AN ANALYSIS OF HEAT CONDUCTION IN POLAR BEAR HAIRS USING ONE-DIMENSIONAL FRACTIONAL MODEL

by

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Hairs of a polar bear are of superior properties such as the excellent thermal protection. The polar bears can perennially live in an extremely cold environment and can maintain body temperature at around 37 °C. Why do polar bears can resist such cold environment? Its membrane-pore structure plays an important role. In the previous work, we established a 1-D fractional heat conduction equation to reveal the hidden mechanism for the hairs. In this paper, we further discuss solutions and parameters of the equation established and analyze heat conduction in polar bear hairs.

Key words: *heat conduction, fractional derivative, polar bear, membrane-pore structure, fractional model*

Introduction

Polar bears have a special ability to survive in the cold areas of the Arctic. Studies have found that the polar bear hair plays an important role in protection against the bitter cold. The structure of polar bear hair is completely different from the structure of other mammals' hair, figs. 1 and 2. Polar bear hairs possess membrane-pore structure, figs. 3 and 4 [1]. The unique membrane-pore structure makes polar bear hair not only can maintain body temperature, but also can absorb energy from surrounding environment [2]. This is the real reason that the polar bear can perennially live in environment of –45 °C to –50 °C and to maintain body temperature at around 37 °C.

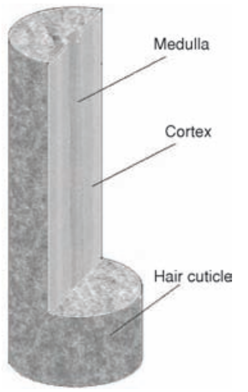

Figure 1. Structure diagram of other mammals' hair

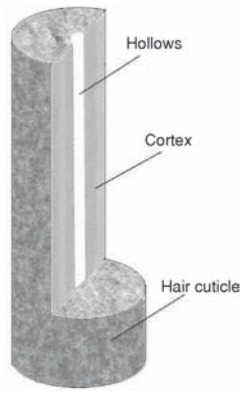

Figure 2. Sturcture diagram of polar bear hair

In the previous work [3], we approximately considered the structure of polar bear hair as fractal space and established 1-D heat conduction equation with the fractional order derivative,

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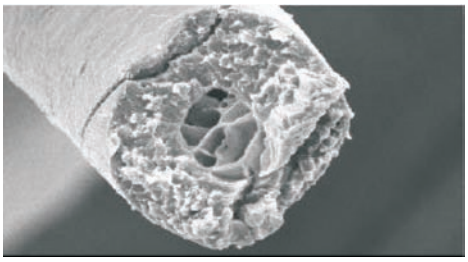

Figure 3. Scanning electron micrographs of polar bear hair-transverse sections

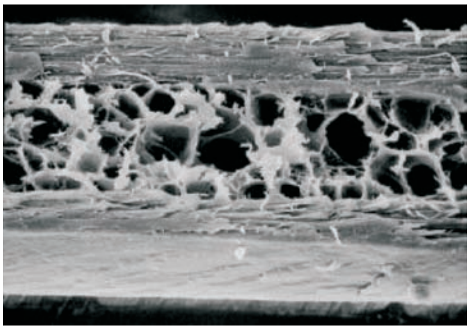

Figure 4. Scanning electron micrographs of polar bear hair-longitudinal sections

but solutions and parameters of the equation ware not discussed. In this paper we further discuss solutions and parameters of the equation and analyze heat conduction in polar bear hairs.

The 1-D heat conduction fractional equation of the polar bear hair and its solutions

The 1-D heat conduction fractional equation of polar bear hair is written [3]:

$$\frac{\partial T}{\partial t} = \frac{\partial^\alpha}{\partial x^\alpha}\left(D\frac{\partial^\alpha T}{\partial x^\alpha}\right), \quad x \in (0, L), \quad t > 0 \tag{1}$$

where $T(x, t)$ is the temperature at the point x and moment t, D – the thermal diffusivity, L – the length of the polar bear hair, α – the fractional dimensions of the polar bear hair, and $\partial^\alpha/\partial x^\alpha$ – He's fractional derivative defined [4]:

$$\frac{\partial T^\alpha}{\partial x^\alpha} = \frac{1}{\Gamma(n-\alpha)}\frac{\mathrm{d}^n}{\mathrm{d}x^n}\int_{t_0}^{t}(s-x)^{n-\alpha-1}[T_0(s) - T(s)]\mathrm{d}s \tag{2}$$

where $T_0(x)$ can be the solution of its continuous partner of the problem with the same boundary/initial conditions of the fractal partner.

The polar bear's body temperature is about 37 °C and the environment temperature can be as low as –50 °C, we, therefore, have the following boundary conditions:

$$T(0, t) = 37\ ^\circ\text{C}, \quad T(L, t) = -50\ ^\circ\text{C} \tag{3}$$

By the fractional complex transformation [5-11]:

$$s = \frac{x^\alpha}{\Gamma(1+\alpha)} \tag{4}$$

Equation (1) is converted to a partial differential equation, which reads:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial s}\left(D\frac{\partial T}{\partial s}\right) \tag{5}$$

In order to search for wave solutions of eq. (5), we can introduce the traveling wave transformation:

$$T(s, t) = T(\xi), \quad \xi = s - kt = \frac{x^\alpha}{\Gamma(1+\alpha)} - kt \tag{6}$$

where k denotes vibration wave velocity of the polar bear hair and t denotes time. Substituting eq. (6) into eq. (5), eq. (5) becomes:

$$k\frac{\partial T}{\partial \xi} + \frac{\partial}{\partial \xi}\left(D\frac{\partial T}{\partial \xi}\right) = 0 \tag{7}$$

Solving eq. (7) by integration, we obtain:

$$T(\xi) \quad c_1 \quad c_2 \exp \frac{k\xi}{D} \tag{8}$$

where c_1 and c_2 are integral constants. Incorporating the boundary conditions, eq. (3), we finally obtain solution of eq. (1), which is written:

$$T(x,t) \quad 37\exp \frac{2kL^{\alpha}}{D\Gamma(\alpha)\alpha} \quad 13\exp \frac{kL^{\alpha}}{D\Gamma(\alpha)\alpha} \quad 50 \quad 87\exp \frac{kL^{\alpha} \quad kx^{\alpha}}{D\Gamma(\alpha)\alpha}$$

$$87\exp \frac{kx^{\alpha}}{D\Gamma(\alpha)\alpha} \quad \exp \frac{kL^{\alpha}}{D\Gamma(\alpha)\alpha} \quad 1^{\quad 2} \tag{9}$$

An analysis of heat conduction in polar bear hairs

In eq. (9), $T(x, t)$ is the temperature at the point x and moment t, D – the thermal diffusivity, L – the length of the polar bear hair, α – the fractional dimensions of the polar bear hair, k – vibration wave velocity of the polar bear hair, and $\Gamma(\alpha)$ denotes gamma function. The reference [12] pointed out that $\alpha = 1.625$. We gained numerical values of D, L, k by actual measurement. The results are:

– When the temperature changes from 233.75 K to 310.76 K, namely from –40.6 °C to 37.61 °C, k changes from 480 m/s to 1162 m/s.
– When the temperature changes from 233.75 K to 310.76 K, namely from –40.6 °C to 37.61 °C, D changes from $0.236 \cdot 10^{-8}$ m^2/s to $4.614 \cdot 10^{-8}$ m^2/s.
– The length of the polar bear hair is usually 6 ~ 7 cm, namely 0.06 ~ 0.07 m.

By observing eq. (9), we find that it does not contain parameter t, in other words, parameter t does not exert an influence on temperature at the point x in polar bear hair. In the following, under the condition that temperature is 273.15 K, we consider change in temperature of the polar bear with the change of length of the polar bear hair. In the case that external temperature is 273.15 K, values of D, L, k are:

$$D = 0.2425 \text{ m}^2/\text{s}, \quad L = 0.065 \text{ m}, \quad k = 852 \text{ m/s} \tag{10}$$

In addition:

$$\alpha = 1.625 \tag{11}$$

Substituting eqs. (10) and (11) in eq. (9) with the help of MAPLE software results in:

$$T(x,t) \quad 37 \quad 1.88 \quad 10^{\ 23} \exp(2412x^{13/8} \quad 28.4) \quad 1.88 \quad 10^{\ 23} \exp(2412x^{13/8}) \tag{12}$$

Using MAPLE software, we draw the graph of eq. (12), namely fig. 5.

From fig. 5, we obtain:

– The temperature of polar bear hair $T(x, t)$ is 36.54 °C at the point $x = 0$ m, namely, the surface temperature of body of polar bear hair is 36.54 °C. The temperature can be maintained from $x = 0$ m to $x = 0.057$ m.
– The temperature of polar bear hair $T(x, t)$ is –50 °C (environment temperature) at the point $x = 0.065$ m.
– From $x = 0.057$ m to $x = 0.065$ m, the temperature of polar bear hair $T(x, t)$ has a fast reduction from 36.54 °C to –50 °C.

On the base of the previous analysis, we find that body surface temperature of polar bear can be maintained from $x = 0$ m to $x = 0.057$ and the heat of polar bear is not lost. Accord-

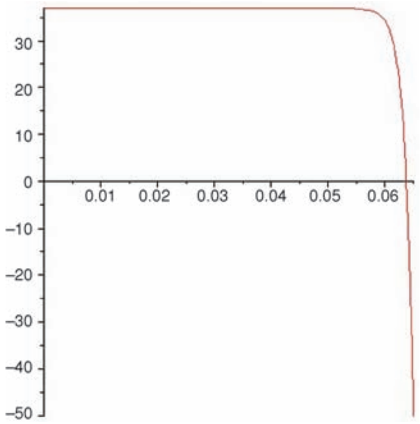

Figure 5. The graph of change in temperature of the polar bear with the change of length of the polar bear hair

ing to usual theory, the heat of polar bear goes to the surroundings, but in fact the opposite is true. We speculate that the membrane-pore structure of polar bear hair plays an important role in keeping body temperature.

Conclusions

The polar bear hairs have unique membrane-pore structure. The structure enables the animal live in extremely cold climate as low as –50 °C. On the base of previous work, we further discuss solutions and parameters of the equation established and analyze heat conduction in polar bear hairs. The main results are: body surface temperature of polar bear can be maintained from $x = 0$ m to $x = 0.057$ m and the heat of polar bear is not lost; the membrane-pore structure of polar bear hair plays an important role in keeping body temperature.

Acknowledgment

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