MAGNETOHYDRODYNAMIC EFFECTS ON UNSTEADY DYNAMIC, THERMAL AND DIFFUSION BOUNDARY LAYER FLOW OVER A HORIZONTAL CIRCULAR CYLINDER

by

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This paper is devoted to the analysis of unsteady 2-D dynamic, thermal and diffusion magnetohydrodynamic laminar boundary layer flow over a horizontal circular cylinder of incompressible and electrical conductivity fluid, in a porous medium, in the presence of a heat source or sink, and chemical reactions. The present magnetic field is homogenous and perpendicular to the body surface. It is assumed that the induction of the outer magnetic field is the function of the longitudinal coordinate and time. Fluid electrical conductivity is constant. The outer electric field is neglected and the magnetic Reynolds number is significantly lower than one i.e. the considered problem is in induction-less approximation. Free stream velocity, temperature and concentration on the body are arbitrary differentiable functions. The developed governing boundary layer equations and associated boundary conditions are converted into a non-dimensional form using a suitable similarity transformation and similarity parameters. The system of dimensionless equations is solved using the finite difference method and iteration method. Numerical results are obtained and presented for incompressible fluid for different numbers, such as Sc, Pr, Ec and magnetic number, and the parameter of the porous medium, temperature parameters, thermal parameter, diffusion parameters and chemical reaction parameter. The solutions for the flow, temperature and diffusion transfer and other integral characteristics, boundary layer, are evaluated numerically for different values of the magnetic field. Transient effects of velocity, temperature and diffusion are analyzed. A part of obtained results is given in the form of figures and corresponding conclusions.

Key words: magnetohydrodynamic boundary layer, mass and heat transfer, porous medium, heat source or sink, circular cylinder

Introduction

The problem of boundary layer separation and control has attracted considerable attention over several decades because of the fundamental flow physics and technological applications. Some of the essential ideas related to boundary layer separation, and the need to prevent the same from occurring, have been addressed by Prandtl [1]. A number of methods may be employed to control the boundary layer separation that occurs due to: adverse pressure gradient, possibility of body motion in stream wise direction, increasing the boundary

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layer velocity, boundary layer suction, second gas injection, body cooling, introducing a transverse magnetic field, etc. Boundary layer flow over a circular cylinder has been the subject of intense studies since the early work of Prandtl in 1904 (Schlichting [2]). It is well known that the first solution of the steady forced convection momentum (velocity) boundary layer flow over a circular cylinder was obtained by Blasius [3]. Using the series method, Frossling [4] also solved the thermal equation of this problem for the case when the surface temperature of the cylinder is subjected to a constant temperature.

This problem, boundary layer flow over a horizontal circular cylinder, has been extended by many investigators to a large number of different cases in viscous, viscoelastic or micropolar fluids, in which the boundary condition that is usually applied is either constant or variable wall temperature or constant or variable heat flux. The problems investigated are mixed convection [5], natural-free convection flow [6], free convection boundary layer in a porous medium [7], mixed convection boundary layer flow of a viscoelastic fluid [8], and forced convection boundary layer flow [9]. Of particular interest is a need for research of magnetic field effects on the flow of electrical conductivity fluid in a boundary layer. The study of magnetohydrodynamic (MHD) incompressible viscous flow has many important engineering applications in devices such as MHD power generators, cooling of nuclear reactors, geothermal systems, aerodynamic processes and the heat exchange designs, pumps and flow meters, in space vehicle propulsion, control and re-entry, in creating novel power generating systems and developing confinement schemes for controlled fusion. The interest in the effect of an outer magnetic field on heat-physical processes first appeared sixty years ago [10]. Many researchers have investigated the effects of MHD free and forced convection flow both experimentally and theoretically [11-13]. A number of different physical models of MHD boundary layer have been investigated using different methods giving exact or approximated solutions [14-22]. Special attention has been paid to various forms of the body profile, whereby the special interest has been shown in the analysis of MHD flow around a horizontal cylinder [23-27].

The subject of the present study is an analytic investigation on the effects of heat and mass transfer in 2-D unsteady laminar, dynamic, thermal and diffusion MHD laminar boundary layer flow over a horizontal circular cylinder of incompressible and electrical conductivity fluid, in a porous medium, in the presence of a heat source or sink, and chemical reactions. The external magnetic field is homogeneous and perpendicular to the body. The outer electric field is neglected and the magnetic Reynolds number is significantly lower than one i.e. the considered problem is in induction-less approximation. The system of dimensionless equations is solved using the finite difference method. Numerical results are obtained and presented for different numbers such as Sc, Pr, Ec, and magnetic number, and also the parameter of the porous medium, temperature parameters, thermal parameter, diffusion parameters and chemical reaction parameter. The solutions for the flow, temperature, diffusion transfer and other integral characteristics of boundary layer are evaluated numerically for different values of the magnetic field.

Formulation of the problem, mathematical analysis

A mathematical model, unsteady two-dimensional dynamics, thermal and diffusion MHD laminar boundary layer of incompressible fluid on horizontal circular cylinder, when the uniform external magnetic field $B(x, t)$ is applied perpendicular to the surface of the body and outer electric field is neglected (magnetic Reynolds number is significantly lower then one-considered problem is in induction-less approximation), of free stream velocity $U(x, t)$, and am-
bient temperature $T_\infty$ and concentrations $C_\infty$, is defined with system of four simultaneous equations: continuity equation and equations of dynamic, thermal and diffusion boundary layer.

In the previous system of equations and boundary conditions, the parameter labeling used is common for the MHD boundary layer theory: $x, y$ – the longitudinal and transversal co-ordinate; $u, v$ – the longitudinal and transversal velocity components, on the outer edge of boundary layer; $\nu$ – the kinematic viscosity of fluid, $K$ – the permeability, $\sigma$ – the fluid electrical conductivity; $\rho$ – the fluid density, $\lambda$ – the thermal conductivity, $c_p$ – the specific heat at constant pressure, $D$ – the effective diffusion coefficient, $k_b$ – the chemical reaction parameter, $T$ – the temperature and $c$ – the concentration of ionic species in solution.

(a) continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(b) momentum equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u - U - \frac{\nu}{K} u - U$$

(c) energy equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{1}{c_p} \left( \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} \right) U - u + \frac{\mu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B^2}{\rho c_p} u - U^2 + Q \left( T - T_\infty \right)$$

(d) mass diffusion equation:

$$u \frac{\partial c}{\partial x} + \nu \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} - k \ c - c_\infty$$

and corresponding boundary and initial conditions:
The term \( Q(T - T_w) \) is assumed to be the amount of heat generated or absorbed per unit volume. \( Q \) is a heat generation/absorption constant, which may take either positive or absorption when \( Q < 0 \). \( K \) is the permeability of the porous medium, \( D \) is the coefficient of chemical molecular diffusivity, and \( k \) is the reaction rate constant.

If we introduce stream function \( \psi(x, y, t) \) in the usual manner, the governing boundary layer equations and associated boundary conditions (1-5) are converted into a non-dimensional form using a suitable similarity transformation. Introducing new general similarities variable [28]:

\[
x = x, \quad t = t, \quad y = \frac{h x, t}{D_0} \eta, \quad x, y, t = \frac{U x, t}{D_0} \phi, \quad x, t, \eta
\]

Prandtl number, \( Pr = \frac{\nu \rho c_p}{\lambda} \),

\[
C x, t, \eta = \frac{c_w - c}{c_w - c_\infty}, \quad \Theta x, t, \eta = \frac{T_w - T}{T_w - T_\infty},
\]

Eckart number, \( Ec = \frac{U^2}{c_p T_w - T_\infty} \),

the system of eqs. (1)-(4) is transformed into a form:

\[
D_0^2 \phi_{\eta \eta} + \frac{f x, t + 2 f_1 x, t}{2} \phi_{\eta \eta} + \frac{\eta Z x, t}{2} \phi_{\eta \eta} + f_1 x, t 1 - \phi_\eta^2 + [f_0 x, t + g x, t + p x, t] 1 - \phi_\eta = Z \phi_{\eta t} + Z x, t U x, t \phi_\eta \phi_\eta x - \phi_\eta \phi_{\eta \eta}
\]

\[
\frac{D_0^2}{Pr} \Theta_{\eta \eta} + \frac{f x, t + 2 f_1 x, t}{2} \phi \Theta_{\eta \eta} + \frac{\eta Z x, t}{2} \Theta_{\eta \eta} - g x, t 1 - \phi_\eta^2 - \frac{Ec}{2} [f_1 x, t + f_0 x, t] 1 - \phi_\eta + \left[l_0 x, t + l_1 x, t \phi_\eta + q x, t\right] 1 - \Theta + D_0^2 Ec \phi_{\eta \eta}^2 = Z \Theta + Z \left[c_{l/2} x, t \right] \phi_{\eta t} - \phi_\eta \Theta_\eta
\]

\[
\frac{D_0^2}{Sc} C_{\eta \eta} + \frac{f x, t + 2 f_1 x, t}{2} \phi C_{\eta \eta} + \frac{\eta Z x, t}{2} C_{\eta \eta} + s x, t - C x, t \right] Z C + Z \left[c_{l/2} x, t \right] C_{\eta t} - \phi_\eta C_\eta
\]

with boundary conditions:

\[
\phi = \frac{\partial \phi}{\partial \eta} = \phi_\eta = 0, \quad \Theta = 0 \quad \text{and} \quad C = 0 \quad \text{for} \quad \eta = 0
\]

\[
\phi_\eta \to 1, \quad \Theta \to 1, \quad \text{and} \quad C \to 1 \quad \to 1 \quad \text{for} \quad \eta \to \infty
\]
and dimensionless parameters of similarities are introduced in the following form:

- dynamical parameters:

\[ f_{\text{x},t} = U_{\text{x},t} Z_{\text{x},t}, \quad f_1_{\text{x},t} = U'_{\text{x}} Z_{\text{x},t}, \quad f_0_{\text{x},t} = \frac{U}{U_{\text{x},t}} Z_{\text{x},t} \]

- magnetic parameter: \( g_{\text{x},t} = N_{\text{x},t} Z_{\text{x},t}, \quad N_{\text{x},t} = \frac{\sigma B_{\text{x},t}^2}{\rho} \)

- parameter of the porous medium:

\[ p_{\text{x},t} = \frac{V}{K} Z_{\text{x},t}, \quad (9) \]

- temperature parameters:

\[ l_1_{\text{x},t} = \frac{T_{w_{\text{x},t}}}{T_{w_{\text{x},t}}} U_{\text{x},t} Z_{\text{x},t}, \quad l_0_{\text{x},t} = \frac{\hat{T}_{\text{w}_{\text{x},t}}}{\hat{T}_{\text{w}_{\text{x},t}}} Z_{\text{x},t} \]

- thermal parameter: \( q_{\text{x},t} = Q Z_{\text{x},t} \)

- diffusion parameter: \( c_1_{\text{x},t} = \frac{c'_{w_{\text{x},t}}}{c_{\text{w}_{\text{x},t}} - c_{\infty}}, \quad U_{\text{x},t} Z_{\text{x},t} \)

- chemical reaction parameter: \( s_{\text{x},t} = k Z_{\text{x},t} \)

Sets of these independent parameters, with \( h(x, t) \) as the characteristic of boundary layer thickness, reflect the nature of free stream velocity change \( U = U(x, t) \), influence of magnetic field \( N(x, t) \), porosity media, temperature \( T_{w}(x, t) \) and diffusion \( c_{w}(x, t) \) boundary conditions on the body, as well as the influence of chemical reactions or a heat source or sink \( Q \), and apart from that, in the integral form \( Z(x, t) \), the pre-history of flow in the boundary layer.

\( D_0 \) is the normalizing constant, which is determined from the condition that the first dynamics equation is reduced to the case of flow past a flat plate: \( \phi_{0\eta\eta} + \phi_{0\eta} = 0, \) from which follows the constant value \( D_0 = \xi_0^{0.5} = 0.469. \)

To the system of eqs. (7), where the unknown functions are \( \phi_{0}(x, t, \eta), \Theta(x, t, \eta), C(x, t, \eta), \) and \( Z(x, t) \), it is necessary to add one of the appropriate integral equations. The first integral equation is obtained by integrating the momentum equation perpendicular to the boundary layer, the second integral equation for temperature is obtained by integrating the energy equation, and the third integral equation for concentration is obtained by integrating mass diffusion equation. In this paper, as integral equations, we use the impulse equation, with initial and boundary conditions: \( \partial Z/\partial t = 0, \) for \( t = 0 \) and \( \partial Z/\partial x = 0 \) for \( x = 0. \)

\[
\frac{H^*}{2} \frac{\partial Z}{\partial t} + \frac{U}{2} \frac{\partial Z}{\partial x} = \zeta - \left\{ \frac{U}{U_{\text{x},t}} \frac{\partial Z}{\partial t} + U'_{\text{x}} x, t + N_{\text{x},t} + \frac{V}{K} \right\} \frac{H^*}{2} + 2U'_{\text{x}} x, t H^{**} \right\} Z_{\text{x},t} \quad (10)
\]
First, the initial condition means the pre-history of the boundary layer. These conclusions are valid also for the values of the porous parameter stationary boundary layer or the previous state, the situation stagnancy sleep mode. The second boundary condition is the usual condition in the theory of the stationary boundary layer, and it means that all the boundary layer thickness in front of the stagnation point has a tangent parallel to the x-axis.

In these relations, characteristic functions of the boundary layer are introduced in the following form: shear stress on the body $\zeta = [\partial(uU)/\partial(y/h)]$, and $H = \delta/h$, $H^{**} = \delta^{**}/h$, $H_T = \delta_T/h$, $H_T = \delta_T/h$, where

$$\delta^{**} = \int_0^\infty 1 - \phi_1 \, dy \quad \text{displacement thickness}, \quad \delta^{**} = \int_0^\infty \phi_2 \, dy \quad \text{momentum thickness},$$

$$\delta_T = \int_0^\infty 1 - \Theta \, dy \quad \text{temperature thickness}, \quad \delta_c = \int_0^\infty 1 - C \, dy \quad \text{diffusion thickness}.$$

In this paper, $\delta^{**}$ - momentum thickness is taken as $h(x, t)$ - characteristic boundary layer thickness.

A system of eqs. (7, 10) is obtained in the paper, and it is a generalized system, that can equalize the certain parameters similarities (9) to zero, and is reduced to simple physical models of the boundary layer. This is the case when $g = 0$ (non-conducting fluid flow), for $f_0 = 0$, gets a stationary MHD boundary layer, and when the thermal or diffusion parameters are equal to zero, it is a flow around the body surface constant temperature and concentration, etc.

Derived eqs. (7, 10), with corresponding boundary conditions, with sets of independent parameters (9), can now be applied to any concrete example, defined profile of the body, given boundary and initial conditions of temperature and concentration on the surface of the body and velocity outer flow.

As a concrete example of the application of systems of eqs. (7, 10), with corresponding boundary conditions, the paper further considers the unsteady flow around a horizontal circular cylinder. In this case, the analysis will be carried in a non-dimensional form, where longitudinal coordinate and velocity will be scaled in relation to the velocity of outer flow $U(x, t)$ and the radius of the cylinder $a$, and the coordinate and velocity, perpendicular to the boundary layer, will be divided with $R_c^{0.5} - (R_c = U_\infty a/\nu)$. With defined free stream velocity, externally uniform magnetic field strength $B(x, t)$, which is in this case taken as constant, and boundary conditions for temperature and concentration and friction on the body $\tau_w = \pi(x, t)$, determined with the following expressions:

$$U \bar{x}, \bar{t} = 1 + a_t \bar{t}^n \sin \bar{x}, \quad \tau_w = T_a + T_0(1 + a_z \bar{x}^\mu)(1 + a_{2z} \bar{x}^{m_z})$$

$$c_w \bar{x}, \bar{t} = c_w + c_0(1 + a_3 \bar{x}^\nu), \quad \tau_w = \mu \partial u/\partial y$$

where the $\bar{x}$ are the angular coordinates of arbitrary points on the cylinder measured from the stagnation point in radians, and $t$ is the dimensionless size of the time.

Size in terms (11) $a_1$, $a_2$, $a_3$, $a_2$, are arbitrary positive or negative dimensionless constants. Positive constant value $a_1$ corresponds to rapid fluid flow, and the negative corresponds to slow fluid flow. Positive or negative value constants $a_2$, $a_3$, indicate and increase or decrease in temperature and concentration, respectively, along the body, and positive or negative value constants $a_2$, indicate an increase or decrease in temperature with time. Exponents $n, m, m_z$, and $p$ are positive integer constants. For this case, the introduced similarity parameters (9) are transformed into the following form:
Method of solution, results, and discussion

It is known that numerical integration gives the results that are characterized by high accuracy. In this respect, the non-dimensional system of parabolic differential eq. (7) is solved numerically, with respect to the boundary conditions (8) and integral momentum eq. (10), with the most efficient and accurate method known as the finite difference method with an adequate block scheme.

Getting highly accurate solutions is achieved, on the one hand, as a result of wide-ranging capabilities provided by the method of finite differences, and, on the other, as a result of application iterative processes that allows for approximate solutions with arbitrary accuracy given. Approximation of nonlinear differential equations is performed by a system of algebraic equations, defined on a discrete set of points of integration network as defined in the 3-D co-ordinate system: \( x, t, \eta \). Linearization problem for nonlinear members of equations is solved using the iteration method, such that the values, which now represent coefficients, are taken from the previous network layer or from the previous iteration for the current line and the current iteration, as a known value. In this paper, the obtained system of algebraic equations, that is suitable for further computations, is solved using the three-diagonal method, known in the Eastern literature as the “progonka” method.

Numerical results are obtained and presented for different values of numbers \( Ec, Sc, Pr \) and also for the parameter of the porous medium, temperature parameters, thermal parameter, diffusion parameters and chemical reaction parameter. The solutions for the flow, temperature and diffusion transfer, and other integral characteristics boundary layer, are evaluated numerically for different values of the magnetic number \( N \). Transient effects of velocity, temperature and diffusion are analyzed.

A part of the obtained results is presented in fig. 2 to fig. 7. Figures 2 to 4 show the variations of the variables \( \tau_0, \theta_0, \) and \( C_0 \), as functions of the coordinate \( x \), and fig. 5 to fig. 7 show the variations dimensionless velocity, temperature and concentration, in a certain section of the boundary layer, for several values of the magnetic parameter \( N, Pr, Ec, Sc \) number and also for several values of the \( K \) – permeability parameter, \( Q \) – heat source or sink parameter, \( k \) – chemical reaction parameter. In this paper, all coefficients in relations (11, 12), take the values \( m_i = a_{ij} = p_i = 1 \), except the coefficients \( a_i = \pm 1 \) and \( n = 2 \).

It can be noticed (fig 2), (where \( Pr = 0.73, Ec = 0.7, Sc = 7.0, Q = k = K = 0.1 \)), that the magnetic parameter \( N \), have a considerable influence on the position of the boundary layer separation point. The increase in the magnetic parameter moves the point of boundary layer separation downstream and, in that sense, its influence can be considered positive. These conclusions are also valid for the values of the porous parameter.
It can be noticed from these figures that free stream acceleration causes a delay in boundary layer separation, i.e., it moves the boundary layer separation point downstream ($a_1 = 1$), while free stream deceleration moves the boundary layer separation point upstream ($a_1 = -1$). Thus, the influence of free stream acceleration is positive and the influence of free stream deceleration is negative.

Figure 3 (where $E_c = 0.7$, $Sc = 7.0$, $N = 0.2$, $k = K_p = 0.1$, $a_1 = 1$, $t = 0.1$), and fig. 4 (where $E_c = 0.7$, $Sc = 7.0$, $N = 0.2$, $k = K_p = 0.1$, $a_1 = 1$, $t = 0.1$), show the graphs of characteristic functions $\theta_{\phi \theta}$ and $C_{\phi \theta}$, respectively, as a function of co-ordinate $x$, for different values of the parameter $Q$ and Pr – number, and $k$ and Sc – number, respectively.

Figures 5, 6, and 7 show the graphs of variations of dimensionless velocity, temperature and concentration in a certain section of the boundary layer ($x = 1.0$).

Dimensionless velocity in boundary-layer is shown in fig. 5 as a function of dimensionless transversal coordinate $\eta$, for different values of magnetic parameter and time for two values of constants $a_1 = \pm 1$. We observe that with the increase in magnetic parameter this ve-
locity also increases and the minimal value is obtained for the case of magnetic field absence. This analysis indicates the significant influence of magnetic field on increasing velocity in the boundary-layer. The results clearly show that the magnetic field tends to delay or prevent separation.

Figure 6. Dimensionless temperature $\theta$, for different values of Pr and $Q$

Figure 7. Dimensionless concentration $C$, for different values of Sc and $k$

Figure 6 shows the effects of Pr and parameter $Q$, on the dimensionless temperature (for fixed values of Ec = 0.7, Sc = 7.0, $N = 0.2$, $k = K = 0.1$, $a_1 = 1$, $t = 0.02$). It is clear that the dimensionless temperature at a point increases (fluid temperature decreases) with the increase in Prandtl number. The increase in Pr means that the thermal diffusivity decreases. Thus, the rate of heat transfer is decreased due to the decrease in fluid temperature in the boundary-layer. The increase in heat generation/absorption constant $Q$, from negative to positive values, has the opposite effect, since in this case, the temperature in the boundary layer grows while the dimensionless temperature decreases. The same conclusion applies to the change in size $\theta_p$, fig. 3.

Figure 7 shows the effects for different Sc numbers and parameter $k$ on the dimensionless concentration (for fixed values of Ec = 0.7, Sc = 7.0, $N = 0.2$, $Q = K = 0.1$, $a_1 = 1$, $t = 0.02$). The increase in the Schmidt number and value of the parameter $k$ increases dimensionless concentration. The same conclusion applies to the change in size $\theta_p$, fig. 4.

In the case of absence of a magnetic field or the non-conducting fluid model presented in this paper, the size of the dynamic boundary layer, for the case where $N = 0.0$ – nonconductive fluid, or the size of the temperature layers is in high agreement with the corresponding results [9] and [4].

Conclusions

This paper is devoted to the analysis of unsteady two-dimensional dynamic, thermal and diffusion MHD laminar boundary layer flow over a horizontal circular cylinder of incompressible and electrical conductivity fluid with effects of thermal radiation, in a porous medium, in the presence of a heat source or sink, and chemical reactions. The developed governing boundary layer equations and associated boundary conditions are converted into a dimensionless form using a suitable similarity transformation and similarity parameters. Numerical solu-
tion of dynamic, temperature and diffusion boundary layer equations, with the integral impulse equation, are obtained by using the finite difference method, combined with the method of iteration. Numerical results for the solutions of the flow, temperature and diffusion transfer, and other integral characteristics boundary layer, are obtained and presented for different parameters such as Sc, Pr, Ec, magnetic-N number, and also the parameter of the porous medium, temperature parameters, thermal parameter, diffusion parameters and chemical reaction parameter. Transient effects of velocity, temperature and diffusion are analyzed. A part of obtained results is given in the form of figures and corresponding conclusions. In this paper, the results of integral and differential characteristics of the boundary layer show the effects of the magnetic field and other introduced similarity parameters on the flow development around the circular cylinder surface.

Nomenclature

\[ B \] – magnetic field induction, [T]
\[ C \] – dimensionless concentration function
\[ c \] – concentration of the fluid
\[ H \] – characteristic function, [-]
\[ N \] – characteristic function, [s^{-1}]
\[ t \] – time, [s]
\[ U \] – free stream velocity, [ms^{-1}]
\[ u, v \] – velocities in boundary layer, [ms^{-1}]
\[ x, y \] – longitudinal, transversal ordinate, [m]
\[ Z \] – characteristic function, [s]

Greek symbols

\[ \zeta \] – characteristic function, [-]
\[ \phi \] – dimensionless stream function, [-]
\[ \theta \] – dimensionless temperature function, [-]
\[ \eta \] – dimensionless transversal coordinate, [-]
\[ \tau \] – shear stress, [Pa]
\[ \Psi \] – stream function, [m^{2}s^{-1}]

Subscripts

\[ w \] – surface conduction
\[ \infty \] – conduction far away from the surface
\[ 0 \] – initial time moment \( t = t_0 \)
\[ 1 \] – boundary layer cross-section \( x = x_0 \)
\[ \eta, x, t \] – differentiation with respect to \( \eta, x, t \)

References

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