NUMERICAL ANALYSIS OF COMBINED NATURAL CONVECTION-INTERNAL HEAT GENERATION SOURCE-SURFACE RADIATION

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Numerical study of combined laminar natural convection and surface radiation with internal heat generation is presented in this paper and computations are performed for an air-filled square cavity whose four walls have the same emissivity. Finite volume method through the concepts of staggered grid and SIMPLER algorithm has been applied, and the view factors are determined by analytical formula. A power scheme is also used in approximating advection–diffusion terms. Representative results illustrating the effects of emissivity and the internal heat generation on the streamlines and temperature contours within the enclosure are reported. In addition, obtained results for local and average convective and radiative Nusselt, for various parametric conditions, show that internal heat generation modifies significantly the flow and temperature fields.

Key word: Natural convection; Heat generation; Surface radiation; Numerical simulation

1. Introduction

Due to its many engineering applications and impact on both flow structure and heat transfer processes in double pane windows, solar collectors, building insulation, nuclear engineering, ovens and rooms; combined natural convection and radiation exchange between surfaces involving a radiatively non-participating medium inside enclosures has been a very important research topic. During the last decades, significant attention was given to the study of natural convection in enclosures subjected to volumetric internal heat generation, ranging from the mantle convection in the earth [1,2] to the cooling of a molten nuclear reactor core [3,4]. Although the surface radiation is inherent in natural convection, the interaction between the two phenomena has received a little attention. Many studies on laminar and turbulent natural convection heat transfer in a rectangular enclosure are carried out including radiation [5–11]. In their investigation, Lauriat and Desrayaud [12] studied numerically the heat transfer by natural convection and surface radiation in a two-dimensional vented enclosure in contact with a cold external environment and a hot internal one. They found that the radiative contributions to the heat transfer along the facing surfaces were the dominant heat transfer mode for all of the considered cases.

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Amraqui et al. [13] analysed computation of the radiation–natural convection interactions in an inclined «T» form cavity. They conclude that the heat transfer decreases with increasing \( \varphi \) (inclination angle). Moreover, they noted that the Rayleigh number and the presence of radiation produce a considerable increase of the heat transfer. Ramesh and Merzkirch [14] made an experimental study of the combined natural convection and thermal radiation heat transfer, in a cavity with top aperture; they found out that the surface thermal radiation heat transfer in cavities with walls of high emissivities had a significant change in the flow and temperature patterns and therefore influence the natural convection heat transfer coefficients. Nouanegue et al. [15] investigated conjugate heat transfer by natural convection, conduction and radiation in open cavities in which a uniform heat flux is applied to the inside surface of the solid wall facing the opening. They noticed that the surface radiation affected the flow and temperature fields considerably. Effect of radiation on natural convection flow around a sphere in presence of heat generation was investigated by Miraj [16].

The governing equations were transformed into dimensionless non-similar equations by using a set of suitable transformations and solved numerically by the finite difference method with Newton's linearization approximation. They found that the increase in the values of radiation parameter or in surface temperature parameter, leads to increase in the velocity profile, the temperature profile, the local skin friction coefficient and the local rate of heat transfer. Rao et al. [17] investigated the interaction of surface radiation with mixed convection in a vertical channel. The natural convection component was driven by symmetrically deployed discrete volumic heat sources in the channels walls. The sources span the full thickness of the wall. The wall heat conduction coupling was treated as a boundary condition with a fin-type equation and heat transfer correlations were obtained from computational results covering a wide parametric space. The effect of radiative transfer and the aspect ratio on the 3D natural convection has been studied by Kolsi et al. [18]. They showed that the principal flow structure is considerably modified when the radiation-conduction parameter was varied. However, the peripheral spiraling motion is qualitatively insensitive to these parameters. The complete conjugate heat conduction, convection and radiation problem for a heated block in a differentially heated square enclosure was solved by Liu and Phan-Tien [19]. It was noticed that the conduction and the emission of the block have a substantial effect on the heat transfer situation. Rahman and Sharif [20] studied the laminar natural convection in differentially heated inclined rectangular enclosures of aspect ratios from 0.25 to 4. They considered a rectangular cavity with and without internal heat generation showing that the uniform internal heat generation increases the local heat flux ratio along the hot wall and decreases it along the cold wall. Recently, Pal and Mondal [21] have investigated the combined convection flow of an optically dense viscous incompressible fluid past a magnetized vertical plate.

Elbashbeshy et al. [22] studied the effect of heat generation or absorption and thermal radiation on free convection flow and heat transfer over a truncated cone in the presence of pressure work. They concluded that an increasing in the values of radiation parameter and heat generation/absorption parameter leads to increases in the value of the skin fiction coefficient while the local Nusselt number decreases. Ashraf et al. [23] investigated the effect of radiation on fluctuating hydro-magnetic natural convection flow of viscous, incompressible, electrically conducting fluid past a magnetized vertical plate.
The purpose of this paper is to study the combined effects of surface radiation and heat generation on
the flow and heat transfer in the cavity. The surface emissivity $\varepsilon$ and the internal Rayleigh number $Ra_i$ are parameters to be varied.

2. Mathematical formulation

Details of the geometry are shown in fig.1. The flow is assumed to be incompressible, laminar and two
dimensional in an enclosure of square cavity with internal heat generation $q$, the two vertical walls are
maintained at two different temperatures $T_H$ and $T_C$ ($T_H > T_C$) while the two horizontal walls are
submitted to a radiative heat flux $q_r = k \frac{\partial T}{\partial y}$. It will be further assumed that the temperature
differences in the domain under consideration are small enough to justify the employment of the
Boussinesq approximation.

$\begin{align*}
    u = v = 0, \\
    -k \frac{\partial T}{\partial y} + q_r = 0
\end{align*}$

**Figure 1: The flow configuration and coordinate system**

The fluid is the air and its properties are assumed constant at the average temperature $T_0$, except for
the density whose variation with the temperature is allowed in the buoyancy term. The inner surfaces,
in contact with the fluid, are assumed to be gray and diffuse, and could emit and reflect radiation with
identical emissivities.

The fluid flow state is given by the velocity vector $\vec{V}$, the density $\rho$, the pressure $P$ and the
temperature $T$. The balance equations describing the motion of a fluid in a region of space form a set
of three conservation laws:

- The equation of continuity expresses the mass conservation of the fluid particle.
- The equation of linear momentum reflects the fundamental principle of dynamics applied to the fluid
  particles. It states that the total change of momentum in a given volume control is equal to the sum of
  the forces acting in this volume and surface forces acting on its surface.
- The energy equation requires that the energy cannot be created or destroyed. It expresses the
  conservation of the energy of the fluid particle.

Taking into account the assumptions mentioned above, the governing equations for the problem in two
dimensions unsteady states can be written in dimensional form as:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
(1)

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]  
(2)

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g \beta (T - T_0)
\]  
(3)

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{q}{\rho c_p}
\]  
(4)

Introducing the following non-dimensional variables:

\[ \tau = \frac{t}{H^2/\alpha}, \quad X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{uH}{\alpha}, \quad V = \frac{vH}{\alpha}, \quad P = \frac{pH^2}{\rho \alpha^2}, \quad \theta = \frac{T - T_0}{\Delta T}, \quad \Delta T = T_H - T_C, \]

Dimensionless governing equations (1-4) can be written as:

\[
\frac{\partial U}{\partial \tau} + \frac{\partial V}{\partial Y} = 0
\]  
(5)

\[
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \text{Pr} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]  
(6)

\[
\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \text{Pr} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra_e \text{ Pr} \theta
\]  
(7)

\[
\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) + \frac{Ra_i}{Ra_e}
\]  
(8)

\[ Ra_e \] and \[ Ra_i \] are the external and the internal Rayleigh numbers defined respectively as:

\[ Ra_e = \frac{g \beta \Delta T H^3}{(\nu \alpha)} \]

\[ Ra_i = \frac{g \beta q H^5}{(\nu \alpha k)} \]

The corresponding initial and boundary conditions are:

\[
U = V = 0, \quad \theta = \theta_i \quad \text{for} \quad \tau = 0
\]

\[
U = V = 0, \quad \theta = \theta_c = -0.5 \quad \text{for} \quad 0 \leq Y \leq 1 \text{ at } X = 0
\]

\[
U = V = 0, \quad \theta = \theta_H = 0.5 \quad \text{for} \quad 0 \leq Y \leq 1 \text{ at } X = 1
\]

\[
U = V = 0, \quad \frac{\partial \theta}{\partial Y} - Nr Q_r = 0 \quad \text{for} \quad 0 \leq X \leq 1 \text{ at } Y = 0
\]

\[
U = V = 0, \quad \frac{\partial \theta}{\partial Y} - Nr Q_r = 0 \quad \text{for} \quad 0 \leq X \leq 1 \text{ at } Y = 1
\]
Where \( Nr = \sigma T_0^4 H / k \Delta T \), is the dimensionless parameter of conduction-radiation and \( Q_c = q_c / \sigma T_0^4 \), is the dimensionless net radiative heat flux.

Therefore, the dimensionless net radiative flux density along a diffuse-gray and opaque surface “A_i” is expressed as:

\[
Q_{r,i,j} = R_i - \sum_{j=1}^{N} R_j F_{i,j} \tag{9}
\]

\( R_i \) is the dimensionless radiosity of surface \( A_i \), obtained by resolving the following system:

\[
\sum_{j=1}^{N} (\delta_{ij} - (1-\varepsilon_i)F_{i,j})R_j = \varepsilon_i \Theta_i^4 \tag{10}
\]

Where the dimensionless radiative-temperature \( \Theta_i \) is given by:

\[
\Theta_i = \frac{T_i}{T_0} = [(T_H - T_C)\theta_i + T_0]/T_0 = \theta_i \frac{\Delta T}{T_0} + 1 \tag{11}
\]

\[
\Theta_i = \frac{\theta_i}{\theta_0} + 1 \tag{12}
\]

The average convective Nusselt number is calculated by integrating the temperature gradient over the vertical wall:

\[
Nu_{avg} = \frac{1}{A} \int_0^A \frac{\partial \theta}{\partial X} dX \tag{13}
\]

The average radiative Nusselt number is obtained by integrating the net radiative flux:

\[
Nu_{avg} = N_r \frac{1}{A} \int_0^A Q_r dX \tag{14}
\]

The total average Nusselt number is calculated by summing the average values of convective and radiative Nusselt numbers:

\[
Nu_{avg} = \frac{1}{A} \int_0^A \left( -\frac{\partial \theta}{\partial X} \right)_{\partial Y} + N_r Q_r (0,Y) dY \tag{15}
\]

3. Numerical Procedure

The numerical solution of the governing differential equations for the velocity, pressure and temperature fields is obtained by using a finite volume technique. A power scheme was also used in approximating advection–diffusion terms. The SIMPLER algorithm whose details can be found in Patankar [24], with a staggered grid is employed to solve the coupling between pressure and velocity. The governing equations are cast in transient form and a fully implicit transient differencing scheme was employed as an iterative procedure to reach steady state. The discretised equations are solved using the line by line Thomas algorithm with two directional sweeps. The radiosities of the elemental wall surfaces are expressed as a function of elemental wall surface temperature, emissivity and the shape factors. The radiosity (R_i) and temperature (\Theta_i) are connected by
equation (10) whose resolution is performed by the Gauss elimination method. In 2D, the view factors are analytic [25]:

\[
F_{ij} = \frac{-1}{2(x_i - x_j)} \left[ \sqrt{x_i^2 + y_i^2} - \sqrt{x_j^2 + y_j^2} \right]_{y_i}^{y_j}
\]  

(16)

\[
F_{i-k} = \frac{1}{2(x_2 - x_1)} \left[ \sqrt{(x_2 - x)^2 + H^2} \right]_{x=x_1}^{x=x_2} - \sqrt{(x_1 - x)^2 + H^2} \right]_{x=x_1}^{x=x_2}
\]  

(17)

For the calculations reported in this study a 120 x 120 grid points was chosen to optimise the relation between the accuracy required and the computing time. In order to obtain good convergence solutions, the convergence criterion for the residuals was set at $10^{-5}$.

The outer iterative loop is repeated until the steady state is achieved which occurs when the following convergences are simultaneously satisfied:

\[
|\phi^{\text{old}}_i - \phi_i| \leq \varepsilon_\phi
\]

where $\phi$ represents the variables $U$, $V$ or $\theta$. In most of the cases, the velocity components and temperatures were driven to $\varepsilon_U = \varepsilon_V = \varepsilon_\theta \leq 10^{-6}$.

3. Validation

In order to verify the numerical code, the model is reduced to the classical case of natural convection and surface radiation in a square cavity without heat generation. The convective, radiative and total Nusselt numbers of the active walls are compared with the ones of Wang [26]. The results presented in tab. 1 and tab. 2 show an excellent agreement.

Table 1: Nusselt numbers at hot wall with $T_0 = 293.5$ K and $\Delta T = 10$ K.

<table>
<thead>
<tr>
<th>$\text{Ra}_E$</th>
<th>$H$</th>
<th>$\varepsilon$</th>
<th>Ref. [26]</th>
<th>Present study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nu_c</td>
<td>Nu_r</td>
<td>Nu_t</td>
<td>Nu_c</td>
</tr>
<tr>
<td>$10^4$</td>
<td>0.021</td>
<td>0</td>
<td>2.246</td>
<td>0</td>
</tr>
<tr>
<td>$10^5$</td>
<td>0.021</td>
<td>0.2</td>
<td>2.260</td>
<td>0.507</td>
</tr>
<tr>
<td>$10^4$</td>
<td>0.021</td>
<td>0.8</td>
<td>2.249</td>
<td>2.401</td>
</tr>
<tr>
<td>$10^5$</td>
<td>0.045</td>
<td>0</td>
<td>4.540</td>
<td>0</td>
</tr>
<tr>
<td>$10^5$</td>
<td>0.045</td>
<td>0.2</td>
<td>4.394</td>
<td>1.090</td>
</tr>
<tr>
<td>$10^5$</td>
<td>0.045</td>
<td>0.8</td>
<td>4.189</td>
<td>5.196</td>
</tr>
<tr>
<td>$10^6$</td>
<td>0.097</td>
<td>0</td>
<td>8.852</td>
<td>0</td>
</tr>
<tr>
<td>$10^6$</td>
<td>0.097</td>
<td>0.2</td>
<td>8.381</td>
<td>2.355</td>
</tr>
<tr>
<td>$10^6$</td>
<td>0.097</td>
<td>0.8</td>
<td>7.815</td>
<td>11.265</td>
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</tbody>
</table>
Table 2: Nusselt numbers at cold wall with $T_0 = 293.5$ K and $\Delta T = 10$ K.

<table>
<thead>
<tr>
<th>$Ra_e$</th>
<th>$H$</th>
<th>$\varepsilon$</th>
<th>Ref. [26]</th>
<th>Present study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{Nu}_c$</td>
<td>$\text{Nu}_t$</td>
<td>$\text{Nu}_t$</td>
<td>$\text{Nu}_c$</td>
</tr>
<tr>
<td>$10^4$</td>
<td>0.021</td>
<td>0</td>
<td>2.246</td>
<td>0</td>
</tr>
<tr>
<td>$10^4$</td>
<td>0.021</td>
<td>0.2</td>
<td>2.268</td>
<td>0.499</td>
</tr>
<tr>
<td>$10^4$</td>
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<td>0.8</td>
<td>2.278</td>
<td>2.372</td>
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<tr>
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<td>0</td>
<td>4.540</td>
<td>0</td>
</tr>
<tr>
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<td>4.411</td>
<td>1.073</td>
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<tr>
<td>$10^6$</td>
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<td>0</td>
<td>8.852</td>
<td>0</td>
</tr>
<tr>
<td>$10^6$</td>
<td>0.097</td>
<td>0.2</td>
<td>8.417</td>
<td>2.319</td>
</tr>
<tr>
<td>$10^6$</td>
<td>0.097</td>
<td>0.8</td>
<td>7.930</td>
<td>11.150</td>
</tr>
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</table>

4. Results and Discussions

This section is devoted to analyse the effects of the internal heat generation parameter ($Ra_i$) on the flow and heat transfer on the combined natural convection surface radiation flow considering $Pr = 0.70$, $T_0 = 300$ K, $Ra_e = 10^6$, $\Delta T = 10K$ and $\varepsilon = (0.2, 0.5)$.

For low internal Rayleigh number $Ra_i$ ($\leq 10^7$), considerations are given to the cases when the effects of external heating and internal heat generation are comparable. Fig. 2 and Fig. 3 illustrate the sequences of flow and thermal fields for $\varepsilon = 0.2$ and $\varepsilon = 0.5$ respectively. Performing order of magnitude analysis on $Ra_i = 0$ and $10^6$, implies that the relative impact of internal heat generation is minor. The flow is attributed by the presence of a single clockwise circulation cell, which occupies much of the cavity and a secondary and a tertiary vortices are formed inside the cavity (vortices of surface radiation effects without internal heat generation, $Ra_i = 0$).

As the heat generation increases, ($Ra_i \geq 10^7$), the total thermal energy in the cavity is on increase, those small vortices are merged to the primary vortex of relatively higher intensity of circulation than that at low $Ra_i$.

Consequently, a counter-clockwise small cell appeared at the upper left corner (for $Ra_i = 2 \times 10^7$), and is shifted left with increasing $Ra_i$. The flow strength in this new cell also increases when the internal heat generation increases in magnitude.

For large values of internal Rayleigh number, the whole cavity is occupied by two recirculating cells; i.e. both counter-clockwise and clockwise cells near the hot and cold side walls respectively due to the positive buoyancy effect. The isotherms tend to be horizontally uniform and vertically linear at the upper portion of the enclosure. However, in the bottom part of the cavity, the isotherms are divided into two groups.
This effect of internal heat generation on the flow field is reasonable since internal heat generation assists buoyancy forces by accelerating the fluid flow, (fig.4a, 4b and 4c). On the other hand, the presence of heat source within the enclosure causes an increase in the fluid temperature, (Fig. 4d), leading to a reduction of convective and radiative heat transfer on the hot wall (Fig. 5 and Fig. 6).
Figure 3: (a) Streamlines; (b) isotherms for $\epsilon = 0.5$ and various values of $Ra_I$

From Fig. 5, we can also note that for weak heat generation, local convective Nusselt number has positive values at the upper section of the cavity, and from $Ra_I = 2 \times 10^7$ all values are negative. This means that, the heat is transferred from the fluid to the hot wall (the hot wall absorbs the heat from the interior higher temperature fluid). The same behaviour is observed for the local radiative Nusselt number at $Ra_I = 5 \times 10^7$ (Fig. 6).
Figure 4: Variations of: (a) Maximum values of the stream function, (b) vertical velocity, (c) horizontal velocity (d) temperature with internal Rayleigh number for $Ra_E = 10^6$

Figure 5: Local convection Nusselt number on the hot wall for $Ra_E = 10^6$
Figure 6: Local radiation Nusselt number on the hot wall for $Ra_F = 10^6$

The negative sign of $Nu_c$ and $Nu_r$ corresponds to the apparition of the small counter-clockwise cell shown previously in Fig. 2. It is noticeable that the absolute value for the temperature gradient has a maximum value at this position, since this cell is coming to the hot wall at the upper corner; so therefore, the values of $Nu_c$ and $Nu_r$ along the hot side wall are governed by the direction and strength of the flow adjacent to the hot wall.

Fig. 7 illustrates the variation of average convective and radiative Nusselt numbers for different values of internal heat generation and emissivity. The positive values of $Nu_{cavg}$ and $Nu_{ravg}$ mark that there is ascending motion near the hot wall though the circulation feels retardation due to the buoyancy effect generated by internal heat generation. Therefore, as $Ra_i$ increases, the average convective and radiative Nusselt numbers decrease indicating the descending motion near the hot side wall.

Figure 7: Variations of the average (a) convection and (b) radiation Nusselt number as a function of internal Rayleigh number

5. Conclusion
In the present numerical investigation, calculations have been made for the combined natural convection and surface radiation in a differentially cavity with the presence of an internal heat generation.

The study shows that internal heat generation modifies significantly the flow and temperature fields. The increase in the value of the heat generation parameter leads to increase in the flow rates in the secondary cell as well an increase in its size until it occupies the half of the total cavity space. Further increase in the value of heat generation causes for development of more cells in the cavity. The temperature of the fluid in the cavity also increases due to the increase of internal heat generation and hence that negates the heat transfer from the heated surface.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$A_i$</td>
<td>radiative surface number $i$</td>
</tr>
<tr>
<td>$C_p$</td>
<td>specific heat at constant pressure, [J Kg$^{-1}$ K$^{-1}$]</td>
</tr>
<tr>
<td>$F_{ij}$</td>
<td>view factor between surfaces $S_i$ and $S_j$</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration, [m.s$^{-2}$]</td>
</tr>
<tr>
<td>$H$</td>
<td>height of the enclosure, m</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity, [W m$^{-1}$ K$^{-1}$]</td>
</tr>
<tr>
<td>$N$</td>
<td>total number of radiative surfaces</td>
</tr>
<tr>
<td>$N_r$</td>
<td>radiation number, $\sigma T_0^4 (k \Delta T / H)$</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>$P$</td>
<td>dimensionless pressure,</td>
</tr>
<tr>
<td>$p$</td>
<td>fluid pressure, Pa</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number, $\nu / \alpha$</td>
</tr>
<tr>
<td>$q$</td>
<td>internal heat generation [W m$^{-3}$]</td>
</tr>
<tr>
<td>$qr$</td>
<td>net radiative flux, [W m$^{-2}$]</td>
</tr>
<tr>
<td>$Qr$</td>
<td>dimensionless net radiative-flux</td>
</tr>
<tr>
<td>$R_{aE}$</td>
<td>dimensionless radisosity</td>
</tr>
<tr>
<td>$R_{aI}$</td>
<td>internal Rayleigh number</td>
</tr>
<tr>
<td>$t$</td>
<td>time, [s]</td>
</tr>
<tr>
<td>$T$</td>
<td>dimensional temperature, [K]</td>
</tr>
<tr>
<td>$T_0$</td>
<td>reference temperature, $(T_c - T_H) / 2$, K</td>
</tr>
<tr>
<td>$u, v$</td>
<td>dimensional velocity-components, [m.s$^{-1}$]</td>
</tr>
<tr>
<td>$U, V$</td>
<td>dimensionless velocity-components</td>
</tr>
<tr>
<td>$x, y$</td>
<td>cartesian coordinates, [m]</td>
</tr>
<tr>
<td>$X, Y$</td>
<td>dimensionless coordinates</td>
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**Greek symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>thermal diffusivity, [m$^2$.s$^{-1}$]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>thermal expansion coefficient [K$^{-1}$]</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>temperature difference, $T_c - T_f$, [K]</td>
</tr>
<tr>
<td>$\epsilon_i$</td>
<td>emissivity of surface $A_i$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity of the fluid, [Kg m$^{-1}$.s$^{-1}$]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity, [m$^2$.s$^{-1}$]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>fluid density, [Kg.m$^{-3}$]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan–Boltzmann constant, [W m$^{-1}$.K$^{-4}$]</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>dimensionless temperature, $T / T_0$</td>
</tr>
<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker symbol</td>
</tr>
<tr>
<td>$\tau$</td>
<td>dimensionless time</td>
</tr>
</tbody>
</table>

**Subscripts**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>avg</td>
<td>average value</td>
</tr>
<tr>
<td>$C$</td>
<td>cold</td>
</tr>
<tr>
<td>$c$</td>
<td>convective</td>
</tr>
<tr>
<td>$H$</td>
<td>hot</td>
</tr>
<tr>
<td>max</td>
<td>maximum value</td>
</tr>
<tr>
<td>$r$</td>
<td>radiative</td>
</tr>
<tr>
<td>$t$</td>
<td>total</td>
</tr>
<tr>
<td>0</td>
<td>reference state</td>
</tr>
</tbody>
</table>

**References**


