MODELING ON THE BREAKUP OF VISCO-ELASTIC LIQUID FOR EFFERVESCENT ATOMIZATION

by

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Short paper
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A primary atomization model for visco-elastic liquid is established based on the ligament formation assumption and dispersion relationships. Two linear instability analyses are used in presented paper. Both of them are involved in the primary atomization model and the calculated results are compared with published experimental data. After validation, the model is used to predict the effects of various operating conditions and visco-elastic liquid physical properties on atomization performance.

Key words: atomization, visco-elastic, mathematical model

Introduction

Atomization is a dynamic multiphase flow that involved complex interactions between gas and fluid. Compared to other liquid atomization mechanisms, effervescent atomization, a method of twin-fluid process that involves bubbling gas into the liquid internally, has so far been found available in lots of applications. Recently, Nielsen et al. [1] evaluated the actual potential of effervescent atomizer in atomization of high viscosity aqueous solutions for coating applications. Esfarjani et al. [2] simulated the internal liquid-gas flow in an effervescent nozzle for suspension plasma spray. While in the process of atomization, these nanoparticles will transport, disperse, collide, coagulate, agglomerate and so on [3-10], which makes the atomization be complex.

The study of atomization process is firstly conducted on the breakup of inviscid liquid, then the viscous liquid, recently extension to liquids with complex physical properties, such as visco-elastic or thixotropic [11]. Under shear flow, the visco-elastic fluid is more difficult to deformation due to the intrinsic material time scales. This is because the initial response of visco-elastic fluid to an imposed deformation involves a material-dependent transient that may be many orders of magnitude longer than time scales associated with instrument inertia. The breakup mechanism of visco-elastic liquid is more complex than the Newtonian liquid. The purpose of present paper is to develop a model for the breakup of visco-elastic liquid. Different stability analysis methods with different mathematical treatments are investigated and their advantages and drawbacks are discussed. The influences of visco-elastic physical properties and operating conditions on atomized droplet size are indicated.

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Mathematical description

Compared to conventional pressure, rotary and twin-fluid atomizers, the effervescent atomizer, using a method of twin-fluid process that involves bubbling gas into the liquid internally offers advantages of smaller drop sizes, reduced injection pressure, lower gas flow, larger exit orifice, higher viscosity. The gas-liquid two phase flow pattern closed to the effervescent nozzle orifice is annular. After being discharged from the atomizer orifice, the rapidly expanding gas phase shatters the annular sheath liquid into a number of cylindrical ligaments, the ligaments are instability and then breakup into ligament fragments and drops.

The momentum equation of the annular flow and the expression of interface velocity slip ratio are used to calculate the diameter of the typical cylindrical ligament as in [12]. Secondly, the ligaments will breakup into ligament fragments at the wavelength of the most rapidly growing wave. In present study, two linear instability dispersion relationships proposed by Coren et al. [13] and Joseph et al. [14] respectively are used to determine the wave number at the maximum growth rate occurs. Coren and Gorttlieb’s dispersion relation can be expressed by non-dimensional form as:

\[
\zeta \gamma^2 + \frac{3 \pi^2 \gamma (1 + \phi_1 \gamma)}{1 + \phi_2 \gamma} - \frac{\pi^2 (1 - \pi^2 \gamma^2)}{2} - T_e \pi^2 (1 - \phi_1 \gamma) = 0
\]

(1)

where \( \gamma \) is the non-dimensional growth rate, \( \zeta \) – the inertial forces relative to surface tension forces, \( \zeta = 2\pi^2 \rho_1 \sigma / \eta_0 k^2 \delta_{ig} \), \( k \) – the dimensional wave number of a disturbance, \( \eta_0 \) – the zero shear viscosity, \( \rho_1 \) – the liquid density, \( \sigma \) – the liquid surface tension, \( \eta = -\sigma_{ij} k^2 /2\pi \) – the modified non-dimensional wave number, \( \phi_1 \) and \( \phi_2 \) are measures of viscoelasticity that include the fluid relaxation time \( \lambda_1 \) and retardation time \( \lambda_2 : \phi_1 = 2 \lambda_2 \sigma / \eta_0 \delta_{ig} \phi_2 = 2 \lambda_2 \sigma / \eta_0 \delta_{ig} \). For a dilute polymer solutions, the relationship of retardation time and relaxation time is \( \lambda_2 = \lambda_1 \eta_{solvent} / \eta_{solution} \). \( \eta_{solvent} \) and \( \eta_{solution} \) are the viscosity of solvent and solution, respectively. \( T_e \) is the un-relaxed axial time normal to the direction of the flow which would cause the visco-elastic jet to be stabilized, as suggested by Bousfield et al. [16], \( T_e \) can be expressed as: \( T_e = 32(1 - \lambda_1 / \lambda_2) (2 \lambda_2 \gamma V) \), where \( V \) is the average velocity of the liquid. Then, eq. (1) can be written as a fourth-degree polynomial in the growth rate \( \gamma \) and solved analytically.

The ligament with thickness \( \delta_{ig} \) that grows rapidly and accelerated by explosion bubbles is similar to a flattened drop in a high-speed gas stream. Subjected to a R-T instability, the non-Newtonian fluids dispersion relation can be given as Joseph et al. [14] proposed:

\[
- \left[ 1 + \frac{1}{\gamma^2} \left( -ak + \frac{\sigma k^3}{\rho_l} \right) - \frac{4 k^2 \Delta}{\gamma \rho_l} + 4 \frac{k^3}{\gamma^2} \left( \frac{\Delta}{\rho_l} \right)^2 \left( k^2 + \frac{\gamma \rho_l}{\Delta} - k \right) = 0 \right. \quad \left( \rho_l \Box \rho_g \right)
\]

(2)

Equation (3) is an approximate analysis of Rayleigh-Taylor instability based on viscoelastic potential flow which gives the critical wavelength and growth rate within less than 10% of the exact theory. In eq. (3), \( k \) is the magnitude of the wave number, \( \gamma \) is the growth rate, and \( \sigma \) – the liquid surface tension. \( \Delta \) is the effective shear viscosity of liquid: \( \Delta = \eta_0 (1 + \lambda_2 \gamma) / (1 + \lambda_1 \gamma) \). \( a \) is the acceleration of liquid ligament: \( a = F_e / m_{ig} = c_d \rho_g (u_g - u_l)^2 / (\rho_l \delta_{ig}) \). Then, eq. (3) reduces to be a three-degree polynomial of \( \gamma \) and can be solved analytically. Based on dispersion relations eqs. (1) and eq. (2), the critical breakup wave number that corresponds to the maximum growth rate can be used to calculate the wavelength of a typical ligament fragment. Finally, assuming that each fragment is stabilized to one
spherical droplet under the influence of surface tension, the Sauter mean drop diameter (SMD) (defined as the ratio of volume-to-surface area) can then be calculated from the conservation of mass: \( \text{SMD} = \left(1.5 \delta_{\text{lig}} \lambda\right)^{1/3} \).

**Results and discussions**

**Validation**

The experiments conducted by Geckler et al. [17] are used to validate the primary breakup model. Both the dispersion relationships proposed by Coren et al. [13] and Joseph et al. [14] are solved analytically. The predicted results and experimental data are compared in fig. 1, in which “Model A” employed Coren’s linear analysis, and “Model B” used Joseph’s. In fig. 1, both the model A and B predict that with an increase in air-to-liquid mass flow ratio (ALR), droplet Sauter mean diameters will decline significantly, especially at lower ALR. The agreement between the predictions and measurements is qualitative achieved.

**Parameter study**

In figs. 2, 3 and 4 the operating conditions of basic case are: liquid mass flow rate 2.2 g/s, nozzle exit diameter 3 mm, inject pressure 0.35 Mpa, ALR 0.08, surface tension \(22 \times 10^{-3} \text{ J/m}^2\), zero shear viscosity of suspension \(25 \times 10^{-3} \text{ Pa/s}\), and relaxation time \(1.0 \times 10^{-4} \text{ s}\). All the cases have the same operating conditions as the base case, except the specified condition in the legend of the figure. The liquid breakup is a process that the magnitude of the aerodynamic force exceeds the liquid surface tension and viscosity force. Therefore, the increase of rheological properties (\(\lambda_1\) and \(\eta_0\)) makes the visco-elastic liquid more stringy to resist the disruptive force and increases the wavelength of the disturbances, and finally causes drop size increasing. This phenomenon is more distinct at lower ALR, since the reduction of aerodynamic force will accelerate the atomization performance deterioration.

It can be seen from fig. 3-4 that increase of ALR has the most impact on the decrease of initial atomized droplet size. This is because larger gas flow rate and/or smaller liquid flow ratio lead to finer primary breakup and larger ALR indicates large velocity difference between the gas and liquid phases, which enhance the drop distortion and breakup. Besides ALR, larger nozzle exit size gives rise to an obvious increase in droplet size, since larger noz-
zzle exit will result in thick liquid sheets. The effect of injection pressure on effervescent performance is appreciably but not significantly. Results also shows that the visco-elastic fluid has much higher atomized droplet size than that of pure ethanol.

![Figure 3. Effects of liquid properties on atomization performance for different zero shear viscosity](image1.png)

![Figure 4. SMD vs. ALR for different operating conditions and liquid type](image2.png)

**Conclusions**

A model with different stability analysis methods for the primary atomization of visco-elastic liquid is established and validated in the present paper. Results show ALR is the most important factors for the breakup of visco-elastic liquid. The decrease in fluid physical properties (zero shear viscosity and relaxation time) is benefit for finer atomization performance.

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**References**


