HEAT TRANSFER ENHANCEMENT WITH ELLIPTICAL TUBE UNDER TURBULENT FLOW TiO$_2$-WATER NANOFUID

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Heat transfer and friction characteristics were numerically investigated, employing elliptical tube to increase the heat transfer rate with a minimum increase of pressure drop. The flow rate of the tube was in a range of Reynolds number between 10000 and 100000. FLUENT software is used to solve the governing equation of CFD (continuity, momentum and energy) by means of a finite volume method (FVM). The electrical heater is connected around the elliptical tube to apply uniform heat flux (3000 W/m$^2$) as a boundary condition. Four different volume concentrations in the range of 0.25% to 1% and different TiO$_2$ nanoparticle diameters in the range of 27 nm to 50 nm, dispersed in water are utilized. The CFD numerical results indicate that the elliptical tube can enhance heat transfer and friction factor by approximately 9% and 6% than the circular tube respectively. The results show that the Nusselt number and friction factor increase with decreasing diameters but increasing volume concentrations of nanoparticles.

Keywords: nanofluid, CFD, elliptical tube, friction factor, FLUENT

1. Introduction

Very small particles suspension in saturated liquids (water, ethylene glycol, engine oil) is defined as nanofluids may constitute a very interesting alternative for advanced thermal applications. It has been found that important heat transfer enhancement may be achieved while using nanofluids compared to the use of conventional fluids; furthermore, some oxide nanoparticles exhibit an excellent dispersion properties in traditional cooling liquids [1].

Turbulent forced convection heat transfer of water and nanofluid inside tube under single phase approach was carried out by [2-6]. Pak and Cho [7] were investigated the effect of two different metallic oxide particles, titanium dioxide and alumina, with mean diameters of 27 and 13 nm, respectively, on base fluid (water) experimentally; Results showed that significant heat transfer enhancement with nanofluids as compared with the base fluid. A theoretical model proposed by Sharma et al [8] to predict friction and heat transfer coefficients for different nanofluids containing Cu, CuO, TiO$_2$, SiC, ZrO$_2$, and Al$_2$O$_3$ nanoparticles of different sizes, concentration and temperatures dispersed in water; results showed deviation of theoretical with experimental data. CFD modeling of heat transfer enhancement of Al$_2$O$_3$ nanofluid using low volume fraction under turbulent pipe flow with constant wall temperature has been studied by Kumar [9]; Nusselt number (Nu) and friction factor have been predicted for the low volume fractions (i.e. 0.02%, 0.1 and 0.5%). Numerical study of turbulent forced convection flow of alumina nanofluid through circular tube was subjected by Bianco [10]; results showed that Heat transfer increased with the particle volume concentration and Reynolds number. Experimental study of turbulent forced convection heat transfer through pipe employing twisted tape with and without Al$_2$O$_3$–water nanofluid
was conducted by S. Sundar and Sharma [11], generalized correlation has been developed for the estimation of Nusselt number and friction factor in a pipe with and without inserts.

Wide variety of practical and industrial applications of forced convection in an elliptical tube between two horizontal cylinders such as heat exchangers, heating processes, power generation, chemical processes, microelectronics and cooling processes have been led to the interest this type of studies[12-16].

Duangthongsuk and Wongwises [17] observed the heat transfer coefficient and friction factor of the nanofluid TiO$_2$-water flowing in a horizontal double tube counter flow heat exchanger under turbulent flow conditions, experimentally and showed that the heat transfer coefficient of nanofluid is higher than that of the base liquid. Forced convection turbulent flow of Al$_2$O$_3$-water nanofluid inside an annular tube with variable wall temperature has been investigated experimentally by Prajapati [18], results showed the enhancement of heat transfer due to the nanoparticle presence in the fluid. The forced convection flow between two corrugated cylinders has been studied by Kittur [19]; results found friction factor and heat transfer on the boundaries. Horizontal double-tube heat exchanger counter turbulent flow was studied numerically by Bozorgan [20]. Al$_2$O$_3$ – water nanofluid with particle size of 7 nm with volume concentrations up to 2% are selected as a coolant used, results showed that the pressure drop of nanofluid is slightly higher than water and increases with the increase of volume concentrations. Forced convection flow of nanofluids of TiO$_2$-water in a double-tube counter flow heat exchanger using CFD simulation FLUENT software has been investigated by Demir [21]. A double tube coaxial heat exchanger heated by solar energy using Aluminum oxide nanofluid presented experimentally and numerically by Luciu [22], results showed that nanofluids have a higher performance of heat transfer than the base fluid.

Practically, in compact heat exchangers may be necessary using non-circular flow-duct, to increase the heat transfer with a minimum increase of pressure drop Heris [23]. The non-circular flow duct has been used in the study undertaken to achieve to this application with minimum pressure drop.

In this work, CFD simulation by FLUENT software is used to predict friction factor and Nusselt number, with turbulent forced convection of TiO$_2$-water nanofluid through elliptical tube under constant heat flux, then compared results of CFD with experimental data available in the literature.

2. Theoretical Analysis

2.1 Physical model

cylindrical geometry of problem undertaken has been shown in fig. 1, Dimensions of elliptical tube are major and minor diameter (a=27mm and b=9mm), the length (L) and hydraulic diameter (D$_h$) of the elliptical tube are 1000 mm and 24mm. Reynolds number has been calculated regarded hydraulic diameter (D$_h$) as:

\[ D_h = 4 \times \text{Area} / \text{Perimeter} \]  \hspace{1cm} (1)

Where: \( D_h \) = hydraulic diameter of elliptical tube, \( \text{Area} = \frac{\pi \times a \times b}{4} \) \) and \( \text{Perimeter} = \pi \sqrt{\frac{a^2 + b^2}{2}} \)

Reynolds number (Re) is:

\[ Re = \frac{\rho_{nf} \times D_h \times u}{\mu_{nf}} \]  \hspace{1cm} (2)

The problem under taken is assumed to be two dimensional, steady, incompressible and Newtonian turbulent fluid flow, constant thermo physical properties of nanofluid, no effect of gravity and heat conduction in the axial direction and wall thickness of tubes are neglected.
2.2 Governing equations

Infinitesimal (less than 100 nm) solid particles are considered to be able using single phase approach, so single phase approach is adopted for nanofluid modeling. The thermal properties of nanofluid is estimated by equations below Sharma et al [8]

\[ \rho_{nf} = \left( \frac{\phi}{100} \right) \rho_p + \left( 1 - \frac{\phi}{100} \right) \rho_f \]  \hspace{1cm} (3)

\[ C_{nf} = \frac{\phi}{100} (\rho C)_p + \left( 1 - \frac{\phi}{100} \right) (\rho C)_f \]  \hspace{1cm} (4)

\[ k_r = \frac{k_{nf}}{k_f} = \left( 0.8938 \left( 1 + \frac{\phi}{100} \right)^{1.37} \left( 1 + \frac{T_{nf}}{70} \right)^{0.2777} \left( 1 + \frac{d_p}{150} \right)^{-0.036} \left( 0.01737 \alpha \right)^{0.061} \right) \]  \hspace{1cm} (5)

\[ \mu_r = \frac{\mu_{nf}}{\mu_f} = \left( 1 + \frac{\phi}{100} \right)^{11.3} \left( 1 + \frac{T_{nf}}{70} \right)^{0.038} \left( 1 + \frac{d_p}{170} \right)^{-0.061} \]  \hspace{1cm} (6)

Where \( \rho \), \( C \), \( k \) and \( \mu \) are density, specific heat capacity, thermal conductivity and viscosity respectively, but subscripts \( nf \), \( f \), and \( p \) are represented nanofluid, fluid and solid properties. On the other hand, thermal properties of solid particles are: \( k_p = 8.4 \) W/m °C, \( \rho_p = 4175 \) kg/m³, \( C_p = 692 \) J/kg K Sharma et al [8].

Equations (3-6) are valid for TiO₂ nanoparticles suspended in water with base temperature 25°C and size diameter is (20-50) nm. For all these assumptions the dimensional conservation equations for steady state mean conditions are as follows: continuity, momentum and energy equations: Bejan [24]

\[ \frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho_{nf} u \right) = 0 \]  \hspace{1cm} (7)

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} \left( \rho_{nf} u \right) = - \frac{\partial P}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \nu + \epsilon u \right) \frac{\partial u}{\partial r} \right) \]  \hspace{1cm} (8)

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho u T \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \alpha + \epsilon u \right) \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial x} \left( k_{nf} \frac{\partial T}{\partial x} \right) \]  \hspace{1cm} (9)

High Reynolds number was taken as input parameter, pressure treatment was adopted SIMPLE scheme and turbulent viscous k-ε model has been employed, converged solutions were considered for residuals lower than \( 10^{-6} \) for all the governing equations. The results of simulation for nanofluid were compared with Blasius equation (10) for friction factor and Dittus-Boelter equation (11) for \( Nu \) as:

\[ f = \frac{0.316}{Re^{0.25}} \]  \hspace{1cm} (10)

\[ Nu = \frac{h_f}{k_f} D_{eff} = 0.023 Re^{0.8} Pr^{0.4} \]  \hspace{1cm} (11)

2.3 Boundary conditions

Volume concentration nanofluids (0.25, 0.5, 0.75 and 1%) at 25°C base temperature were used for TiO₂-water as input fluids. For comparison purposes, water was also employed as working fluid. CFD studies were carried out with uniform velocity profile at the inlet and pressure outlet condition has been used at the outlet of elliptical tube. Turbulent intensity (\( I \)) was specified for an initial guess of turbulent quantities (\( k \) and \( \epsilon \)). The turbulent intensity was estimated for each case based on the formula
\[ I = 0.16Re^{-1/8} \]. The walls of the tube were assumed to be perfectly smooth. The constant heat flux condition has been specified on the inside tube wall with a value of 3000 W/m². Reynolds number was varied from \(10^4\) to \(10^5\) at each step of iterations as input data. The friction factor and \(Nu\) were introduced as output data.

2.4 Grid independence test

Grids independence have been chosen in GAMBIT software for elliptical tube narrow as 50000 cells and 1000x50, subdivisions in the axial, and radial directions, respectively. To find the most suitable size of mesh faces, grid independent test was performed for the physical model. In this study, rectangular cells were used to mesh the surfaces of tube wall but triangular cells were used to mesh the surfaces of gap as shown in Fig. 2. The grid independence has been checked by using different grid systems and four mesh faces considered 50000, 40000, 30000 and 20000 for pure water. Friction factor and Nusselt number was determined for all four mesh faces and results were proper. However, any number of mesh faces for these four cases can be used, but in this study, mesh faces with 50000 has been adopted as the best in terms of the accuracy as shown in Fig. 3 and Fig. 4.

3. CFD simulation

CFD simulations were used FLUENT software with solver strategy. To analyze problems Gambit software has been used. To make possible numerical solution of governing equations, single phase conservation equations were solved by control volume approach then converting them to a set of algebraic equations. Simulation results were tested by comparing the predicted results of Pak and Cho [7] who used circular heated tube in experimental work. FLUENT software was used in CFD analysis in the literature and a detailed description of the mathematical model can be found in the Fluent User's Guide [25]. CFD modeling region could be classified into few major step: preprocess stage, the geometry of problem undertaken was constructed as elliptical narrow and computational mesh was generated in GAMBIT. It followed by the physical model, boundary conditions and other parameters appropriate were defined in models setup and solving stage. All scalar values and velocity components of the problem are calculated at the center of control volume interfaces where the grid schemes are used intensively. Throughout the iterative process accurately monitor of the residuals has been done. When the residuals for all governing equations were lower than \(10^{-6}\), all solutions were assumed to be converged.

Finally the results could be obtained when FLUENT iterations have been led to converged results defined by a set of converged criteria. The friction factor and Nusselt number inside the elliptical tube could be obtained throughout the computational domain in the post-process stage.

4. Results and Discussion

4.1 Velocity and Temperature

Velocity and temperature have been represented as contour in Fig. 5. It shows that the difference between minimum and the maximum values of temperature is depended on velocity inlet, as the maximum value of velocity 0.415 m/s and 3.95 m/s have led to temperature deference 6k and 1k respectively. it seems that same behavior of velocity and temperature with different values depending on Reynolds number.

4.2 The effect of nanofluid volume concentrations

This section includes the effects of different nanoparticle concentrations of TiO₂ nanofluid on the friction factor and Nusselt number. The effect of different volume concentration on friction factor is
shown in Fig. 6, at the range of (0 – 1%) nanoparticle volume concentration. The results are revealed that TiO$_2$ nanofluid with highest concentration of volume 1% has the highest friction factor at all Reynolds numbers. The reason of increasing of friction factor with the increasing of concentration of volume of nanofluid is the increasing of fluid viscosity which reduced the moving of fluid; also validation of friction factor results from simulation for nanofluid with Blasius equation for water is studied. On the other hand, Fig. 7 shows that Nusselt number is increasing with increasing in volume concentrations of nanofluid because of the high thermal properties of TiO$_2$ which enhancement of heat transfer and hydrodynamic flow. The results are revealed that TiO$_2$ nanofluid with highest concentration of volume 1% has the highest Nusselt number at all Reynolds numbers. The irregular and random movements of particles increase the energy exchange rates in the fluid with penalty on the wall shear stress and consequently enhance the thermal dispersion of the flow. It is also found that the friction factor is decreased while Nusselt number is increased with the increase of Reynolds number. Also validation of Nusselt number results from simulation for nanofluid with Dittus-Boelter equation for water is showed; similarly, increasing in volume concentrations of nanofluid was led to increase in Nusselt number and heat transfer enhancement.

### 4.3 The effect of nanofluid size diameter

The effect of nanoparticle size diameters of TiO$_2$-water nanofluid on the friction factor and Nusselt number with different Reynolds numbers was examined in this section. The ranges of nanoparticle diameter are varied from 27 nm to 50 nm with 1% concentration of TiO$_2$ nanofluid. Fig. 8 shows the effect of nanoparticle size diameters of TiO$_2$ on the friction factor with different Reynolds numbers. It appears that the friction factor has minor change when nanoparticle diameters of TiO$_2$ nanofluid are varied. As shown in this figure the nanofluid with 27 nm nanoparticle size diameter has the highest friction factor, whereas, the nanoparticle with a diameter of 50 nm has the lowest friction factor. The 27nm size TiO$_2$ particles give rise high friction factor over the same particles of 50nm size related to two reasons which are the high specific surface area of the nanoparticles and the Brownian motion. As the particle size reduces, the surface area per unit volume increases, the heat transfer is being dependent on the surface area, and thus the effectiveness of nanoparticles in transferring heat to the base liquid increases. However, reducing the particle size means increasing the Brownian motion velocity, which again adds up to the contribution by the nanoparticles to the total heat transfer by continuously creating additional paths for the heat flow in the fluid.

Fig. 9 revealed that Nusselt number has lower value with higher particle size diameter of TiO$_2$ nanofluid. May be the high specific surface area of the nanoparticle is the reason of the effect of size diameter; the heat transfer is being dependent on the surface area. Also Fig. 8 and Fig. 9 are concluded that comparisons between CFD analysis and experimental data of Pak and Cho [7] for both the friction factor and Nusselt number of size diameter 27nm of TiO$_2$ nanofluid and there is good agreement.

### 4.4 Comparison CFD results with experimental data

This section analyzes the comparison between CFD data and experimental data for TiO$_2$-water nanofluid with particle size is 27nm and base temperature is 25°C. Fig. 10 shows the CFD analysis of friction factor versus Reynolds number compared with Pak and Cho [7] experimental data. There is same behavior and good agreement with deviation 2%. On the other hand, Nusselt number versus Reynolds number compared with Pak and Cho [7] experimental data as shown in Fig. 11. There is good agreement with deviation 1.4%. Results show that elliptical tube has enhancement of heat transfer and hydrodynamic flow more than circular tube.
5. Conclusions

In this paper, forced convection heat transfer inside elliptical tube under turbulent flow by numerical simulation with uniform heat flux boundary condition around elliptical tube has been studied. The heat transfer enhancement resulting from various parameters such as nanoparticle concentration of volume, and Reynolds number is reported. The finite volume methods have been used to solve the governing equations with certain assumptions and appropriate boundary conditions. The Nusselt number, friction factor were obtained through the numerical simulation.

The study is concluded as follows:
• The enhancement of friction factor and Nusselt number are 4% and 9% for enhanced tube than that of the circular tube at all Reynolds numbers.
• The concentration of volume (1%) of TiO$_2$ nanofluid has the highest Nusselt number and friction factor values, followed by (0.75, 0.5 and 0.25%) finally pure water has the lowest values of them.
• There is good agreement between simulation results and experimental data of Pak and Cho [7] with deviation 2% and 1.4% for Nu and friction factor respectively.

References


Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$C$</td>
<td>specific heat capacity [W/ kg.°C]</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter [m]</td>
</tr>
<tr>
<td>$E$</td>
<td>energy [W]</td>
</tr>
<tr>
<td>$f$</td>
<td>friction factor</td>
</tr>
<tr>
<td>$htc$</td>
<td>convection heat transfer coefficient [W/m$^2$.°C]</td>
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<tr>
<td>$k$</td>
<td>thermal conductivity [W/m.°C]</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt Number [$htc .D/K_{nf}$]</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure [N/m$^2$]</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtle Number [$C.\mu/K_{nf}$]</td>
</tr>
<tr>
<td>$Re$</td>
<td>Renolds Number [$\rho_{nf}D_h u / K_{nf}$]</td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity [m/s]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Viscosity [N.s /m$^2$]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density [kg/m$^3$]</td>
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<tr>
<td>$\tau$</td>
<td>Shear stress [N/m$^2$]</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Volume concentration</td>
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Subscripts
liquid phases
solid particle
nanofluid
hydraulic

Acknowledgment
The financial support to the authors by University Malaysia Pahang is gratefully acknowledged.

Fig 1. Schematic of problem undertaken
Fig 2. Mesh generated by GAMBIT

![Friction Factor vs Reynolds Number]

Fig 3. Optimum mesh grid size for friction factor with Reynolds Number at 25°C

![Nusselt Number vs Reynolds Number]
Fig 4. Optimum mesh grid size of Nusselt Number with Reynolds Number at 25°C

A-velocity and temperature contours at Re=10000

B-velocity and temperature contours at Re=100000

Fig 5. Contour of velocity and temperature

Fig 6. Effect of volume concentration of nanofluid on friction factor
**Fig 7.** Effect of nanofluid volume concentration on heat transfer enhancement

**Fig 8.** Effect of particle size diameter on friction factor with Reynolds number
Fig 9. Effect of particles size diameter on Nusselt number with Reynolds number

Fig 10. Comparison CFD results of friction factor with Pak & Cho experimental data [7]
Fig 11. Comparison CFD results of $Nu$ with Pak & Cho experimental data [7]