A STUDY OF SENSING HEAT FLOW THROUGH THERMAL WALLS BY USING THERMOELECTRIC MODULE

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Demands on heat flow detection at a plane wall via a thermoelectric module have drawn researchers’ attention to quantitative understanding in order to properly implement the thermoelectric module in thermal engineering practices. Basic mathematical models of both heat transfer through a plane wall and thermoelectric effects are numerically solved to represent genuine behaviors of heat flow detection by mounting a thermoelectric module at a plane wall. The heat transfer through the plane wall is expected to be detected. It is intriguing from simulation results that the heat rejected at the plane wall is identical to the heat absorbed by the thermoelectric module when the area of the plane wall is the same as that of the thermoelectric module. Furthermore, both the area sizes of the plane walls and the convective heat transfer coefficients at the wall influence amount of the heat absorbed by the thermoelectric module. Those observational data are modeled for development of sensing heat flow through a plane wall by a thermoelectric module in practical uses.

Key words: heat flow detection, thermoelectric module, heat transfer, thermal plane wall

1. Introduction

There are a lot of demands on sensing heat flow in many advanced thermal engineering applications, such as green buildings, oven/furnace monitoring, solar-radiation detecting, etc. Recently, researchers have developed new techniques that lead to efficient heat flow detection. For example, a micro heat flux sensor developed by [1] is used to measure a small amount of heat flux where it has quite high sensitivity for a micro-machined heat flux sensor produced in microelectromechanical systems [2,3]. In another motivating research of [4], thermoelectric modules are studied for practical implementation on heat flow detection even though they are widely used as cooling/heating devices in various thermal engineering applications [5-7]. For heat flow detection, they are regarded as semiconductor-based electronic devices that principally function as inherent interactions between heat transfer through the thermoelectric module from one side to the other (Fourier’s law) and electric voltage generation induced by temperature difference across the thermoelectric module (Seebeck effect). The amount of the heat flow is determined by the magnitude
of the electric voltage while the direction of the heat flow is indicated by the electric polarity when the terminal voltage of the open thermoelectric circuit is measured.

As mentioned earlier, the experimental studies in the research of [4] are performed to show the viability of the heat flow detection by using the thermoelectric modules. In fact, the heat flow is detected by observing the electric voltage of the thermoelectric module. This principle can be applied properly in the case that the heat to be detected is the same as the heat directly flowing through the thermoelectric module. For instance, the thermoelectric module is used to detect solar radiation within an outdoor air environment while a side of the surface area of the thermoelectric module directly faces the sunlight. In this case, such thermal radiation on the facing side is conducted through the thermoelectric module. On the other hand, the amount of the heat flow might be altered from that of the detected source once the detected media is attached to the thermoelectric module. A common thermal engineering application is to determine the heat rejected from or absorbed by thermal plane walls. There is still no analytical study of this proposed detection in open literature. To obtain quantitative understanding, the solutions of various scenario studies in this paper are numerically determined by applying ANSYS Mechanical™ simulation software [8]. The numerical parameters of a real built-in heat sink thermoelectric module are used in mathematical models for reflecting genuine representation of the heat flow detection.

This paper is organized as follows. The introductory motivation of this study is given in section 1. Materials and their parameters used in this work are discussed in section 2. In section 3, mathematical models of the built-in heat sink thermoelectric module are developed as overall governing principles of the heat flow detection. Simulated results under various conditions are reported in section 4. The conclusion is stated in section 5.

2. Experimental setup

A schematic diagram of a commercial 4×4 cm² thermoelectric module as shown in Fig. 1 consists of elements of n-type and p-type doped semiconductor material, which is mounted between two ceramic substrates made from aluminum oxide of 0.9 mm thickness. The physical properties of the thermoelectric module [4] are: the thermal conductivity of 0.831 W K⁻¹ and the Seebeck coefficient of 0.0488 V K⁻¹. For usage, the thermoelectric module is coupled with an aluminum heat sink. The scenarios considered in this study are varied within air-cooling environment. The overall convective heat transfer coefficient between the heat sink and the cold air as discussed in appendix is to be derived from the experiment setup as illustrated in Fig. 2. A 4×4×0.1 cm³ copper plate, covered by insulator, is attached to the aluminum heat sink. A J-type thermocouple is mounted on the copper plate for temperature measurement. The copper plate is first heated up at a certain temperature and then it is freely cooled down via the heat sink in an atmospheric environment. The data of decreasing temperature against time is recorded until the temperature reaches the air temperature.

![Fig. 1 Architecture of thermoelectric modules](image)
3. Analysis of heat flow detection via thermoelectric module

In this section, mathematical models are developed in order to quantitatively understand how the thermoelectric module detects heat flow through a thermal wall to which it is attached. Fig. 3 shows a schematic diagram of a typical heat transfer through a single wall exposed to the cold fluid. For brief interpretation, the following consideration can be applied to other cases of heating conditions by reversing the direction of heat flow. Without loss of generality, it is assumed that a given amount of heat is inputted to one side of the wall. Afterward, it is rejected at the other side of the wall due to cooling from the cold fluid. According to heat-balancing mechanism, the temperature distribution takes place within the plane wall.

The objective is to detect the amount of the heat flowing through the thermal wall to the cold fluid. In many thermal engineering applications, this amount of the heat is required to be known for resolving particular problems as mentioned earlier. In this study, Fig. 4 shows a research attempt of using the thermoelectric module to detect the heat flow from the wall surface. In practical use, the thermoelectric module is properly attached to the wall surface. With this implementation, it can be seen that the heat is not rejected to the cold fluid; however, it flows through the thermoelectric module and it is rejected at a heat sink to the cold fluid instead. In turn, the amount of heat flowing through the thermal wall with the original convection boundary is not the same as that in the case of the
attachment with the thermoelectric module to the wall surface. The explanation is simply due to the dissimilarity of boundary conditions at the cooling surface. In this section, the phenomena of heat transfer through the thermal wall with/without the thermoelectric module are technically investigated in order to determine those quantitative relations of heat flow detection in various physical conditions.

Fig. 4 Heat flow detection on wall via thermoelectric module

3.1. A thermoelectric module

Fig. 5 shows the thermoelectric effect of a thermoelectric module during heat detection. According to Fourier’s law, the rate of the heat absorbed by the thermoelectric module at the hot side can be determined in terms of temperature difference between the temperature at the hot side and the temperature at the cold side [4]:

\[ Q = k_T (T_h - T_c) \]  \hspace{1cm} (1)

where \( k_T \) is the thermal conductivity of the thermoelectric module, \( T_h \) is the temperature at the hot side, and \( T_c \) is the temperature at the cold side.

Fig. 5 Schematic diagram of heat flow: a) without heat flow detection and b) with heat flow detection via thermoelectric module
The heat directly transfers from the hot side to the cold side because the thickness of the thermoelectric module is relatively small compared to the size of the surface of the thermoelectric module. Additionally, the heat sink is designed to be well-conducted so as to reject the heat to the cold fluid efficiently. Therefore, the amount of heat transfer from the hot side to the cold side is dependent on the temperature difference and the thermal conductivity of the thermoelectric module. Consequently, it is known as the Seebeck effect; an electric voltage across the thermoelectric module is generated as:

\[ v = \beta(T_h - T_c) \]  

(2)

where \( \beta \) is the Seebeck coefficient.

By manipulating Eqs. (1)-(2), the rate of heat absorbed by the thermoelectric module can be determined from the amount of the electric voltage as shown in Eq. (3).

\[ Q = \frac{k_w}{\beta}v \]  

(3)

The more heat that is absorbed, the higher the electric voltage that is measured. It should be remarked that the direction of heat transfer can be observed by the polarity of the electric voltage. The thermal conductivity and the Seebeck coefficient of the thermoelectric modules can be derived from experiments of parametric determination in few steps [5]. With this working principle, the thermoelectric module can be used to detect the heat transfer through the thermal wall in order to determine both the amount and the direction of the heat flow while the thermoelectric module is attached to the wall at desired location as shown in Fig. 4.

3.2. A plane wall

To investigate the amount of the heat transfer through the thermal wall, the homogenous plane wall is applied as a system in one direction of the heat flow at the plane wall. In turn, four edges of the wall are proposed to be insulated. The governing differential equations of heat transfer within the plane wall can be written from Eq. (4) as:

\[ \rho c \frac{dT}{dt} = k_w \left( \frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} \right) \]  

(4)

where \( T \) is the temperature distributed within the plane wall, \( t \) is the time, \( \rho \) is the density of the wall material, \( c \) is the specific heat of the wall material, and \( k_w \) is the thermal conductivity of the wall material.

The heat flow outward at the wall surface under steady conditions is considered in this study. The transient temperature distribution within the plane wall has no effects on heat flow detection. In turn, the differential term with respect to time on the left side of Eq. (4) is omitted. It should be remarked that a typical heat transfer in a plane wall can be considered in one-dimensional heat conduction due to the temperature difference between two sides of the plane wall. However, in next discussion, the heat conduction transfer near the wall surface where the thermoelectric module
attaches to the plane wall is to be the case of two-dimensional heat flow. The amount and direction of heat flow can be determined once the boundary conditions are specified. On the right side of the plane wall in Fig. 5, the heat flow through the wall is required to be determined. It is considered the heat \( Q^* \) rejected at the plane wall. A common situation in heat transfer is to have fluid convection occurring at this boundary. The heat conducted to the surface of the plane wall is convected away by the cold fluid. This boundary condition is mathematically written as:

\[ -k_w \frac{dT}{dx} \bigg|_{x=L} = h_j (T_{x=L} - T_f) \]  \hspace{1cm} (5)

where \( h_j \) is the convective heat transfer coefficient between the wall surface and the cold fluid, and \( T_f \) is the temperature of the cold fluid.

On the other hand, the thermoelectric module is attached to the plane wall for heat flow detection instead. The boundary condition is determined by:

\[ T_{x=L} = T_h \]  \hspace{1cm} (6)

The boundary condition in Eq. 6 is based on the fact that thermoelectric module is tightly mounted on the wall surface in practical use. However, without loss of generality, the conduction heat transfer, that takes place across thermal resistance due to contact of two surfaces in reality, is not considered in numerical analysis under steady conditions.

On the left side of the plane wall, the heat input \( Q' \) is subjectively set in order to result in the temperature distribution within the plane wall. The mathematical expression of the boundary condition is given as:

\[ -k_w \frac{dT}{dx} \bigg|_{x=0} = Q' \]  \hspace{1cm} (7)

Accordingly, the expected heat \( Q'' \) is generated for various detecting scenarios. The comparisons of the heat \( Q'' \) rejected at the plane and the absorbed heat \( Q \) are mainly concerned via simulation studies in next section.

4. Results and discussion

In practices of heat flow detection, a heat sink coupled with the thermoelectric module is exposed to ambient air. The opposite side of the thermoelectric module is properly attached to the plane wall. To represent physical performance, actual values of properties and parameters are employed in the proposed mathematical models for simulation studies. The overall convective heat transfer coefficient between the heat sink and the air is determined by carrying out the experiment as described in section 2. The copper plate is initially heated to the temperature of 100 °C. Fig. 6 shows the temperature measurements of the copper plate against time during free cooling. It is observed that the temperature of the copper plate reduces to the air temperature of 25 °C within 5 minutes. Those data of the copper temperature are used to plot the linear relation of Eq. (A3). In Fig. 7, a linear
regression model is applied so as to obtain a line of best fit to the temperature data. This mathematical manipulation yields the slope of -0.0109 and the intercept of 0.002. The corresponding coefficient of the determination $R^2$ is 0.99885, which indicates the regression line quite well fits the data. By substituting the values of the heat capacitance and the area of the heat sink as given in the appendix into the slope of Eq. (A3), the overall convective heat transfer coefficient is 15.5629 W m$^{-2}$ K$^{-1}$.

![Fig. 6 Temperature data of copper plate during cooling](image)

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![Fig. 7 Plot of copper temperature against time](image)

**Fig. 7 Plot of copper temperature against time**

With determined parameters, the mathematical models derived in section 3 are applied by the commercial software of ANSYS Mechanical$^\text{TM}$ in order to numerically solve Eqs. (1) and (4) under given boundary conditions from Eq. (5) to Eq. (7). Without loss of generality, the thermoelectric module coupled with a heat sink as described in section 2 is assumed to be attached to a concrete plane wall in case studies. The thermal conductivity of the concrete wall is 0.72 W m$^{-1}$ K$^{-1}$. In a basic scenario, the 4x4 cm$^2$ square plane wall with the thickness of 10 cm is subjected to the heat input of 500 W m$^{-2}$ at one side and the free convection from the wall to the air of 25 °C takes place at
the other side. The convective heat transfer coefficient between the wall and the air is subjectively set to be 15.5629 Wm⁻²K⁻¹. As expected, the temperature distribution indicates tendency of cooling the heated wall through the air. Fig. 8(a) shows the mean temperature within the plate wall without the thermoelectric module. The temperature of 126.57 °C is developed at the left surface of the plane wall. The temperature decreases to 57.13 °C at the right surface of the plane wall. Fig. 8(b) illustrates the heat transfer through the plane wall. With the steady state condition, the same amount of the 500 Wm⁻² heat flux is convected to the air. In Table 1, the simulated results of temperatures at both sides of the plane wall are presented when the thickness is varied at 10 cm, 7.5 cm, 5 cm, 2.5 cm, and 1 cm. It is observed that the temperatures at the convection side maintain at 57.13 °C. Therefore, the thickness of the wall does not affect the amount of the heat \( Q^* \) of 0.8 W (\((500 \text{ Wm}^{-2})\times(4\times4 \text{ cm}^2)\)) at the cooling surface. Excluding the wall thickness, the size of the plane wall and the convective heat transfer coefficient are considered variables, which influence the amount of the absorbed heat. In Fig. 9, when the size of the plane wall increases from the size of the 4×4 cm² thermoelectric module, the heat \( Q \), which is absorbed by the thermoelectric module, is greater than the heat \( Q^* \) for a given convective heat transfer coefficient of 15.5629 Wm⁻²K⁻¹ at the plane wall. After the plane wall is sufficiently large (greater than fiftyfold area of the thermoelectric module), the absorbed heat more or less remains at a certain value of 2.6 W, which is significantly higher than the expected heat \( Q^* \) of 0.8W. It can be observed from Fig. 10 that heat flux is more competently convected through the thermoelectric module than the plane wall. Furthermore, Fig. 11 illustrates analogous tendencies of the absorbed heat according to various convective heat transfer coefficients at the plane wall. The higher the convective heat transfer coefficient, the less the heat is absorbed by the thermoelectric module and vice versa. The explanation is simply due to upsurge of capacity in heat convection at the wall. In Fig.12, it can be observed that the absorbed heat varies proportionally as the heat input increases from 100 Wm⁻² to 500 Wm⁻² for given size of the plane wall and convective heat transfer coefficient.

Table 1 Temperature at two surfaces of wall with varying wall thickness

<table>
<thead>
<tr>
<th>Thickness [cm]</th>
<th>Temperature at left surface [°C]</th>
<th>Temperature at right surface [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>126.57</td>
<td>57.13</td>
</tr>
<tr>
<td>7.5</td>
<td>109.21</td>
<td>57.13</td>
</tr>
<tr>
<td>5</td>
<td>91.85</td>
<td>57.13</td>
</tr>
<tr>
<td>2.5</td>
<td>74.49</td>
<td>57.13</td>
</tr>
<tr>
<td>1</td>
<td>64.07</td>
<td>57.13</td>
</tr>
</tbody>
</table>

5. Conclusion

The thermoelectric modules are used to detect the amount and the direction of the heat flow through the thermal plane wall. However, there are two main factors influencing amount of the detected heat: the size of the plane wall and the convective heat transfer coefficient. The absorbed heat is identical to the heat rejected from the plane wall when the cross-sectional area of the plane wall is equal to that of the thermoelectric module. In other cases, the absorbed heat varies according to the size of the plane wall and the convective heat transfer coefficient. For heat flow detection, the size of the plane wall and the convective heat transfer coefficient are considered to be constant when the thermoelectric module is implemented on the plane wall. In turn, the absorbed heat is proportional to the heat input. With this fact, the thermoelectric module can be exploited to monitor the change of the heat flow in thermal engineering practices.
Fig. 8 Heat transfer within plane wall: (a) temperature and (b) heat flux

Fig. 9 Plot of absorbed heat flux against cross-sectional wall area
Fig. 10  Heat flux convected from plane wall and heat sink

Fig. 11  Effects of convective heat transfer coefficient on absorbed heat flux with $Q'$ of 500 Wm$^{-2}$
Fig. 12 Effects of heat input on absorbed heat flux with $h_f$ of 15.5629 Wm$^{-2}$K$^{-1}$

6. Appendix

This section is to determine the overall convective heat transfer coefficient between a heat sink and cold air. The copper plate is initially heated up at a certain high temperature before it is attached to a heat sink as shown in Fig 2. During the cooling process, the internal energy of the hot copper plate decreases as the heat of the heat sink is rejected to the air. The governing equation of the energy balance for small interval of time is written as:

$$C_c \frac{dT_c}{dt} = h_A (T_c - T_a) \frac{dT_c}{dt}$$  \hspace{1cm} (A1)

where $C_c$ is the heat capacitance of the copper plate (32.725 JK$^{-1}$), $dT_c$ is the temperature increment of the copper plate, $T_a$ is the temperature of the air, $A_s$ is the surface area of the heat sink (22920 mm$^2$), $h$ is the overall convective heat transfer coefficient, and $dt$ is the time increment.

Eq. (A1) can be arranged as:

$$\frac{1}{T_c - T_a} d(T_c - T_a) = -\frac{hA_s}{C_c} dt$$  \hspace{1cm} (A2)

With the initial condition $T_c = T_c(t = 0)$, the analytical solution of Eq. (A2) can be obtained as:
\[
\ln \left( \frac{T_c - T_a}{T_i - T_a} \right) = - \frac{hA_i}{C_c} t
\]

(A3)

In Eq. (A3), the slope of the linear function of \( \ln \left( \frac{T_c - T_a}{T_i - T_a} \right) \) against \( t \) yields the value of the overall convective heat transfer coefficient.

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References


