NEW MULTI-SOLITON SOLUTIONS FOR GENERALIZED BURGERS-HUXLEY EQUATION

by

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The double exp-function method is used to obtain a two-soliton solution of the generalized Burgers-Huxley equation. The wave has two different velocities and two different frequencies.

Key words: solitary solution, non-linear evolving equation, double exp-function method

Introduction
The Burgers-Huxley equation is encountered in the description of many non-linear wave phenomena \cite{1}. It can be written \cite{1}:

\[ u_t + \alpha u^n u_x - u_{xx} - \beta u(u^n - \gamma)(1 - u^n) = 0 \]  \hspace{1cm} (1)

where $\alpha$, $\beta$, $\gamma$, and $n$ are constants.

This equation can be solved by various analytical methods, such as the variational iteration method \cite{2}, the homotopy perturbation method \cite{3-5}, and the exp-function method \cite{6, 7}. A complete review on various analytical methods is available in \cite{8, 9}. In this paper the double exp-function method \cite{10} is adopted to elucidate the different velocities and different frequencies in the travelling wave.

Double exp-function method
The multiple exp-function method was first proposed in \cite{10}, and the double exp-function method was used to search for double-soliton solutions in \cite{11}. Assume that the solution of eq. (1) can be expressed in the form:

\[ u = \frac{a_1 e^\xi + a_2 e^{-\xi} + a_3 e^\eta + a_4 e^{-\eta}}{k_1 e^\xi + k_2 e^{-\xi} + k_3 e^\eta + k_4 e^{-\eta}} \] \hspace{1cm} (2)

where $\xi = c_1 x + c_2 t$, and $\eta = c_3 x + c_4 t$.

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Substituting eq. (2) into eq. (1) and equating all coefficients of $e^{i(\xi + \eta)}$ to zero, we have a set of algebraic equations. Solving the resulting system with the aid of some mathematical software, we can identify the constants in eq. (2).

**Case 1. One-soliton solution**

$$u(x,t) = \frac{a_5}{k_1 e^{\xi} + a_5 + k_4 e^{-\xi}}$$

(3)

where $k_1, k_4,$ and $a_5$ are free parameters, and:

$$\xi = \frac{\alpha}{4} - \frac{\sqrt{\alpha^2 + 8\beta}}{4} x + c_2 t$$

**Case 2. Two-soliton solution**

$$u(x,t) = \frac{sk_3 k_4 e^{\xi} + k_2 e^{-\xi} + s k_3 e^{\eta} + k_4 e^{-\eta}}{k_1 e^{\xi} + k_2 e^{-\xi} + k_3 e^{\eta} + k_4 e^{-\eta}}$$

(4)

where

$$\xi = \left(\frac{\alpha}{4} - c_3 - \frac{\alpha \gamma}{4} + \frac{\delta}{4}\right) x + (2c_3 - \alpha) \left(\frac{\alpha}{4} - c_3 - \frac{\alpha \gamma}{4} + \frac{\delta}{4}\right) t$$

$$\eta = c_3 x + \left\{-\frac{1}{2} \alpha c_3 - \frac{\beta - 1}{2} \alpha \gamma c_3 + \frac{\beta \gamma^2}{2} + \frac{1}{2} \alpha - 2c_3 - \frac{1}{2} \alpha \gamma \right\} \left(\frac{\alpha}{4} - c_3 - \frac{\alpha \gamma}{4} + \frac{\delta}{4}\right) t$$

$$\delta = 8 \beta \gamma^2 - 16 \beta \gamma + 8 \beta + \alpha^2 \gamma^2 - 2 \alpha^2 \gamma + \alpha^2$$

Alternatively by the following transformation:

$$u = \frac{1}{v}$$

eq. (1) becomes:

$$v v_t + \alpha n^2 v_x + \left(1 - \frac{1}{n}\right) v_x - v v_{xx} + \beta n^2 (v - 1)(v - \gamma) = 0$$

(6)

By the similar solution process as above, we have:

**Case 1. One-soliton solution**

$$u(x,t) = \frac{\gamma k_5}{k_3 e^{\xi} + k_5}$$

(7)
where $k_3$ and $k_5$ are some free parameters, and:

$$
\xi = c_3 x + c_4 t
$$

$$
c_3 = \left[ \frac{\alpha n - \sqrt{(\alpha n)^2 + 4\beta n^2(1+n)}}{1+n} \right]^{1+n}
$$

$$
c_4 = \frac{n\lambda[\beta\gamma + c_3 \alpha - \beta(1+n)]}{1+n}
$$

**Case 2. One-soliton solution**

$$
\eta = c_1 x + c_2 t
$$

$$
c_1 = \left[ \frac{\alpha n - \sqrt{(\alpha n)^2 + 4\beta n^2(1+n)}}{1+n} \right]^{1+n}
$$

$$
c_2 = \frac{-n\lambda[\beta\gamma + c_3 \alpha - \beta(1+n)]}{1+n}
$$

**Case 3. Two-soliton solution**

$$
\eta = c_1 x + c_2 t
$$

$$
c_1 = \frac{-2c_3 (1+n) + \alpha n^2 + \sqrt{\delta}}{2(1+n)}
$$

$$
c_2 = -c_4 + \beta n\gamma \left[ \frac{\alpha n^2 - 2c_3 (1+n) + \sqrt{\delta}}{2(1+n)} \right] + (-\beta\lambda^2 n - \gamma n\alpha c_3)(1+n)^{-1}
$$

$$
\delta = \gamma^2 n^2 \left[ \alpha^2 \gamma^2 n^2 + 4\beta(1+\gamma n) \right]
$$

**Conclusions**

Using the double exp-function method, new two-soliton solutions are obtained for generalized Burgers-Huxley equation. This method can also be applied to solve other types of non-linear evolution equations.

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