initial moisture is uniformly distributed and the diffusion coefficient of moisture is constant.

In this case we obtain the following expression:

$$\frac{\partial X}{\partial t} = D\frac{\partial^2 X}{\partial x^2} \tag{1a}$$

where D is the diffusion coefficient and X – the moisture content.

To solve eq. (1) it is necessary to know the initial moisture content and the boundary conditions.

Initially the moisture content is uniform through the whole medium:

$$X(x,0) = X_\mathrm{i} \tag{1b}$$

The brick is put on a grid posed on the balance. Condensation is done in the same way a cross the two faces of the sample, and the moisture content tends rapidly to its equilibrium value on the two faces, consequently we have:

$$\begin{gathered} X(0,t) = X_\mathrm{e} \\ X(e,t) = X_\mathrm{e} \\ \left(\frac{\partial X}{\partial x}\right)_{x=\frac{e}{2}} = 0 \end{gathered} \tag{1c}$$

where X_e refers to the equilibrium moisture content.

The solution of eq. (1) is carried out by several methods such as the method of separation of variables, the change of variables method and the Laplace transform.

Equation (1) made linear by introducing the transformation:

$$X^* = \frac{X - X_\mathrm{i}}{X_\mathrm{e} - X_\mathrm{i}} \tag{2}$$

where X^* is the dimensionless moisture content and X, X_i, and X_e are the moisture content in kg of water by kg dry mass at t and at $t = 0$ (initial value) and at equilibrium, respectively.

Substituting eq. (2) into eq. (1), the transformed differential equation becomes as:

$$\frac{\partial X^*}{\partial t} = D\frac{\partial^2 X^*}{\partial x^2} \tag{3a}$$

The initial and the boundary conditions become:

$$X^*(0,0) = 0 \quad X^*(0,t) = 1 \quad \text{and} \quad X^*(e,t) = 1 \quad \left(\frac{\partial X^*}{\partial x}\right)_{x=e/2} = 0 \tag{3b}$$

Separation of variables method

We apply the separation of variables method (MOD1) by assuming that the solution can be expressed as the product of two functions, one depending only on x while the other depending only on t:

$$X^*(x, t) = X(x)Y(t) \tag{4}$$

The general form of the solution can be expressed as:

$$X^*(x, t) = [A_1\cos(kx) + A_2\sin(kx)]\exp(-Dk^2t) \tag{5}$$

The final solution of the moisture content is expressed as [7]:

$$X^*(x,t) = 1 - \sum_{n=0}^{\infty}\frac{4}{(2n+1)\pi}\exp\left[-\left(\frac{(2n+1)\pi}{e}\right)^2 Dt\right]\sin\left(\frac{(2n+1)\pi}{e}x\right) \tag{6}$$

The average of the moisture content noted is defined by the following expression:

$$\bar{X}^*(t) = \frac{1}{e}\int_0^e X^*(x,t)\mathrm{d}x$$

thus

$$X^*(t) = 1 - \sum_{n=0}^{\infty}\frac{8}{(2n+1)^2\pi^2}\exp\left[-\left(\frac{(2n+1)\pi}{e}\right)^2 Dt\right] \tag{7}$$

Using only the first term, it may be shown that:

$$\bar{X}^*(t) = 1 - \frac{8}{\pi^2}\exp\left(-\frac{\pi^2}{e^2}Dt\right) \tag{8}$$

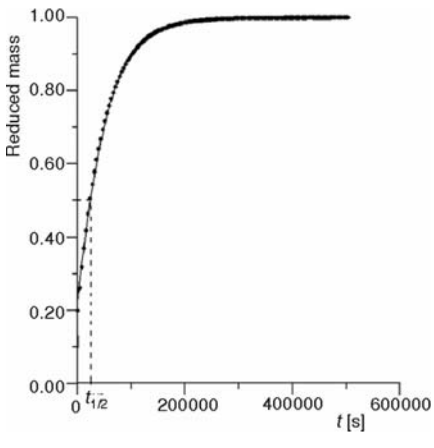

Figure 2. The evolution of the reduced moisture content according to time

Figure 2 shows an example of the evolution of the dimensionless moisture content as function of time. The quantity $t_{1/2}$ corresponds to the condition $\bar{X}^* = 0.5$ so that:

$$0.5 = 1 - \frac{8}{\pi^2\exp}\left[-\frac{\pi^2}{e^2}Dt_{1/2}\right] \tag{9}$$

Thus the diffusion coefficient is determined as:

$$D = \frac{e^2}{\pi^2 t_{1/2}}\ln\left(\frac{16}{\pi^2}\right) \tag{10}$$

Knowing the sample thickness used and the time $t_{1/2}$ obtained from the experimental curve of the reduced mass *vs.* time, it is possible to determine the diffusion coefficient.

Change of variables method

Equation (3) can also be solved by the change of variables method MOD2. The solution may be obtained by introducing a new variable η satisfied by:

$$\eta = \frac{x}{\sqrt{4Dt}}$$

which transforms the diffusion equation from a partial differential equation involving two independent variables to an ordinary differential equation expressed by the single variable.

Substituting into eq. 3, the diffusion equation becomes:

$$\frac{\partial^2 X^*}{\partial \eta^2} + 2\eta \frac{\partial X^*}{\partial \eta} = 0 \tag{11a}$$

The initial condition and the boundary conditions become then:

$$X^*(\eta \to \infty) = 0 \tag{11b}$$

$$X^*(\eta = 0) = 1 \tag{11c}$$

The analytical solution can be obtained as [17]:

$$X^* = \mathrm{erfc}(\eta) = \mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) \tag{12a}$$

in which the erfc(η) is called the complimentary error function. This function is a monotone decreasing function that goes from 1 to 0 and it is defined by the following equation:

$$\mathrm{erfc}(\eta) = 1 - \mathrm{erf}(\eta) \tag{12b}$$

where erf(η) is the error function defined by the following expression:

$$\mathrm{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta} \exp(-\eta^2)\mathrm{d}\eta \tag{12c}$$

The average of the dimensionless moisture content is determined by supposing that the vapor diffusion in the medium is carried by the two faces with the same quantity. In this case we have obtained the following expression of the average of the dimensionless moisture content [7]:

$$\overline{X}^*(t) = \frac{4}{e}\sqrt{\frac{Dt}{\pi}} \tag{13}$$

The reduced mass of the medium is the same as the dimensionless moisture content:

$$\overline{X}^*(t) = \overline{m}^*(t) = \frac{\Delta m(t)}{\Delta m_{\mathrm{total}}} \tag{14}$$

The diffusion coefficient is given by the slope of the linear part of the evolution curve of the reduced mass according to the square of time when using eq. 13. This linear part is often between $0.1 < m^*(t) < 0.6$ [7] (fig. 3).

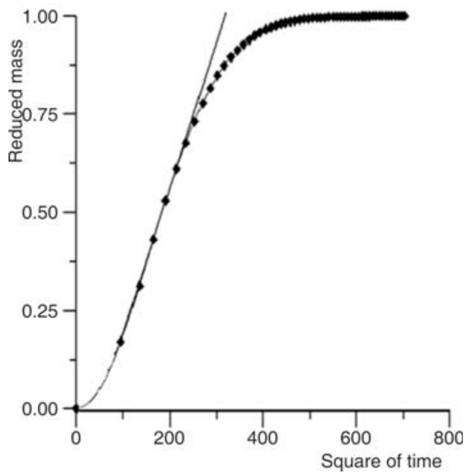

Figure 3. The evolution of the mass reduced according to the square of time

Results and discussion

General description

The determination of the moisture diffusivity in unsaturated porous media is necessary to avoid the problem of condensation inside the walls.

Measurements of the moisture diffusivity were conducted for a total of 12 insulating materials five of which are new original insulators and the rest are commercially available. The new insulators are home-made materials composed of short palm fibers in a ceramic matrix. Five mass percentages of palm fiber are considered namely 5, 10, 20, 40, and 60%. The moisture diffusivity for different samples in the unsteady-state was obtained by MOD1 according to eq. (10) and by MOD2 according to eq. (13).

Data for commercial insulators

We have used two known insulators already studied by Agoua *et al.* [7] and calculated the mass diffusion coefficients using the two models. The obtained values are presented in tab. 1.

Table 1. Determination of the diffusion coefficient for two known samples

Samples	Coefficient of the diffusion obtained by MOD1 $D_1 \cdot 10^{-9}$ [m^{2}s^{-1}]		Coefficient of the diffusion obtained by MOD2 $D_2 \cdot 10^{-9}$ [m^{2}s^{-1}]		Relative difference, [%] $\left\lvert \frac{D_1 - D_2}{D_1} \right\rvert$	
	Calculated	Obtained by Agoua *et al*. [7]	Calculated	Obtained by Agoua *et al*. [7]	Calculated	Obtained by Agoua *et al*. [7]
Beech (type of wood)	1.51	1.42	1.86	1.94	23	36
Pin sylvestre	1.74	1.65	1.92	1.86	10	12

We have found that the obtained values of these materials, using the MOD1, are in satisfactory agreement with typical values given by Agoua *et al.* [7] within 6% for the first and 5% for the second.

By applying the MOD2 we obtained values for these two insulators which are the same or even better than the values given by MOD1. In fact the relative differences between the obtained values and the typical values given by Agoua *et al.* [7] are, respectively, within 4% for the first one and 3% for the second.

Comparing the two obtained results, we have found a relative difference of 23% for the first sample and 10% for the second one. Agoua *et al.* [7] has obtained almost the same values namely 36% for the first and 12% for the second one. Hence, we can conclude that the obtained values of the two models agree well with the literature values.

In tab. 2, we present the values of the mass diffusion coefficients of five commercial insulators calculated by two models.

The relative difference obtained by these two models is between 9% and 44%. These values are comparable with the values mentioned above (12 % and 36%).

Table 2. Determination of the diffusion coefficient of industrial samples

Samples	Coefficient of the diffusion obtained by MOD1 $D_1 \cdot 10^{-9}$ [m^{2}s^{-1}]	Coefficient of the diffusion obtained by MOD2 $D_2 \cdot 10^{-9}$ [m^{2}s^{-1}]	Relative difference, [%] $\left\lvert \frac{D_1 - D_2}{D_1} \right\rvert$
Rockwool density (40 kg/m^3)	1.623	1.962	21
Recycled paper	2.387	2.61	9
Product foams polyurethane agglomerated	1.151	1.661	44
Rockwool covered with a kraft paper	1.037	1.411	36
Rockwool covered with an aluminum foil	1.042	1.484	42

This difference is due to the fact that in the MOD1 we have considered only the first term, whereas the MOD2 is general. In this case we have considered the linear part of the curve of evolution of the reduced mass.

Figure 4. Examples of new bricks

Characterization of new bricks

The new insulators are home-made materials composed of short palm tree fibers in a cement matrix. Five mass percentages of palm fibers are considered, namely: 0, 20, 30, 40 and 60% (fig. 4).

Table 3 presents the values of the diffusion coefficients *D* of the new insulating bricks calculated by the two models. The results suggest that the diffusion coefficient increases when increasing the fibers content. In fact the sample becomes more absorbing when the quantity of fibers increases. We notice that the relative difference obtained by the two models is less than 15% (tab. 3). Also when the fiber rate reaches the value of about 40%, the values of the diffusion coefficient becomes fairly constant (fig. 5).

Table 3. The diffusion coefficient as function of mass ratio

Mas ratio, [%]	0%	20%	30%	40%	60%
Coefficient of the diffusion obtained by MOD1, $D_1 \cdot 10^{-10}$ [m^{2}s^{-1}]	0.926	2.187	2.450	2.768	2.782
Coefficient of the diffusion obtained by MOD2, $D_2 \cdot 10^{-10}$ [m^{2}s^{-1}]	0.785	2.126	2.338	2.456	2.488
Relative difference, [%] $\left\lvert \frac{D_1 - D_2}{D_1} \right\rvert$	15	2	4	11	10

Figure 6 shows a comparison of the time evolution of the moisture content obtained by two models and experimental data. It is seen that the results obtained by the second model is more satisfactory.

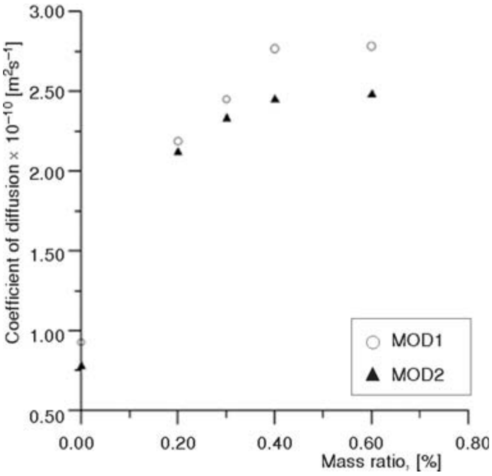

Figure 5. The evolution of coefficient of diffusion according to mass ratio of fibers

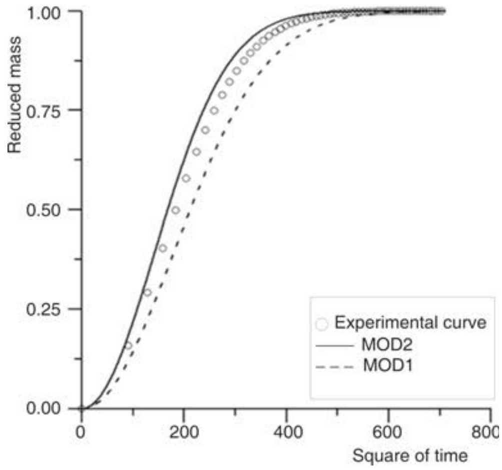

Figure 6. Comparison of fitted models with experimental data

Conclusions

The diffusion coefficient of several insulators was experimentally determined based on the solution of Fick's law equation of diffusion with appropriate boundary conditions. Two analytical methods are used to determine the diffusion coefficient. The results suggest that the values of the measured diffusion coefficients of various materials obtained are acceptable. The relative difference in the diffusion coefficient obtained by the two models is the same as those obtained by Agoua *et al*. [7].

Concerning the new bricks, the diffusion coefficient increases with an increase in the mass ratio. For higher values of the mass ratio this coefficient becomes constant.

Nomenclature

D – diffusion coefficient, [m^2s^{-1}]
e – thickness of sample, [m]
m – mass, [kg]
m^* – relative mass, [–]
t – time, [s]
X – moisture content, (kg of water/kg of solid)]
X^* – dimensionless moisture content, [–]
$\overline{X}^*$ – average dimensionless moisture content, [–]
x – transverse co-ordinate, [m]

Greek symbols

η – dimensionless constant, [–]

Subscripts

e – equilibrium
i – initial

References

[1] Zohoun, S., Perre, P., Measurement of the Moisture Diffusivity in the Steady-State Method of PVC-CHA, (in French), *ARBOLOR, 1* (1997), pp. 171-180

[2] Comstock, G. L., Moisture Diffusion Coefficients in Wood as Calculated from Adsorption, Desorption and Steady-State Data, *Forest Products Journal, 13* (1963), 3, pp. 97-103
[3] Richards, R. F., Steady-Flux Measurements of Moisture Diffusivity in Unsaturated Porous Media, *Building and Environment, 29* (1994), 4, pp. 531-535
[4] Olek, W., Weres, J., The Inverse Method for Diffusion Coefficient Identification during Water Sorption in wood, *Proceedings*, 3rd COST E15 Workshop on Softwood Drying to Specific end-Uses, 2001, Helsinki, Paper No. 27, 2001
[5] Philip, J. R., De Vries, D. A., Moisture Movement in Porous Materials under Temperature Gradients, *Transaction of American Geophysical Union, 38* (1957), 2, pp. 222-232
[6] De Freitas, V. P., Abrantes, V., Crausse, P., Moisture Migration in Building Walls-Analysis of the Interface Phenomena, *Building and Environment, 31* (1996), 2, pp. 99-108
[7] Agoua, E., Zohoun, S., Perre, P., A Double Climatic Chamber Used to Measure the Diffusion Coefficient of Water in Wood in Unsteady-State Conditions: Determination of the Best Fitting Method by Numerical Simulation, *International Journal of Heat and Mass Transfer*, *44* (2001), 19, pp. 3731-3744
[8] Wadso, L., Studies of Water Vapor Transport and Sorption in Wood, Building Materials, Lund University, Lund, Sweden, 1993
[9] Siau, J. F., Transport Processes in Wood, Springer-Verlag Berlin Heidelberg, New York, Tokyo, 1984
[10] Hjor, S., A Two-Dimensional Computer Program for Calculating the Moisture Conditions in Painted Wooden Structures, *Surface Coatings International, 81* (1998), 7, pp. 330-336
[11] Hjor, S., A New Method to Determine the Moisture Permeability of Painted Wood, Dept of Building Materials, Chalmers University of Technology, Gothenburg, Sweden, 1997
[12] Crank, J., The Mathematics of Diffusion, Oxford University Press, Oxford, UK, 1975
[13] ***, ASTME 96/E 96M-05, Standard Test Methods for Water Vapor Transmission of Materials ASTM International, West Conshohocken, Penn., USA, 2005
[14] ***, THALES Underwater Systems, Determination of the Conventional Coefficient of Permeability to Water Vapor (in French), ISO 1663 de 1999 (ex NF T 56-105) ASTM E96-80, ASTM F372-73, 1984
[15] ***, EN 1931:2000 Flexible Sheets for Waterproofing, Bitumen, Plastic and Rubber Sheets for Roof Waterproofing, Determination of Water Vapor Transmission Properties, CEN, 2000
[16] ***, EN ISO 12572:2001, Hygrothermal Performance of Building Materials and Products – Determination of Water Vapour Transmission Properties, CEN, 2001
[17] Bird, R. B., Stewart, W. E., Lightfoot, E. N., Transport Phenomena, John Wiley and Sons, Inc, 2nd ed., N. Y., USA, 2002, pp. 614-621

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