SIMULATION OF NATURAL CONVECTION HEAT TRANSFER USING NANOFLUID IN A CONCENTRIC ANNULUS

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Abstract

In the present study, natural convection of nanofluids in a concentric horizontal annulus enclosure has been numerically simulated using the lattice Boltzmann method. A water-based nanofluid containing Al$_2$O$_3$ nanoparticle has been studied. Simulations have been carried while the Rayleigh number ranges from $10^3$ to $10^5$ and solid volume fraction varies between 0 and 0.04. The effects of solid volume fraction of nanofluids on hydrodynamic and thermal characteristics such as average and local Nusselt numbers, streamlines and isotherm patterns for different values of solid volume fraction, annulus gap width ratio and Rayleigh number are investigated and discussed in detail.

Keywords: Natural convection, Nanofluid, Concentric annulus, Lattice Boltzmann method

1. Introduction

The problem of natural convection in annulus between two horizontal cylinders has attracted much attention due to its wide applications in solar collector-receiver, underground electric transmission cables, vapor condenser for water distillation and food process. The literature survey shows that there is numerous works on both experimental and numerical investigation natural convection heat transfer between two concentric circular cylinders.(Kuhen and Goldsein [1–2]; Glakpe et al. [3]; Guj et al. [4]; etc.). Heat transfer enhancement in these systems is of great importance from the industrial and energy saving perspectives. The low thermal conductivity of conventional heat transfer fluids, such as water, is considered a primary limitation in enhancing the performance and the compactness of such thermal systems. An innovative technique to improve the heat transfer is adding the nano-scale particles to the base fluid, which has been used extensively in the past decade (Choi et al. [5]; Oztop and Abu-Nada [6]; Abu-Nada et al. [7]; Sheikhzadeh et al. [8]; Khanafer and Vafai [9]; Salari et al. [10-11]; etc). The corresponding results show that the existence of the nanoparticles can reduce or increase the rate of heat transfer based on the geometry and boundary conditions of the
problem, type, size, shape, and volume fraction of the nanoparticles. Therefore, the effects of nanofluids on the heat transfer must be studied.

One of the useful numerical methods that have been used in the recent years is the Lattice Boltzmann Method (LBM). LBM has several advantages over other conventional CFD methods due to its particular nature and local dynamics. The major advantages of the LBM are due to the fact that the solution for the particle distribution functions is explicit, easy for parallel computation and implementation of boundary conditions on complex boundaries is simple. It has been used for simulating the flow field in wide ranges of engineering applications such as natural convection [12], nanofluid [13-14], unsteady flow [15-19], and so on.

The aim of this study is to study heat transfer characteristics of natural convection in the annulus between horizontal concentric cylinders filled with Al₂O₃–water nanofluid. The Lattice Boltzmann Method with single-relaxation-time collision model is employed. The study is carried out at a various range of annulus gap width ratio (0.5≤σ≤5) and solid volume fractions (0% to 4%) at Rayleigh numbers 10³-10⁵.

2. Numerical method

The LB model implemented here is the same as that employed in Fattahi et al. [12] and Mei et al. [16]. In this work, the most popular model for two dimensional problems, the D₂Q₉ model, which consists of 9 distribution functions, has been used.

2.1. Lattice Boltzmann method for the flow field

The lattice Boltzmann method solves the microscopic kinetic equation for the particle distribution function \( f(x,v,t) \). These particle distribution functions are normally calculated by solving the following equation:

\[
\frac{\partial f}{\partial t} + V \cdot \nabla f = \Omega(f)
\]  

(1)

Where \( \Omega(f) \) is the collision operator.

The lattice Boltzmann equation of velocity field with the BGK collision operator and external force can be discretized in space \( x \) and time \( t \) into the following form:

\[
f_i(x + c_i \Delta t, t + \Delta t) = f_i(x, t) + \frac{\Delta t}{\tau_v} \left[ f_i^{eq}(x, t) - f_i(x, t) \right] + \Delta t c_i F_i
\]

(2)

Where \( F_i \) is the external force in the direction of lattice velocity, \( \tau_v \) denotes the lattice relaxation time for the flow field and \( f_i^{eq} \) stands for the equilibrium distribution function. The kinematic viscosity \( \nu \) is defined in terms of its respective relaxation time, i.e. \( 3\nu=\tau_v-0.5 \).

The equilibrium distribution function, which depends on the local density and velocity, is computed as:

\[
f_i^{eq} = w_i \rho \left[ 1 + \frac{c_i \nu}{c_i^2} + \frac{1}{2} \frac{(c_i \nu)^2}{c_i^2} - \frac{1}{2} \frac{u^2}{c_i^2} \right]
\]

(3)

In the Eq. (3), \( c_i \) and \( w_i \) denote the particle velocity and the weighting factor for the \( i \)th direction in the discrete velocity space.

Based on conservation laws of mass and momentum, the macroscopic density \( \rho \) and velocity \( \mathbf{u} \) in terms of the density distribution functions are calculated by:

\[
\rho = \sum_{i=0}^{8} f_i
\]

(4)
\[ \rho u = \sum_{i=0}^{8} f_i c_i \]  \hspace{1cm} (5)

In order to incorporate buoyancy forces into the model, the force term in Eq. (2) needs to be calculated in the vertical direction (y) as:

\[ F = 3 w_i g_y \beta \theta \]  \hspace{1cm} (6)

For natural convection the Boussinesq approximation is applied and radiation heat transfer is negligible. To ensure that the code works in a near incompressible regime, the characteristic velocity of the flow for natural \( (V_{\text{Natural}} = (\beta g_y \Delta T H)^{1/2}) \) regimes must be small compared with the fluid speed of sound. In the present study, the characteristic velocity was selected as 0.1 of sound speed.

2.2. Lattice Boltzmann equation for the temperature field

In natural convection problems, the effect of viscous heat dissipation can be neglected for applications in incompressible flow (Mei et al. [16]). So, the Bhatnagar–Gross–Krook (BGK) equation for the temperature field is defined as:

\[ g_i(x + c_i \Delta t, t + \Delta t) = g_i(x, t) + \frac{\Delta t}{\tau_D} \left[ g_i^{eq}(x, t) - g_i(x, t) \right] \]  \hspace{1cm} (7)

Where \( \tau_D \) denotes the lattice relaxation time for the temperature field. The thermal diffusivity \( \alpha \), is defined in terms of its respective relaxation time, \( 3\alpha = (\tau_D - 0.5) \). Furthermore, the local equilibrium distribution function of temperature field is calculated as follows:

\[ g_i^{eq} = w_i \frac{T}{1 + \left( \frac{c_i u}{c_f} \right)} \]  \hspace{1cm} (8)

Also, the temperature is recovered using the summation on \( g_i \):

\[ T = \sum_{i=0}^{8} g_i \]  \hspace{1cm} (9)

2.3. Curve Boundary conditions

In computational fluid dynamics the ability to handle complex geometries accurately and efficiently is the primary discussion. Various methodologies have been proposed for simulating the flow over complex geometries while using LBM. In this work, the method proposed in [14] has been used for treating curved boundaries in the velocity and temperature fields.

2.4. Lattice Boltzmann model of the nanofluid

Nanofluids have different behavior from pure liquids due to inter-particle potentials and other forces act on them. Therefore, some governing equations should be modified for modeling the nanofluid flows. That is because of changes in the fluid thermal conductivity, density, heat capacitance and thermal expansion which are discussed in this section.

The thermal diffusivity in nanofluids is given by:

\[ \alpha_{nf} = k_{nf} / (\rho \ c_p)_{nf} \]  \hspace{1cm} (10)

The effective density of a nanofluid is written as:

\[ \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s \]  \hspace{1cm} (11)

where \( \phi \) is the solid volume fraction of nanoparticles and \( f, s \) and \( nf \) denote base fluid, solid nanoparticle and nanofluid properties, respectively.

The heat capacitance of the nanofluid and thermal expansion coefficient in the Boussinesq equation can be written as:

\[ (\rho \ c_p)_{nf} = (1 - \phi) (\rho \ c_p)_f + \phi (\rho \ c_p)_s \]  \hspace{1cm} (12)
\[
(\rho\beta)_{nf} = (1 - \phi) (\rho\beta)_f + \phi (\rho\beta),
\]

The dynamic viscosity of the nanofluid containing a dilute suspension of small rigid spherical particles is given by Khanafer and Vafai [9] as:
\[
\mu_{nf} = -0.4491 + \frac{28.837}{T} + 0.574 \phi - 0.1634\phi^2 + 23.053 \frac{\phi^2}{T^2} + 0.0132 \phi^3
\]

\[
-2354.735 \frac{\phi}{T^3} + 23.498 \frac{\phi^2}{d_p^2} - 3.0185 \frac{\phi^3}{d_p^3}
\]

The thermal conductivity of the nanofluids can also be defined as [18]:
\[
\frac{k_{nf}}{k_f} = 1 + 64.7 \phi^{0.764} \left( \frac{d_f}{d_p} \right)^{0.369} \left( \frac{k_f}{k_s} \right)^{0.7476} \frac{Pr_T \ Re_T^{1.2321}}{d_p}
\]

Where \(Pr_T\) and \(Re_T\) are given by:
\[
Pr_T = \frac{\mu_f}{\rho_f \sigma_f}, \quad Re_T = \frac{\rho_f k_b T}{3 \pi \mu_f l_f}
\]

Where \(l_f\) is the mean path of the fluid particle that is assumed as 17 nm similar to ref. [22] and \(k_b\) is the Boltzmann constant. It should be noticed that, this model is based on experimental measurements of Chon et al. [21] for \(Al_2O_3\) suspension in water at volume fraction up to 4% and includes the nanoparticle size and the work temperature effects. However, Minsta et al. [23] showed that this model is appropriate for thermal conductivity prediction of both \(Al_2O_3\) and \(Cu\) nanoparticles up to a volume fraction of 9% by experimental test.

The dimensionless relaxation time for flow and temperature fields which are determined by the nanofluid properties are as follows:
\[
\tau_u = \frac{3}{2} \nu_{nf(lbm)} + 0.5
\]
\[
\tau_D = \frac{3}{2} \sigma_{nf(lbm)} + 0.5
\]

That \(lbm\) subscript determines the lattice scale.

### 3. Computational Domain and Numerical Details

The physical model of this work is shown in Fig. 1. A two dimensional horizontal annulus is bounded by an outer radius \(R_o\) and an inner core radius \(R_i\) and \(\theta\) is measured counterclockwise from the upward vertical plane through the center of the cylinders. \(T_b\) and \(T_c\) are the constant temperatures of the inner and outer cylinders, respectively, and \(T_b > T_c\). The fluid in the annulus is a water-based nanofluid containing \(Al_2O_3\) or \(Cu\) nanoparticles. Also, \(\sigma=2R_o/(R_o-R_i)\) denotes annulus gap width ratio.

![Figure 1 Flow geometry for annulus.](image)

In this problem, the Rayleigh number \((Ra)\) is defined as follows:
\[
Ra = g \beta \Delta T (R_o - R_i)/\alpha v
\]
We assumed that nanofluid is similar to a pure fluid and then nanofluid qualities are gotten and they were applied for the equations. The ratio of nanofluid Rayleigh number to that of the base fluid can then be expressed as:

$$\frac{Ra_{nf}}{Ra_f} = \frac{\beta_{nf} v_f \alpha_f}{\beta_f v_{nf} \alpha_{nf}}$$  \hspace{1cm} (20)

Local Nusselt numbers are named on inner and outer cylinder as $Nu_i, Nu_o$:

$$Nu_i = \frac{\kappa_{nf} R_i}{k_f} \ln \left( \frac{R_0}{R_i} \right) \frac{\partial T}{\partial r} \bigg|_{r=R_i}$$

$$Nu_o = \frac{\kappa_{nf} R_o}{k_f} \ln \left( \frac{R_0}{R_o} \right) \frac{\partial T}{\partial r} \bigg|_{r=R_o}$$  \hspace{1cm} (21)

The mean Nusselt number can be calculated as:

$$\overline{Nu} = \frac{\kappa_{nf}}{k_f} \frac{1}{2\pi} \int_0^{2\pi} (Nu) \varphi \, d\varphi$$  \hspace{1cm} (22)

And the average Nusselt number is:

$$Nu_{av} = \frac{(\overline{Nu}_o + \overline{Nu}_i)}{2}$$  \hspace{1cm} (23)

4. Results and Discussion

In this article the effect of nanoparticle suspension in water is studied for $Ra=10^3$-$10^5$ and solid volume fraction $0$ to $0.05$. Water is the base fluid with $Pr=6.57$ at $22^\circ C$, containing $Al_2O_3$ nanoparticle. The nanofluid is assumed incompressible and flow is considered to be laminar and two dimensional. It is idealized that water and nanoparticles are in thermal equilibrium and non-slip occurs between the two media. Physical and thermal properties of both solid and fluid at the base temperature $20^\circ C$ are listed in tab. 1 [6].

In order to prove that the results are independent of the grid size, the numerical results using four different grids are presented in tab. 2 in a case of $Ra=10^5$, $Pr=0.71$ and $\sigma=1.25$ with no nanoparticles in the flow field. Four successively grids as $41 \times 41$, $61 \times 61$, $81 \times 81$ and $101 \times 101$ are tested for grid independence by calculating the average Nusselt number. As shown in table, it was concluded that a $81 \times 81$ grid can be considered as a good compromise between accuracy and computational costs. The solution was converged when the relative error between the new and old values of velocity components and temperature fields become less than $10^{-4}$ and $10^{-8}$ respectively.

<table>
<thead>
<tr>
<th>Property</th>
<th>fluid phase (water)</th>
<th>solid phase ($Al_2O_3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_p$ ($J/KgK$)</td>
<td>4179</td>
<td>765</td>
</tr>
<tr>
<td>$\rho$ ($Kg/m^3$)</td>
<td>997.1</td>
<td>3970</td>
</tr>
<tr>
<td>$K$ ($W/mK$)</td>
<td>0.613</td>
<td>25</td>
</tr>
<tr>
<td>$\beta \times 10^5$($K^{-1}$)</td>
<td>21</td>
<td>0.85</td>
</tr>
<tr>
<td>$d_p$ ($nm$)</td>
<td>0.384</td>
<td>47</td>
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</table>

<table>
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<tr>
<th>Mesh size</th>
<th>41 $\times$ 41</th>
<th>61 $\times$ 61</th>
<th>81 $\times$ 81</th>
<th>101 $\times$ 101</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Nu_{av}$</td>
<td>3.28</td>
<td>3.50</td>
<td>3.66</td>
<td>3.71</td>
</tr>
</tbody>
</table>

Figure 2 Comparison of streamlines and isotherms lines of the present study and experimental work of Guj et al. [4] for $Ra = 4.59 \times 10^4$, $Pr=0.71$ and $\sigma=1.25$. 

Table 1 Thermophysical properties of different phases.
To validate the present study two different cases are considered. First, natural convection in a cylindrical annulus is simulated at $Ra = 4.59 \times 10^4$ and $\sigma = 1.25$ is considered. Figure 2 shows the obtained streamlines and isotherms lines of the present study in comparison with the experimental results of Guj et al. [4]. The comparison indicates that the results have good agreement with experimental data.

Second, the result of heat transfer enhancement in horizontal annuli using nanofluids has been represented. The normalized average Nusselt number at different void fractions are presented in fig. 3 and are compared with the numerical results of Abu Nada et al. [7] and Sheikhzadeh et al [8] for $Ra = 10^5$, $\sigma = 1.25$ and $Pr = 6.2$. The normalized average Nusselt number is defined as the ratio of average Nusselt number at any volume fraction of nanoparticles to that of pure water and is given as:

$$Nu^* = \frac{Nu}{Nu_{\phi=0}}$$  \hspace{1cm} (24)

It can be clearly seen that the present numerical approach is in good agreement with the previous ones.

In order to obtain insight into the detailed flow structures, initially, streamline and isotherm pattern at different volume fraction, Rayleigh number and annulus gap width ratio for $Al_2O_3$ is presented in Fig. 4 and 5. The streamlines are drawn on the left half of the annulus, while the isotherms are plotted on the right half. Due to the large quantity of data, some results are omitted. Figure 4 illustrates the effects of volume fraction and Rayleigh number on the heat and fluid flow patterns for $\sigma = 0.5$. It is clear that by increasing the volume fraction of nanoparticles the maximum strength of streamlines is reduced due to the increased viscosity of the nanofluids. Also, the figures show that the thickness of the thermal boundary layer decreases with increasing the volume fraction. It is due to the increase in heat conduction which is associated with the presence of the nanofluid. As the volume fraction increases, irregular and random movements of particles increase the energy exchange rates in the fluid. Also, it can be seen that at low $Ra$ number ($10^3$) the fluid flow in the half-annuals is weak and forms a symmetrical recirculation. In this condition, the isotherms are nearly concentric indicating the small effect of the convective flow on heat transfer rate. As $Ra$ number increases, the fluid motion becomes stronger and center of rotation moves upward. Also, a radial temperature inversion appears, indicating separation of the inner and outer boundary layers by increasing $Ra$ number.
Figure 4 Streamlines and isotherm pattern at (a) $Ra=10^3$ and (b) $Ra=10^5$ at different volume fractions for $\sigma=0.5$.

Figure 5 presents the influence of the annulus gap width ratio on the heat and fluid flow patterns within the concentric annulus at fixed volume fraction ($\phi=0.02$) of $Al_2O_3$ nanoparticles for $Ra=10^3$ and $10^5$. At $Ra=10^3$, heat transfer is mainly due to conduction since the fluid motion driven by the buoyancy force is very slow. As shown in Fig. 5 (a) and (b), the isotherms form concentric circles and center of rotation is the midpoint of the annulus that is due to the conduction domination in these cases. In Fig. 5(c), the results show that the center of rotation of the flow is in the top portion of the annulus, and the isotherms form eccentric circles due to the strong convection heat transfer, where $\sigma=1$. As the annulus gap width ratio increases, the flow becomes weaker, and the center of rotation moves toward the midpoint of the annulus, and the isotherms form concentric circles as shown in Fig. 5(d). The effect of the annulus gap width ratio can be described as follows. For a small gap width ratio, the thickness of the velocity boundary layers is less than the gap between the cylinders. Consequently, a free stream velocity exists in the gap leading to an increase in the radial and tangential velocity components. Hence, as shown in Fig. 5(c), the possibility of developing a thermal boundary layer will increase and the cold fluid moves to the lower part of the annulus and hot fluid moves to the upper part of it due to natural convection. As the annulus gap width ratio is augmented, the influence of the viscosity becomes higher. Hence, the thickness of the velocity boundary layers is wider than the gap between the cylinders. Consequently, the thermal boundary layer will decrease due
to reduce in the radial and tangential velocity components and this causes the fluid between the inner and outer surfaces behave similar to a solid insulating material.

Figure 5 Streamlines and isotherm pattern at (a) $Ra=10^3$, $\sigma=1$, (b) $Ra=10^3$, $\sigma=3$, (c) $Ra=10^5$, $\sigma=1$, and (d) $Ra=10^5$, $\sigma=5$ with volume fractions $\phi=0.02$.

The local Nusselt number distribution is one of the important parameters in the present study, since it can reflect the local heat transfer characteristics around the inner and outer cylinder. Hence, the local Nusselt number distribution at various $Ra$, $\sigma$ and $\phi$ for $Al_2O_3$ nanoparticle are presented in fig. 6 and 7. Figure6 illustrates how the addition of nanoparticles influences the Nu number distribution around inner and outer cylinder for different Ra numbers at various volume fractions of $Al_2O_3$ nanoparticles when $\sigma=0.5$. It is observed that Nu number increases with the increasing of $Ra$ number, as expected. It is due to elevation buoyancy forces, which augments thermal convective currents.

The effect of the annulus gap width ratio on the local Nu number along the inner and outer cylinder at $Ra=10^5$ and $\phi=2\%$ is shown in fig. 7. Over the range of annulus gap width ratio, the Nu number of the inner (outer) cylinder starts from a local minimum (maximum) at $\phi=0$ (top of the annulus) and then increases (decreases) to the point $\phi=180$ (at the bottom), indicating that the convection heat transfer is dominant. In the case of $\sigma=5$, because of the conduction effect, the local Nu curve shows no remarkable change around the inner cylinder and it is specially seen for the outer surface.
Figure 6 Local Nusselt number distribution around inner (left) and outer (right) cylinder at various volume fractions of $Al_2O_3$ nanoparticles at (a) $Ra=10^3$, (b) $Ra=10^4$ and (c) $Ra=10^5$ for $\sigma=0.5$.

For better discussion, the values of the averaged Nusselt numbers versus volume fractions of $Al_2O_3$—water nanofluid at different annulus gap width ratios for $Ra=10^3, 10^4$ and $10^5$ are shown in fig. 8 (a), (b) and (c), respectively. Also, the diagrams show that the mean $Nu$ number decreases with increasing the annulus gap width ratio. The maximum decrease in the mean $Nu$ numbers is seen for $Ra=10^5$. Increment of $Ra$ number strengthens the fluid motion and consequently affects the temperature field, creating a higher temperature gradient which leads to a higher $Nu$ number for all cases.
Figure 7 Local Nusselt number distribution around inner (left) and outer (right) cylinder at $Ra=10^5$ and $\phi=2\%$ for different annulus gap width ratio.

Figure 8 Average Nusselt number versus to volume fraction for $Al_2O_3$–water nanofluid at different annulus gap width ratio for (a) $Ra=10^3$, (b) $Ra=10^4$ and (c) $Ra=10^5$.

5. Conclusion

Natural convection flow utilizing nanofluids in a horizontal annulus enclosure was simulated numerically using the lattice Boltzmann method. The results are presented as Rayleigh numbers vary between $10^3$ to $10^5$ and the annulus gap width ratio ranges as $0.5 \leq \sigma \leq 5$ containing $Al_2O_3$ nanoparticle while solid volume fractions vary from 0% to 4%. The conclusion of the present study can be summarized as follows:
I. It is found that LBM is a suitable approach for simulating natural convection using nanofluid in the geometry which includes curved boundaries. The simple implementation of efficient thermal conductivity is the most preference of this method.

II. As solid volume fraction increases in the annulus, the $Nu$ number at the both outer and inner cylinder increases.

III. Results show that the mean $Nu$ number decreases with increasing the annulus gap width ratio at the same $Ra$ number.

IV. By increasing solid volume fraction, the results show a heat transfer enhancement at any Rayleigh number.

V. In all the cases considered volume fraction and gap-width ratio, the mean $Nu$ number augments with increasing the $Ra$ number.

References


<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Greek letters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_L$ Lattice speed</td>
<td></td>
</tr>
<tr>
<td>$C_p$ Specific heat capacity</td>
<td>$\tau$ Relaxation time</td>
</tr>
<tr>
<td>$C_s$ Speed of sound</td>
<td>$\phi$ Volume fraction</td>
</tr>
<tr>
<td>$d_P$ Diameter of solid particles</td>
<td>$v$ kinematic viscosity</td>
</tr>
<tr>
<td>$F$ External force</td>
<td>$w$ Weight function</td>
</tr>
<tr>
<td>$f$ Distribution function for flow field</td>
<td>$\beta$ Thermal expansion coefficient</td>
</tr>
<tr>
<td>$g$ Distribution function for temperature</td>
<td>$\mu$ Viscosity</td>
</tr>
<tr>
<td>$g_Y$ Acceleration due to gravity</td>
<td>$\alpha$ Thermal diffusivity</td>
</tr>
<tr>
<td>$k$ Thermal conductivity</td>
<td>$\varphi$ Tangential direction</td>
</tr>
<tr>
<td>$Pr$ Prandtl number</td>
<td>$\theta$ Dimensionless temperature</td>
</tr>
<tr>
<td>$Ra$ Rayleigh number</td>
<td>$\rho$ Fluid density</td>
</tr>
<tr>
<td>$Nu$ Local Nusselt number</td>
<td></td>
</tr>
<tr>
<td>$Nu_i$ Inner Local Nusselt number</td>
<td>$ave$ Average</td>
</tr>
<tr>
<td>$Nu_o$ Outer Local Nusselt number</td>
<td>$c$ Low temperature</td>
</tr>
<tr>
<td>$\bar{Nu}$ Mean Nusselt number</td>
<td>$f$ Fluid</td>
</tr>
<tr>
<td>$u$ Velocity in the x-direction</td>
<td>$h$ High temperature</td>
</tr>
<tr>
<td>$v$ Velocity in the y-direction</td>
<td>$i$ Direction of single-particle</td>
</tr>
<tr>
<td>$P$ Pressure</td>
<td>$l$ local</td>
</tr>
<tr>
<td>$T$ Macroscopic temperature</td>
<td>$nf$ Nanofluid</td>
</tr>
<tr>
<td>$p$ Particle</td>
<td></td>
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<td>$eq$ Equilibrium</td>
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