NOZZLE DESIGN IN A FIBER SPINNING PROCESS
FOR A MAXIMAL PRESSURE GRADIENT

by

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The thickness of a spinneret is always a geometrical constraint in nozzle design. The geometrical form of a nozzle has a significant effect on the subsequent spinning characteristics. This paper gives an optimal condition for maximal pressure gradient through the nozzle.

Key words: nozzle, spinneret, analytic solution, optimal design

Introduction

The nozzle is one of the most important parts of a spinneret in various fiber spinning processes, its form will greatly affect the morphology of its productions and output. Figure 1 shows a widely used spinneret and its nozzle structure.

The top size of the nozzle section is determined by the number of nozzle in a spinneret, while its low size and its geometrical form are determined by fiber requirements. The thickness of a spinneret is a main geometrical constraint in many practical applications. This paper is to optimally design a nozzle with maximal pressure drop in the nozzle.

Theory

Assuming that the flow through a nozzle follows the Darcy law, that is:

\[ u = \kappa \nabla p \]  

(1)

where \( \kappa \) is the constant, \( u \) – the flow speed, and \( \nabla p \) – the pressure gradient through a nozzle.

In order to improve its output, a high spinning velocity is predicted, that means a higher pressure gradient in a nozzle is an appropriate choice in the design of a nozzle.

For a cone nozzle, the velocity distribution on its section can be expressed as:

\[ u = k(R^2 - r^2) \]  

(2)

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where \( k \) is the constant, and \( R \) – the inner radius of the nozzle.

The constant \( k \) in eq. (2) can be determined by the mass conservation law, which requires:

\[
Q = \frac{2\pi k \rho r(R^2 - r^2)}{2} = \frac{1}{2} \pi R^4 \rho k
\]

where \( Q \) is the flow rate, and \( \rho \) – the density of the flow. From eq. (3), we have:

\[
k = \frac{2Q}{\pi R^4 \rho}
\]

The constant \( k \) in eq. (2) can be determined by the mass conservation law, which requires:

\[
Q = \int_0^R 2\pi k \rho r(R^2 - r^2)dr = \frac{1}{2} \pi R^4 \rho k
\]

The velocity in a nozzle can be obtained, which reads:

\[
u = \frac{2Q}{\pi R^4 \rho} (R^2 - r^2)
\]

Due to geometrical constraint of the thickness of a spinneret, complex nozzles appear in many applications. Assume that the radii of the top and low sections of the nozzle are \( r_1 \) and \( r_3 \), respectively, and its thickness is \( h \) as illustrated in fig. 2.

The velocity distribution in each section of the nozzle can be determined by eq. (5). Assume that the flow in the nozzle is viscous and incompressible; the flow is laminar and there is no acceleration of fluid in the nozzle, the momentum equation becomes:

\[
\frac{1}{\rho} \frac{d^2u}{dz^2} = \mu \frac{d^2u}{dz^2}
\]

where \( \mu \) is the viscosity coefficient.

In practical applications, the nozzle height is thin (e. g. 1 mm), so the second derivative of the velocity can be approximately expressed:

\[
\frac{d^2u}{dz^2} \approx \frac{h_1(u_3 - u_2) - h_2(u_2 - u_1)}{h_1h_2^2}
\]

Equation (6) becomes:

\[
\Delta p = \rho h \mu \frac{h_1(u_3 - u_2) - h_2(u_2 - u_1)}{h_1h_2^2} = \rho h \mu \frac{(h - h_2)(u_3 - u_2) - h_2(u_2 - u_1)}{(h - h_2)h_2^2}
\]

The pressure drop at \( r = 0 \) is:

\[
\Delta p(r = 0) = \rho h \mu \frac{(h - h_2)(u_3 - u_2) - h_2(u_2 - u_1)}{(h - h_2)h_2^2} = \rho h \mu \frac{h(u_3 - u_2) - h_2(u_3 - u_1)}{(h - h_2)h_2^2}
\]

where \( u_1, u_2, \) and \( u_3 \) are maximal flow speed at the top, middle, and low sections of the nozzle, respectively.
In practical applications, \( r_1, r_3, \) and \( h \) are constants, and \( r_2 \) and \( h_2 \) should be such determined that its pressure drop at \( r = 0 \) through the nozzle is maximal, that requires:

\[
\frac{d\Delta p}{dh_2}(r = 0) = \rho h \mu \left( \frac{-(h - h_2)h_2^3(\bar{u}_3 - \bar{u}_1) - (2hh_2 - 3h_2^2)[h(\bar{u}_3 - \bar{u}_2) - h_2(\bar{u}_3 - \bar{u}_1)]}{(h - h_2)h_2^{-7}} \right) = 0 \tag{13}
\]

From eq. (13), \( \bar{u}_2 \) can be determined, which reads:

\[
\bar{u}_2 = \frac{(h - h_2)h_2^3(\bar{u}_3 - \bar{u}_1) + (2hh_2 - 3h_2^2)[h(\bar{u}_3 - h_2(\bar{u}_3 - \bar{u}_1)]}{hh_2(2h - 3h_2)} \tag{14}
\]

According to eq. (12), for a fixed \( h_2 \), its nozzle section can be determined:

\[
r_2 = \sqrt{\frac{2Q}{\pi \rho \bar{u}_2}} = \frac{2Q}{\pi ^2 \rho (h - h_2)h_2^3(\bar{u}_3 - \bar{u}_1) + (2hh_2 - 3h_2^2)[h(\bar{u}_3 - h_2(\bar{u}_3 - \bar{u}_1)]} \tag{15}
\]

Equation (15) can be used for practical design of a nozzle.

Conclusions

In this paper, we adopt approximately a difference definition for the second derivative of the velocity in the derivation, and obtain a formula, eq. (15), for determining the radius of the middle section of a nozzle for a maximal pressure drop in the nozzle.

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