HYDROMAGNETIC FLOW AND HEAT TRANSFER ADJACENT TO A STRETCHING VERTICAL SHEET IN A MICROPOLAR FLUID

by

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An analysis is carried out for the steady 2-D mixed convection flow adjacent to a stretching vertical sheet immersed in an incompressible electrically conducting micropolar fluid. The stretching velocity and the surface temperature are assumed to vary linearly with the distance from the leading edge. The governing partial differential equations are transformed into a system of ordinary differential equations, which is then solved numerically using a finite difference scheme known as the Keller box method. The effects of magnetic and material parameters on the flow and heat transfer characteristics are discussed. It is found that the magnetic field reduces both the skin friction coefficient and the heat transfer rate at the surface for any given K and λ. Conversely, both of them increase as the material parameter increases for fixed values of M and λ.

Key words: *heat transfer, magnetohydrodynamics, micropolar fluid, stretching sheet*

Introduction

Sakiadis [1, 2] was the first to investigate the boundary layer flow past a moving solid surface of a viscous fluid with a constant velocity. Later, the numerical results of Sakiadis were confirmed by Tsou *et al.* [3] analytically and experimentally. Since then many researchers have considered various physical effects of this problem in Newtonian fluid [4-10]. In recent years, the flow and heat transfer over a stretching sheet immersed in a Newtonian fluid in the presence of magnetic field has received great attention because of its important applications in metallurgical industry which involves the cooling of continuous strips and filaments drawn through a quiescent fluid [11-19]. However, many industrial processes involve non-Newtonian fluid such as paints, lubricants, blood, polymers, colloidal fluids, and suspension fluids which cannot be described by traditional Newtonian fluid. Therefore, to investigate those fluids, the researchers applied the theory of micropolar fluid that was first formulated by Eringen [20, 21]. This theory is capable to describe those fluids by taking into account the effect arising from local structure and micro-motions of the fluid elements. Since the publication of Eringen's micropolar fluid theory, many authors [22-31] have investigated various flow and heat transfer problems. Extensive reviews of the theory and applications can be found in the review articles by Ariman *et al*. [32, 33] and the recent books by Lukaszewicz [34] and Eringen [35]. Rahman and Sattar [36] ana-

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lyzed the magnetohydrodynamics (MHD) convective flow of a micropolar fluid past a continuously moving vertical porous plate in the presence of heat generation/absorption. Eldabe and Ouaf [37] studied the heat and mass transfer past a stretching horizontal surface immersed in an electrically conduction fluid which are solved numerically using Chebyshev finite difference method. Chen [38] considered the problem of MHD flow and heat transfer of an electrically conducting of a non-Newtonian power-law fluid past a stretching sheet in the presence of a transverse magnetic field. Recently, Ishak *et al*. [39] studied the steady 2-D MHD stagnation point flow towards a stretching sheet with variable surface temperature. Motivated by the above-mentioned investigations, the present paper considers the problem of hydromagnetic flow and heat transfer past a stretching vertical sheet immersed in an incompressible micropolar fluid.

Problem formulation

Consider a steady, 2-D mixed convection flow adjacent to a stretching vertical sheet immersed in an incompressible electrically conducting micropolar fluid of temperature T_∞. The stretching velocity $U_w(x)$ and the surface temperature $T_w(x)$ are assumed to vary linearly with the distance x from the leading edge, *i. e.* $U_w(x) = ax$ and $T_w(x) = T_\infty + bx$, where a and b are constants with $a > 0$ and $b \geq 0$. A uniform magnetic field of strength B is assumed to be applied in the positive y-direction normal to the stretching sheet. The magnetic Re is assumed to be small, and thus the induced magnetic field is negligible. The boundary layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\frac{\mu + \kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial N}{\partial y} - \frac{\sigma B^2}{\rho}u + g\beta(T - T_\infty) \tag{2}$$

$$\rho j\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \gamma\frac{\partial^2 N}{\partial y^2} - \kappa\left(2N \frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

subject to the boundary conditions

$$u = U_w(x), \quad v = 0, \quad N = -m\frac{\partial u}{\partial y}, \quad T = T_w(x) \quad \text{at} \quad y = 0,$$

$$u \to 0, \quad N \to 0, \quad T \to T_\infty \quad \text{as} \quad y \to \infty \tag{5}$$

where u and v are the velocity components along the x- and y-axes, respectively. Further, $\mu, \kappa, \rho, \gamma, \alpha, \beta, T, j, N$, and g are the dynamic viscosity, vortex viscosity (or the microrotation viscosity), fluid density, spin gradient viscosity, thermal diffusivity, thermal expansion coefficient, fluid temperature in the boundary layer, microinertia density, microrotation vector, and acceleration due to gravity, respectively. It should be noted that m is the boundary parameter with $0 \leq m \leq 1$ [40]. Further, we follow the work of many recent authors by assuming that $\gamma = (\mu + \kappa/2)j = \mu(1 + K/2)j$ where $j = \upsilon/a$ [40]. This assumption is invoked to allow the field of equations predicts the correct behavior in the limiting case when the microstructure effects become negligible and the total spin N reduces to the angular velocity [41].

The continuity eq. (1) is satisfied by introducing a stream function ψ such that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. The momentum, angular momentum and energy equations can be transformed into the corresponding non-linear ordinary differential equations by the following transformations:

$$\eta = \sqrt{\frac{a}{\upsilon}}y, \quad f(\eta) = \frac{\psi}{\sqrt{\upsilon a x}}, \quad h(\eta) = \frac{N}{a\sqrt{\frac{a}{\upsilon}}x}, \quad \theta(\eta) = \frac{T - T_\infty}{T_\mathrm{w} - T_\infty} \tag{6}$$

The transformed non-linear ordinary differential equations are:

$$(1+K)f''' + ff'' - f'^2 + Kh' - Mf' + \lambda\theta = 0 \tag{7}$$

$$\left(1 + \frac{K}{2}\right)h'' + fh' - f'h - K(2h + f'') = 0 \tag{8}$$

$$\frac{1}{\mathrm{Pr}}\theta'' + f\theta' - f'\theta = 0 \tag{9}$$

subject to the boundary conditions eq. (5) which become:

$$f(0) = 0, \ f'(0) = 1, \ h(0) = -mf''(0), \ \theta(0) = 1, \ f'(\eta) \to 0, \ h(\eta) \to 0, \ \theta(\eta) \to 0 \ \text{ as } \ \eta \to \infty \tag{10}$$

In the above equations, primes denote differentiation with respect to η, $K = \kappa/\mu$ is the material parameter, $\upsilon = \mu/\rho$ – the kinematic viscosity, and $\mathrm{Pr} = \upsilon/\alpha$.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number (Nu_x), which are defined as:

$$C_\mathrm{f} = \frac{2\tau_\mathrm{w}}{\rho U_w^2}, \quad \mathrm{Nu}_x = \frac{xq_w}{k(T_\mathrm{w} - T_\infty)} \tag{11}$$

where the wall shear stress τ_w and the wall heat flux q_w are given by:

$$\tau_\mathrm{w} = \left[(\mu + \kappa)\frac{\partial u}{\partial y} + \kappa N\right]_{y=0}, \quad q_\mathrm{w} = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{12}$$

with k being the thermal conductivity. Using the similarity variables, eq. (6), we obtain:

$$\frac{1}{2}C_\mathrm{f}\sqrt{\mathrm{Re}_\mathrm{x}} = [1 + (1-m)K]f''(0)$$
$$\frac{\mathrm{Nu}_x}{\sqrt{\mathrm{Re}_x}} = -\theta'(0) \tag{13}$$

We notice that when $K = 0$ (viscous fluid) and $\lambda = 0$ (forced convection flow), eq. (7) has an exact solution:

$$f(\eta) = \frac{1}{\sqrt{1+M}}(1 - \mathrm{e}^{-\sqrt{1+M}\eta}) \tag{14}$$

while the solution for the energy eq. (9) is given by:

$$\theta(\eta) = \mathrm{e}^{-\frac{\mathrm{Pr}}{\sqrt{1+M}}\eta}\frac{F(\zeta - 1, \zeta + 1, -\zeta \mathrm{e}^{\sqrt{1+M}\eta})}{F(\zeta - 1, \zeta + 1, -\zeta)} \tag{15}$$

where $\zeta = \mathrm{Pr}/(1 + M)$, and $F(a, b, z)$ denotes the confluent hypergeometric function [42], with:

$$F(a,b,z) = 1 + \sum_{n=1}^{\infty}\frac{a_n}{b_n}\frac{z^n}{n!}, \ a_n = a(a+1)(a+2) \ldots (a+n-1), \ \mathrm{b}_n = b(b+1)(b+2) \ldots (b+n-1)$$

Further, from eqs. (14) and (15), the skin friction coefficient $f''(0)$ and the Nusselt number $-\theta'(0)$ can be shown by:

$$f''(0) = -\sqrt{1+M}$$

$$-\theta'(0) = \frac{\Pr}{\sqrt{1+M}}\left[1 - \frac{\zeta-1}{\zeta+1}\frac{F(\zeta,\zeta+2,-\zeta)}{F(\zeta-1,\zeta+1,-\zeta)}\right] \tag{16}$$

Results and discussion

The non-linear ordinary differential eqs. (7-9) subject to the boundary conditions (10) have been solved numerically using an implicit finite-difference scheme known as the Keller--box method as described in the book by Cebeci and Bradshaw [43], for several values of K, M, m, and λ, while the Pr is fixed to unity. Table 1 shows the values of $-\theta'(0)$ which show a favorable agreement with previously reported data available in the literature.

Table 1. Values of $-\theta'(0)$ for different values of K, M and m when Pr = 1 and λ = 0

K	M	m	Grubka and Bobba [6]	Ali [8]	Eq. (16)	Present work
0	0		1.0000	0.9959655	1.0000000000	1.0000
	1		–	–	0.8921467957	0.8921
1	0	0	–	–	–	1.0745
		0.5	–	–	–	1.0484
		1	–	–	–	1.0029
	1	0	–	–	–	0.9975
		0.5	–	–	–	0.9860
		1	–	–	–	0.8873

Figure 1 presents the variation of the skin friction coefficient $f''(0)$ with λ, while the respective heat transfer rate at the surface $-\theta'(0)$ is depicted in fig. 2. Figures 1 and 2 show that the values of $f''(0)$ and $-\theta'(0)$ increase with λ but decrease with increasing values of M. It is observed that the values of $f''(0)$ are negative for small values of λ, which means that the sheet exerts a drag force on the fluid. For larger values of λ, $f''(0)$ becomes positive due to stronger buoyancy force, and for this case, the formation of the boundary layer not depend solely on the stretching sheet. On the other hand, the values of $-\theta'(0)$ are always positive, *i. e.* the heat is trans-

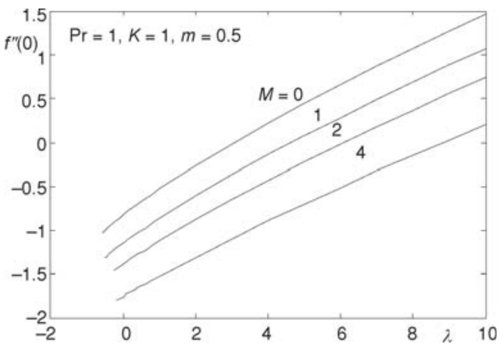

Figure 1. Variation of $f''(0)$ with λ for different values of M when Pr = 1, K = 1, and m = 0.5

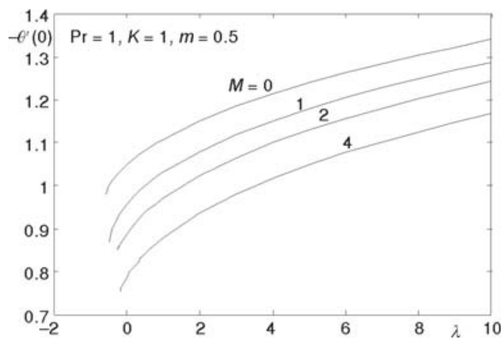

Figure 2. Variation of $-\theta'(0)$ with λ for different values of M when Pr = 1, K = 1, and m = 0.5

ferred from the hot sheet to the cold fluid. Figure 3 shows the variation of the skin friction coefficient $f''(0)$ with M for different values of K when $\lambda = 1$, while the heat transfer rate at the surface $-\theta'(0)$ is depicted in fig. 4. For a particular value of K, the values of $f''(0)$ and $-\theta'(0)$ decrease with an increase in M. This observation is in agreement with the results presented in figs. 1 and 2. Moreover, the effect of the material parameter K is found to increase the values of $f''(0)$ and $-\theta'(0)$.

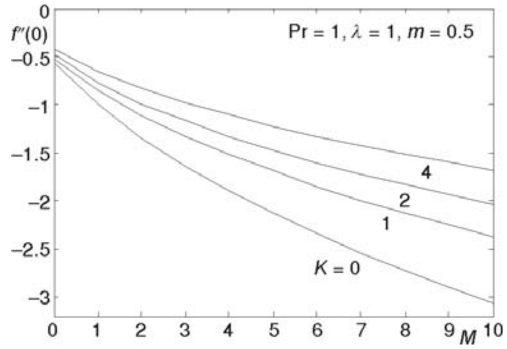

Figure 3. Variation of $f''(0)$ with M for different values of K when Pr = 1, λ = 1, and m = 0.5

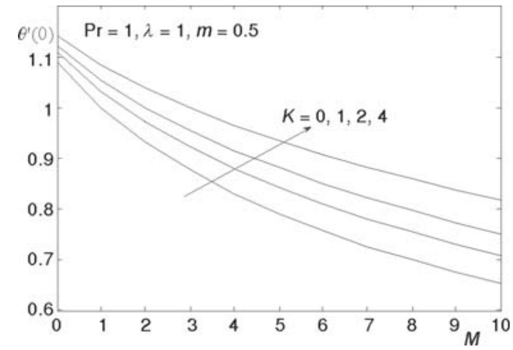

Figure 4. Variation of $-\theta'(0)$ with M for different values of K when Pr = 1, λ = 1, and m = 0.5

The influences of the magnetic parameter M on the velocity and temperature profiles are depicted in figs. 5 and 6, respectively. It can be seen that increasing M is to reduce the velocity distribution in the boundary layer which results in thinning of the boundary layer thickness, and hence induces an increase in the absolute value of the velocity gradient at the surface. The reverse trend is observed for temperature distribution as depicted in fig. 6, where the absolute value of the temperature gradient at the surface decreases with M. Thus, heat transfer rate at the surface decreases with an increase in M. The opposite behaviors are observed for the effect of K as shown in figs. 7 and 8. The sample of microrotation profiles for various values of m, while the other parameters are fixed to unity is presented in fig. 9. As expected, the microrotation effect at the surface $h(0)$ is more dominant for larger values of the boundary parameter m. The velocity, temperature, and microrotation profiles presented in figs. 5-9 show that the far field boundary

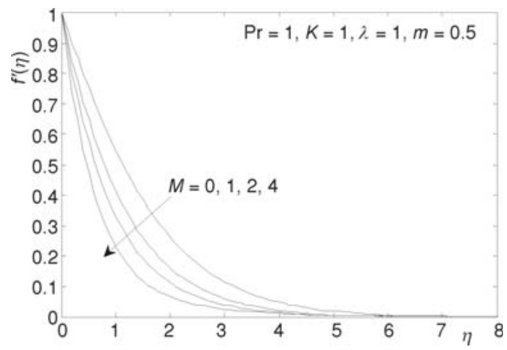

Figure 5. Velocity profiles $f'(\eta)$ for different values of M when Pr = 1, K = 1, λ = 1, and m = 0.5

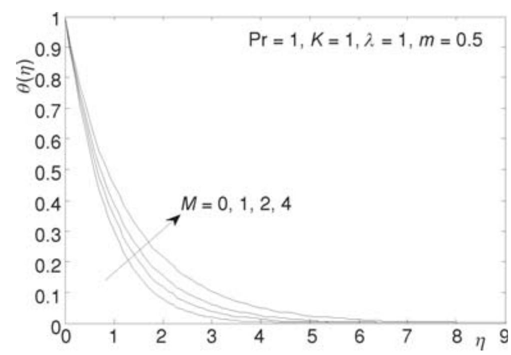

Figure 6. Temperature profiles $\theta(\eta)$ for different values of M when Pr = 1, K = 1, λ = 1, and m = 0.5

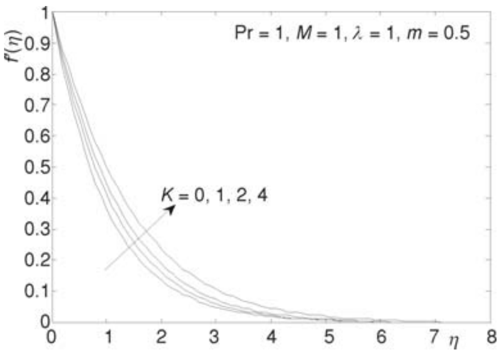

Figure 7. Velocity profiles $f'(\eta)$ for different values of K when Pr = 1, M = 1, λ = 1, and m = 0.5

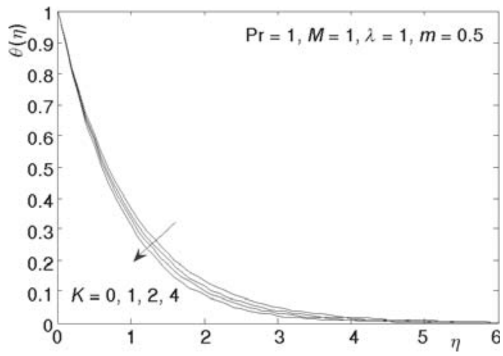

Figure 8. Temperature profiles $\theta(\eta)$ for different values of K when Pr = 1, M = 1, λ = 1, and m = 0.5

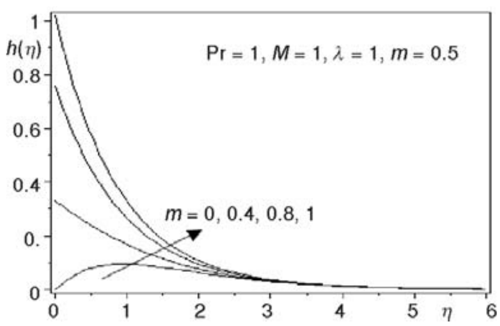

Figure 9. Microrotation profiles $h(\eta)$ for different values of m when Pr = 1, K = 1, M = 1, and λ = 1

conditions are satisfied asymptotically, which support the validity of the numerical results presented in tab. 1 as well as figs. 1-4.

Conclusions

We have theoretically studied the problem of steady 2-D mixed convection flow adjacent to a stretching vertical sheet immersed in an incompressible micropolar fluid. The governing partial differential equations are transformed, using similarity transformation, to a system of non-linear ordinary differential equations, before being solved numerically by the Keller-box method. The effects of the governing parameters on the skin friction coefficient and the heat transfer rate at the surface are discussed. It is found that the skin friction coefficient and the heat transfer rate at the surface decrease in the presence of a magnetic field. Conversely, both of them increase as the material parameter increases.

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Nomenclature

a, b	– constant
B	– uniform magnetic field, [T]
C_f	– skin friction coefficient
F	– confluent hypergeometric function
f	– dimensionless stream function
h	– dimensionless microrotation
Gr_x	– local Grashof numer
g	– acceleration to gravity, [ms^{-1}]
j	– microinertia density, [m^2]
K	– material parameter
k	– thermal conductivity, [Wm^{-1}K^{-1}]
M	– magnetic parameter
m	– boundary parameter
N	– dimensionless angular velocity
Nu_x	– local Nusselt number
Pr	– Prandtl number
q_w	– wall heat flux, [Wm^{-2}]
Re_x	– local Reynolds number
T	– fluid temperature, [K]
$T_w(x)$	– temperature of the stretching sheet, [K]

T_∞ – ambient temperature, [K]
$U_w(x)$ – velocity of the stretching sheet
u, v – velocity components along the x- and y-directions, respectively
x, y – Cartesian co-ordinates along the surface and normal to it, respectively
z – variable

Greek symbols

α – thermal diffusivity, [m^{2}s^{-1}]
β – thermal expansion coefficient, [K^{-1}]
γ – spin-gradient viscosity, [kgms^{-1}]
ζ – constant
η – similarity variable
θ – dimensionless parameter
κ – vortex viscosity, [kgm^{-1}s^{-1}]
λ – buoyancy or mixed convection parameter
μ – dynamic viscosity, [kgm^{-1}s^{-1}]
υ – kinematic viscosity, [m^{2}s^{-1}]
ρ – fluid density, [kgm^{-3}]
σ – electrical conductivity, [Sm^{-1}]
τ_w – shear stress
ψ – stream function

Subscripts

w – condition at the stretching sheet
∞ – condition at infinity

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