BIOMIMIC DESIGN OF MULTI-SCALE FABRIC WITH EFFICIENT HEAT TRANSFER PROPERTY

by

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Wool fiber has a complex hierarchic structure. The multi-scale fibrils are assembled to form a tree-like channel net in wool fiber, providing an efficient heat transfer property. The optimal inner configuration of wool fiber can also be invited to biomimic design of textile fabrics to improve the thermal comfort of cloth. A heat transfer model of biomimic multi-scale fabric using the fractal derivative is established. Theoretical analysis indicates that the heat flux efficiency in the biomimic fabric can be 2 orders of magnitude comparing with that of the continuous medium.

Key words: biomimic, heat transfer, fractal derivative, fabric

Introduction

Thermal property of textile is of major concern for comfort of cloth. A comfortable cloth should be able to assist in the thermo regulatory responses of the body, and this is achieved by maintaining thermal balance of the body with the help of the microclimate produced between the skin and the inner surface of the clothing system.

Wool fiber is called “breathing fiber”, which has a fantastic heat and moisture transport capability comparing with other nature or man-made fibers. The excellent performance of wool fiber was due to its hierarchic micro-structure [1, 2], shown in fig. 1. Wool fiber is composed of size decreasing fibrils, from cortical cell, to microfibril, to protofibril, and at last to $\alpha$-helix [3, 4]. These self-similar components assembled regularly in different hierarchical levels constructing a branching channel net. It is the channel net with higher crystallinity that endows wool fiber with higher heat conductive efficiency [5]. Comparing with the heat transfer in traditional parallel network, heat transfer efficiency in micro channel network of wool fiber was magnified by dozens of times.

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A biomimetic design of wool fiber is much needed, and a fractal structure for heat transport is suggested. The heat transfer efficiency of the fabric will be obviously improved and the biomimic multi-scale fabric would be a promising fabric for sport cloth.

In this paper, we investigate the thermal conductivity of biomimetic fabric using the fractal derivative method [7].

**Heat transfer model of the hierarchic biomimetic fabric**

One-dimensional heat conduction of an isotropic media without inner heat source obeys Fourier’s law, expressed as:

\[
\frac{\partial T}{\partial t} + \frac{\partial}{\partial x}\left( k \frac{\partial T}{\partial x} \right) = 0
\]

(1)

For steady case, eq. (1) becomes:

\[
k \frac{\partial T}{\partial x} = q
\]

(2)

where \( q \) is a constant and \( k \) – the heat conductivity. Equation (2) denotes that heat flux is constant.

Now we consider the heat conduction in the simplest binary tree fractal system of the biomimetic fabric as illustrated in fig. 2. A unit cell established for analysis of heat transfer in a certain hierarchy of binary tree fractal system is given in fig. 2. When measured with the smallest measure \( x_0 \), and any discontinuity less than \( x_0 \) is ignored, the heat flux can be approximately expressed as:

\[
q_0 = k \frac{dT}{dx} \approx k \lim_{\Delta x \to x_0} \frac{\Delta T}{x_0} = k \frac{\Delta T}{x_0}
\]

(3)

Equation (3) describes the heat transfer in the continuous media. However, heat transfer in a discontinuous fractal configuration is different due to the unique structure of the media.
To deal with such complex heat transfer problem, we adopt the fractal derivative [6-8] to model the heat transfer, which is a simple method to deal with heat and mass transfer in discontinuous medium, and the fractal dimensions are equivalent to the order of the fractal derivative.

When measuring with a smaller measure scale $x$, the length of heat transfer is:

$$ds = k_1 x^\alpha$$  \hspace{1cm} (4)

where $\alpha$ is the fractal dimension of the fractal binary tree and $k_1$ – a constant. By fractal derivative method [7], the heat flux in such a discontinuous space is:

$$q = k_2 \frac{DT}{Dx^\alpha} = \frac{dT}{ds} = \lim_{\Delta x \to 0} \frac{\Delta T}{k_1 x_0^{\alpha}} = k_3 \frac{\Delta T}{x_0^{\alpha}}$$  \hspace{1cm} (5)

The definition of the fractal derivative, $DT/Dx^\alpha$, is given in [7]. Comparison of eq. (5) with eq. (3) yields:

$$\frac{q}{q_0} = k_4 \frac{x_0}{x^\alpha} = k_4 \frac{1}{x_0^{\alpha-1}}$$  \hspace{1cm} (6)

where $k_2$, $k_3$, and $k_4$ are constants.

Since the value of $x_0$ is infinitely small, and the fractal dimension $\alpha > 1$, the value of $q/q_0$ is larger than 1, indicating that the efficiency of heat transfer in the fractal tree-shaped discontinuous media is improved comparing with the continuous media.

**Discussion and conclusion**

Brief calculation of the heat transfer efficiency in a multilayer biomimetic fabric is as follows:

Suppose the included angle between the stem and branch is $5\pi/6$, we have:

$$\frac{x_0}{x} = 1 + \frac{\sqrt{3}}{2}$$  \hspace{1cm} (7)

The fractal dimension is:

$$\alpha = \frac{\ln 3}{\ln \left(1 + \frac{\sqrt{3}}{2}\right)} = 1.76$$  \hspace{1cm} (8)

Suppose the multilayer fabric is composed of 20 tex yarn about 140 $\mu$m in diameter, thus the thickness of one layer of fabric is between 280 $\mu$m to 420 $\mu$m, and the value of $x_0$ corresponding to two layer thickness of fabrics is in the ranges from 560 $\mu$m to 840 $\mu$m. Here, we take the average value, i.e. $x_0 = 700$ $\mu$m. Inserting the above values into eq. (8), we obtain:

$$\frac{q}{q_0} = k_4 \frac{1}{(700 \cdot 10^{-6})^{1.76-1}} = 2.5 \cdot 10^2 k_4$$  \hspace{1cm} (9)
The above analysis denotes that the thermal conductive efficiency of biomimic fabric is about 2 orders of magnitude comparing with that of the continuous medium when $k_4$ is closed to 1. The value of $k_4$ is determined geometrically.

In conclusion, we have proposed a fractal derivative model dealing with for the first time heat transfer in a complex hierarchic structure, which is of critical importance for the industry, especially the specialists in biomimic design of wool fiber. The fractal derivative [7] is an efficient method to deal with these complicated heat transport problems of porous media. The biomimic multi-scale design proposed in this letter is also valid for design of functional fabric with super moisture and gas permeability for various special applications. Of course the authors understand that no matter how rigorous, some experimentally verification is needed to validate the model.

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