UNSTEADY FLOW AND LOAD IN 50%
PARTIAL ADMISSION CONTROL STAGE WITH
DIFFERENT ADMISSION ARC DISTRIBUTIONS

by

Ke-Ke GAO, Shun-Sen WANG*, and Dong-Bo SHI

School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an, China

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Full 3-D unsteady numerical investigation on an axial air turbine in 50% partial
admission is conducted. The partial admission turbines are under different un-
steady loading and unloading process, as well as flow parameters, respectively.
The loss coefficient and static pressure distributions at the key position are pre-
sented in detail to analyze the non-uniformity originated from partial admission.
The results show that the non-uniformity decreases along flow direction and the
efficiency of control stage also decreases but with the uniformity improved down-
stream of the rotors with increasing admitting numbers in equal partial admission
degree. The reasons for efficiency decreasing are reasonably explained with
windage and sector end losses presented by static entropy distributions. The per-
odic changes of unsteady forces in amplitude and direction are also compared and
transformed in the frequency domain by fast Fourier transform method. The larg-
est circumferential exciting force factor which is remarkably larger than the cor-
responding axial exciting force factor decreases by 13.2% with the increase of
admitting arc number. Compared with the common distribution of two symmetric
admitting arcs, the maximum exciting force factor of triangle admitting arc dis-
tribution drops 11.3% with the mere efficiency decrease of 1.32%. The multiple a-
mitting arc turbines are more conducive to be applied to submarines which con-
cerns more about exciting force other than efficiency. Efficiency and unsteady
forces are both worth being taken into consideration in the practical applications.

Key words: unsteady, partial admission, flow parameters, exciting force

Introduction
Extremely strong unsteady phenomenon is existed in control stage, especially for
partial admission turbine. The partial admission turbine is necessary to keep dimensions large
through fewer inlet annulus so that the turbine efficiency can be effectively improved. Mean-
while, the rotor blades bear loading and unloading process dramatically owning to partial ad-
mission. Therefore, the investigation of unsteady influence on partial admission turbine is vi-
tal to keep safe and effective operation.

The partial admission turbine performance of impulse design was addressed by
Ohlsson [1] through theoretical method. Simplifications including incompressibility, friction-
less parameters and no leakage were applied. The 2-D numerical investigation by He [2]
pointed out that the enhancing mixing loss can have a positive effect on the overall turbine
performance. The unsteady flow characteristics must be considered to investigate the nature

* Corresponding author; e-mail: sswang@mail.xjtu.edu.cn
of partial admission turbine [3]. The 3-D unsteady performance research with different admission degrees was performed by Xie et al. [4]. The unsteady force relationship of different admission degrees has been revealed. The different numerical models of axial turbine with a low reaction degree were compared by Hushmandi [5]. The 3-D influence on flow parameters was obvious comparing to 2-D model. Experimental investigation on partial admission effects were conducted by Cho et al. [6, 7]. The inhomogeneous flow attenuation and structure in 4-stage turbine was shown by experimental measure and reported by Bohn [8]. Fridh et al. [9] conducted experimental research in partial admission air turbine and the comparison with 2-D compressible model demonstrated the importance of radial components.

The comparable research on the unsteady flow parameters of four vital types of different admitting arc distributions in 50% partial admission has been conducted comprehensively and thoroughly. The loss parameters, efficiency variation, traverse pressure, and operating forces are analyzed in detail to obtain the relationship with the admitting arcs.

Computational geometry and flow conditions

The single stage axial turbine is investigated using ideal air as working fluid in the paper. The air turbine includes 40 straight stators and 65 straight rotors over the full annulus. The blocked arcs with different arc length are included to simulate the unsteady flow of four types of partial admission. The 50% partial admission turbine geometries shown in fig. 1 are achieved by the blocked arcs that are single or evenly divided with optimized admitting distributions. The single blocked arc which is placed upstream of the stator’s leading edge occupies half of whole annulus in case 1. The two blocked arcs are symmetrically arranged upstream of stator’s leading edge in case 2. The three blocked arcs are placed upstream of the stator’s leading edge with triangle distribution in case 3. The four blocked arcs are also symmetrical arranged upstream of stator’s leading edge in case 4. The grids near the wall are refined to obtain accurate flow parameters. Furthermore, the grid independence verification has been done in previous articles to utilize the computational resources effectively [4].

Governing equations and boundary conditions

The equations governing the fluid dynamics are based on the conservation equations of mass, momentum, and energy. The momentum theorem can be expressed:

$$\delta F = \delta m \frac{d\vec{V}}{dt}$$

Considering the gravity and stress tensor, infinitesimal control volume force can be obtained through eq. (2), namely:

$$\delta F = \rho \hat{g} \delta x \delta y \delta z + \nabla \Pi \delta x \delta y \delta z$$

According to Reynolds transport theorem, eq. (3) can be derived, where the speed variation is decomposed into the variation with time and space:
By substituting the eqs. (2) and (3) into eq. (1), the following general form for moment equation can be obtained, namely:

$$\rho \left( \frac{\partial \vec{V}}{\partial t} + u \frac{\partial \vec{V}}{\partial x} + v \frac{\partial \vec{V}}{\partial y} + w \frac{\partial \vec{V}}{\partial z} \right) = \rho \vec{g} + \nabla \vec{\Pi}_j$$

(4)

$$\vec{\Pi}_j = -\rho \delta_{ij} + \mu \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right]$$

(5)

where $\delta_{ij}$ is Kronecker delta function (if $i = j$, $\delta_{ij} = 1$, else $\delta_{ij} = 0$).

Turbulence modeling is the process in which the turbulence stress term solutions in Reynolds equations are closed. Random number generation k-ε turbulence model which is validated to have a good agreement with experiment by Hushmandi [5] is chosen as an acceptable accuracy and reasonable amount of calculation time for the investigation in the paper.

$$\frac{\partial}{\partial t} (\rho k) + \frac{\partial}{\partial x_j} (\rho u_j) = \frac{\partial}{\partial x_j} \left( \alpha_e \mu_{eff} \frac{\partial k}{\partial x_j} \right) + G_k - \rho \varepsilon - Y_M$$

(6)

$$\frac{\partial}{\partial t} (\rho \varepsilon) + \frac{\partial}{\partial x_j} (\rho u_j \varepsilon) = \frac{\partial}{\partial x_j} \left( \alpha_e \mu_{eff} \frac{\partial \varepsilon}{\partial x_j} \right) + \frac{C_\mu}{k} (G_k + C_{\mu \kappa} G_\kappa) - C_{\varepsilon 2} \rho \frac{\varepsilon^2}{k} - R_\varepsilon$$

(7)

where $G_k$ represents the production of turbulence kinetic energy. The compressibility effect can be explained through a dilatation dissipation term $Y_M$, namely $Y_M = 2 \rho \varepsilon M_r^2$.

The term in the transport equation $R_\varepsilon$ can be expressed:

$$R_\varepsilon = \frac{C_\mu \rho \eta^3 \left( 1 - \frac{\eta}{\eta_0} \right)}{1 + \beta \eta^2} \frac{\varepsilon^2}{k}$$

(8)

The transient frozen rotor method is applied to the interface between stators and rotors. High accuracy discretization schemes presented by Barth et al. [10] are applied to the convective term discretization. Virtual and true time step are introduced to the time discretization to avoid the restriction of time step length in explicit scheme and the complicated transformation of implicit scheme [11]. Thus, we have:

$$\frac{U^{q+1} - U^q}{\Delta \tau} + \frac{3U^q - U^{q-1}}{2\Delta \tau} = -R(U^q)$$

(9)

The boundary conditions of four types of admitting arcs distributions in 50% partial admission are similar. Constant inlet total pressure (130 kPa), inlet total temperature (323.15 K), outlet static pressure (97 kPa), rotation speed (5000 rpm), and 5% inlet turbulent intensity are applied for investigations. The time step is set to 0.00001 seconds, which means that 30 steps are needed to pass through a stator passage. Hence 1200 steps are used to cover one revolution of the rotor. The unsteady investigation throughout entire revolution is conducted to capture accuracy results.
Results and discussion

Loss coefficient

In this paper, the loss coefficient in which entropy rise is used to characterize stage loss is defined by [12]:

\[ \zeta = \frac{T_{s,\text{out}} \Delta \varsigma}{h_{\text{out}} - h_{\text{out}}} \]  

(10)

where \( \Delta \varsigma \) means the entropy production and \( T_{s,\text{out}} \) is the static temperature of outlet.

The loss coefficient upstream of stator with four distributions is plotted in fig. 2. The appropriate ranges of loss contours are provided to increase the clarity of results. It is clear that the loss coefficient is affected most strongly for the partial admission with one blockage owing to the stagnation flow. The loss coefficient decreases with the increasing blocked arcs at the position.

The loss coefficient at the cross-section between stator and rotor is plotted in fig. 3. The main loss also exists in the blocked arcs. The loss coefficient of the turbine with four small blocked arcs is lower than the other three blocked arc model comparably at the position. The influence of stagnation flow on the control stage with single blockage is much greater than others.

Figure 2. Loss coefficient at a cross-section upstream of stator with different admitting distributions (for color image see journal web-site)

Figure 3. Loss coefficient at a cross-section between stator and rotor with different admitting distributions (for color image see journal web-site)

The loss coefficient at the cross-section downstream of rotor is plotted in fig. 4. It is easy to find that the loss region influenced by partial admission increases with the augmentation of blocked arcs. The blocked region of case 1 is most seriously affected by partial admission but with a relatively smaller region. Only control stage efficiency considered, case 1 can operate better due to the smallest affected region. But the phenomenon that the uniformity of exit flow is strengthened with the blocked arcs increasing can be beneficial for the stage downstream, relatively.

Traverse static entropy, pressure, and efficiency

Additional aerodynamic losses are generated by partial admission, blade windage...
and sector end losses, respectively. The loss development process can be easily identified from traverse static entropy distribution plotted in fig. 5. The rotor blades corresponding to blocked arcs act as the role of fan resulting in windage loss. The sector end loss occurs in the transitional region when the rotors pass through the admitting arcs from blockage and through blockage from admitting arcs. The empty of rotor passages and extremely unsteady flow from the last stator generate losses, respectively. The sector end loss is in direct proportional to admitting arc number while windage loss is influenced less according to equations [13].

The windage loss is expressed by:

\[ X_{\text{wind}} \propto \left(1 - e\right) \frac{U^3}{C_o} \]  

(11)

The sector end loss is:

\[ X_{\text{segm}} \propto \eta_{bl} \left(\frac{U}{C_o}\right) N_{\text{segm}} \]  

(12)

The results of the static efficiency and total efficiency varying with the number of blocked arcs are shown in fig. 6. It is easy to conclude that the increasing number of blocked arcs reduces the efficiency of control stage in equal admitting degree. The largest efficiency difference occurs between single blockage and two-symmetric blockage. It is worth pointing out that the influence generated by the number of blocked arcs on efficiency gradually decreases with the increasing number of blocked arcs. The total efficiency and the static efficiency of triangle distribution are 1.40%, 0.9% lower than two-symmetric distribution, respectively.

The circumferential static pressure distributions are presented at three cross-sections in the selected two kinds of 50% partial admission turbine in fig. 7. All the static pressure values are normalized with inlet total pressure. Large static pressure drop is found...
when the rotor passes through the blocked arcs upstream of stator and the position between stator and rotor. The static pressure rises rapidly at the entrance to admitting arcs. The non-uniformity originated from the partial admission decreases along the expansion direction, relatively.

**Rotor blade forces**

The axial and circumferential forces of rotor blades traveling along the circumference in the two selected kinds of 50% partial admission turbine are presented in fig. 8. The rotor blades experience unsteady loading including variation of magnitude and direction due to partial admission. The rotor blades are also under the periodic axial and circumferential force approximately in the admitting arcs due to the stator wakes. The circumferential force rises suddenly at the entrance to blocked arcs and then decreases rapidly owing to the stagnation fluid while the axial force decreases directly.

![Figure 8. Transients of circumferential and axial force at rotor blade; (a) case 1, (b) case 3](image)

The fast Fourier transform plays an extremely important role in comparing the frequency components of exciting force. The spectral function is defined as:

$$p_k = \frac{1}{2\pi} \int_0^{2\pi} x(t)e^{-ikt} \, dt$$

The exciting force factor is defined:

$$S_k = \frac{p_k}{\bar{x}}$$

The low frequency exciting force connected with the admitting arcs numbers and rotational speed closely can be defined as eq. (15), for the spacing distribution of admitting arcs:

$$f_k = kN \frac{n}{60}$$

The axial and circumferential forces of case 3 are plotted in fig. 9 as demonstration. The largest exciting force magnitude occurs at the frequency corresponding to the number of blocked arcs and rotation speed. The axial and circumferential exciting forces of 50% partial admission turbine with three blocked arcs are 0.91 N and 2.13 N, respectively. The circumferential exciting force is much more serious than axial direction. The high frequency vibrations caused by stator wakes are also visible although the amplitudes are far less than that of low frequency.
The exciting force factor plotted in fig. 10 is applied to conduct reasonable comparison according to eq. (14). The axial exciting factors increase mainly due to the decreasing average axial force originated from reduction of the pressure gradient before and after rotor blades with the number of admitting arcs in 50% partial admission. The circumferential exciting force factors have declined in general, 0.319, 0.309, 0.274, and 0.277, respectively. The largest decrease of circumferential force factors occurs between two symmetric distribution and triangle distribution. Compared to the common distribution of two symmetric admitting arcs, the maximum exciting force factor of triangle admitting arc distribution drops 11.3%. The largest increase in axial force factor appears at transition from the triangle distribution to four-symmetric distribution while the axial force remains low. And the frequency of vibrations has multiplied compared to that of single admitting arc, 81.7 Hz, 166.7 Hz, 249.9 Hz, and 333.3 Hz, respectively. The decrease of circumferential exciting force factors has important application value on special occasions. The frequency improvement is of benefit to avoid the low natural frequency of turbine in the same partial degree.

Conclusions

Full 3-D unsteady numerical investigations of four types of 50% partial admission distributions have been conducted in the paper. The in-depth comparative study of unsteady aerodynamic parameters in the four partial admission turbines has been performed.

Firstly, the loss coefficients at the position upstream of the stators, the interface of rotors and stators and downstream of the rotors are plotted in detail in the paper, respectively. It can be found that the influence of partial admission is weakened from upstream to downstream passage. The exit flow of control stage with single blockage is under the largest non-uniformity though operating with highest control stage efficiency. The additional losses origi-
inated from partial admission increase with the number of admitting arcs and the decrease of efficiency remains low beginning with symmetrical two admitting arcs.

Secondly, the largest axial and circumferential exciting forces occur at the first order low frequency in the four 50% partial admission. The largest amplitude of circumferential exciting forces which is remarkably larger than the corresponding axial exciting forces decreases by 13.2% with the increasing number of admitting arcs. Compared to the common distribution of two symmetric admitting arcs, the maximum exciting force factor of triangle admitting arc distribution drops 11.3% with the mere efficiency decrease of 1.32%. The frequency of exciting forces has multiplied with the increasing number of admitting arcs compared with that of single admitting arcs. The equal partial admission turbine with more admitting arcs is beneficial to decrease overall exciting force to guarantee the safe operation. And the frequency improvement along with the increasing number of admitting arcs can be used to avoid the low natural frequency of turbine in the same partial admission degree.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tbody>
<tr>
<td>$C_\text{avg}$</td>
<td>Spouting velocity</td>
<td>$\text{m s}^{-1}$</td>
</tr>
<tr>
<td>$C_{1,2}$</td>
<td>Model constant</td>
<td>[-]</td>
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<tr>
<td>$C_{3,4}$</td>
<td>Variables which determine how dissipation rate is affected by buoyancy</td>
<td>[-]</td>
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<tr>
<td>$C_\theta$</td>
<td>Modified constant</td>
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</tr>
<tr>
<td>$e$</td>
<td>Partial admission degree</td>
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<tr>
<td>$F$</td>
<td>Force</td>
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<td>$f$</td>
<td>Frequency</td>
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<tr>
<td>$G_b$</td>
<td>Generation of turbulence due to buoyancy</td>
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<tr>
<td>$h$</td>
<td>Enthalpy</td>
<td>$\text{kJ kg}^{-1}$</td>
</tr>
<tr>
<td>$k$</td>
<td>Kinetic energy of turbulence</td>
<td>$\text{kJ}$</td>
</tr>
<tr>
<td>$M_t$</td>
<td>Turbulent Mach number</td>
<td>[-]</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
<td>$\text{kg}$</td>
</tr>
<tr>
<td>$N$</td>
<td>The number of admitting arcs</td>
<td>[-]</td>
</tr>
<tr>
<td>$n$</td>
<td>Rotation speed</td>
<td>$\text{min}^{-1}$</td>
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<td>$p$</td>
<td>Exciting force</td>
<td>$\text{N}$</td>
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<tr>
<td>$R$</td>
<td>Discrete function</td>
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<td>$S$</td>
<td>Exciting force factor</td>
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<td>$s$</td>
<td>Static entropy</td>
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<tr>
<td>$T$</td>
<td>Temperature</td>
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<tr>
<td>$t$</td>
<td>Time</td>
<td>$\text{s}$</td>
</tr>
<tr>
<td>$U$</td>
<td>Circumferential velocity</td>
<td>$\text{m s}^{-1}$</td>
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<tr>
<td>$u, v, w$</td>
<td>Velocity components in a generalized co-ordinate system</td>
<td>$\text{m s}^{-1}$</td>
</tr>
<tr>
<td>$\bar{V}$</td>
<td>Velocity</td>
<td>$\text{m s}^{-1}$</td>
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<tr>
<td>$X$</td>
<td>Loss factor</td>
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<tr>
<td>$\tau$</td>
<td>Average force</td>
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**Greek symbols**

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<th>Symbol</th>
<th>Description</th>
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<td>$\alpha_k$</td>
<td>Inverse effective Prandtl numbers</td>
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<td>$\beta$</td>
<td>Coefficient of thermal expansion</td>
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<td>$\epsilon$</td>
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<td>Efficiency</td>
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<tr>
<td>$\mu_{\text{eff}}$</td>
<td>Effective dynamic viscosity coefficient</td>
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<tr>
<td>$\Pi_i$</td>
<td>Stress tensor</td>
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<tr>
<td>$\rho$</td>
<td>Density</td>
<td>$\text{kg m}^{-3}$</td>
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<tr>
<td>$\tau$</td>
<td>Virtual time</td>
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**Subscripts**

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<td>in</td>
<td>Inlet</td>
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</tr>
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<td>k</td>
<td>Order of exciting force</td>
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**Superscripts**

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<td>Discrete points serial number</td>
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