CHARACTERISTIC EQUATION METHOD FOR FRACTAL HEAT-TRANSFER PROBLEM VIA LOCAL FRACTIONAL CALCULUS

by

Geng-Yuan LIU$^{a,b,*}$

$^a$ State Key Joint Laboratory of Environment Simulation and Pollution Control, School of Environment, Beijing Normal University, Beijing, China

$^b$ Beijing Engineering Research Center for Watershed Environmental Restoration & Integrated Ecological Regulation, Beijing, China

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In this paper the fractal heat-transfer problem described by the theory of local fractional calculus is considered. The non-differentiable-type solution of the heat-transfer equation is obtained. The characteristic equation method is proposed as a powerful technology to illustrate the analytical solution of the partial differential equation in fractal heat transfer.

Key words: heat-transfer equation, analytical solution, local fractional calculus, characteristic equation method

Introduction

The differential equations involving the local fractional calculus [1] were utilized to investigate the non-differentiable problems, e.g., fractal diffusions [2-7], fractal oscillator [8], fractal wave [9], fractal Laplace [10, 11], fractal heat-conduction [12, 13], fractal Fokker-Planck [14], fractal Helmholtz [15] equations and others [16, 17]. Let us recall the local fractional derivative (LFD) of the function $\Pi(\zeta)$ of order $0<\theta<1$ at $\zeta=\zeta_0$, defined by [10-18]:

$$
D_\zeta^{(\theta)}\Pi(\zeta_0) = \frac{d^{\theta}\Pi(\zeta)}{d\zeta^\theta} \bigg|_{\zeta=\zeta_0} = \lim_{\zeta \to \zeta_0} \frac{\Delta^{\theta}[\Pi(\zeta)-\Pi(\zeta_0)]}{(\zeta-\zeta_0)^\theta}
$$

(1)

where

$$
\Delta^{\theta}[\Pi(\zeta) - \Pi(\zeta_0)] = \Gamma(1+\theta)\Delta[\Pi(\zeta) - \Pi(\zeta_0)]
$$

The LFD of the function $E_\theta(k\zeta^\theta)(k \in \mathbb{R})$ was [1]:

$$
\frac{d^{\theta}E_\theta(k\zeta^\theta)}{d\zeta^\theta} = kE_\theta(k\zeta^\theta)
$$

(2)

The heat-transfer equation involving the LFD in fractal media was written [18]:

$$
\frac{\partial^{\theta}\Omega(\theta, \tau)}{\partial\tau^\theta} + \kappa \frac{\partial^{2\theta}\Omega(\theta, \tau)}{\partial\theta^{2\theta}} + \omega\Omega(\theta, \tau) = 0
$$

(3)

* Author’s e-mail: dliugengyuan@163.com
where $\kappa$ is a heat-diffusive coefficient and $\omega$ – a constant related to the density and specific heat of fractal materials.

There are a lot of numerical and analytical methods for the local fractional partial differential equations, such as the decomposition method [2, 4, 15], differential transform [3], variational iteration method [5, 12, 14], homotopy perturbation method [6], similarity variable method [7], Laplace variational iteration method [9], series expansion method [10], function decomposition method [11], Fourier transform [13], exp-function method [16], Fourier transform [17], and characteristic equation method (CEM) [19]. The main aim of this paper is to present the CEM to solve the heat-transfer equation in fractal media.

**Solve the heat-transfer equation in fractal media**

By using the theory of CEM [19], we set the non-differentiable solution of eq. (3):

$$\Omega(\theta, \tau) = E_0(\rho \tau^\theta)E_0(\sigma \phi^\theta)$$

(4)

In view of eq. (4), we have:

$$\rho + \kappa \sigma^2 + \omega = 0$$

(5)

such that

$$\Omega(\theta, \tau) = \sigma E_0[-(\kappa \sigma^2 + \omega) \tau^\theta]E_0(\sigma \phi^\theta)$$

(6)

where $\kappa$ is a heat-diffusive coefficient, $\sigma$ – a constant, and the corresponding graph is represented in fig. 1.

By changing the dimension from $\theta = \nu (0 < \nu < 1)$ to 1, the conventional heat-transfer equation is written:

$$\frac{\partial \Omega(\theta, \tau)}{\partial \tau} + \kappa \frac{\partial^2 \Omega(\theta, \tau)}{\partial \sigma^2} + \omega \Omega(\theta, \tau) = 0$$

(7)

Then, we obtain:

$$\Omega(\theta, \tau) = \sigma \exp[-(\kappa \sigma^2 + \omega) \tau] \exp(\sigma \phi)$$

(8)

where $\kappa$ is a heat-diffusive coefficient and $\sigma$ – a constant.

Equation (8) represents the heat-transfer equation to account for the radiative loss of heat. The corresponding solutions are illustrated in fig. 2.

**Conclusion**

The fractal heat-transfer problem involving the LFD has been investigated in the work. The non-differentiable solution for
the heat-transfer equation in fractal media was obtained by using the CEM. The results for the fractal and conventional heat-transfer equations were compared. The obtained result is very efficient to show the fractal behaviour of heat transfer.

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Nomenclature

\( \theta \) – fractal order, [-]
\( \vartheta \) – space co-ordinate, [m]
\( \Omega(\vartheta, \tau) \) – temperature, [Km\(^{-3}\)]
\( \tau \) – time, [s]

References


