TURBULENCE INVESTIGATION OF THE NASA COMMON RESEARCH MODEL WING TIP VORTEX

by

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The paper presents high-speed stereo particle image velocimetry investigation of the NASA Common Research Model wing tip vortex. A three-percent scaled semi-span model, without nacelle and pylon, was tested in the 32- by 48-inch In-draft tunnel, at the Fluid Mechanics Laboratory at the NASA Ames Research Center. Turbulence investigation of the wing tip vortex is presented. Measurements of the wing-tip vortex were performed in a vertical cross-stream plane three tip-chords downstream of the wing tip trailing edge with a 2 kHz sampling rate. Experimental data are analyzed in the invariant anisotropy maps for three various angles of attack (0°, 2°, and 4°) and the same speed generated in the tunnel ($V_\infty = 50$ m/s). This corresponds to a chord Reynolds number $2.68 \times 10^5$, where the chord length of 3” is considered the characteristic length. The region of interest was $x = 220$ mm and $y = 90$ mm. The 20 000 particle image velocimetry samples were acquired at each condition. Velocity fields and turbulence statistics are given for all cases, as well as turbulence structure in the light of the invariant theory. Prediction of the wing tip vortices is still a challenge for the computational fluid dynamics codes due to significant pressure and velocity gradients.

Key words: turbulence, invariant maps, vortex, high-speed stereo particle image velocimetry, Common Research Model

Introduction

Experimental and numerical investigations of the NASA Common Research Model (CRM) still capture the attention of many researchers worldwide. It was developed as an open-source contemporary transonic supercritical wing for various studies in aerodynamics, as reported in [1]. Numerous experimental and numerical investigations have been performed since 2008 [1-14]. Literature overview of interest is provided in [11, 15].

Study of the wake of NASA CRM Model in European transonic wind tunnel (ETW), under high Re-number stall conditions for sub- and transonic speeds, with high-speed particle image velocimetry (HS PIV) is presented in [12]. Comparison of the NASA CRM test data in

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ETW to NASA test data obtained in the NASA Langley national transonic facility is given in [13]. Also, in [13] the force and moment, surface pressure and wing bending and twist data are presented.

**Experimental test rig**

Experiments were conducted in the 32- by 48-inch Indraft Tunnel, located at the NASA Ames research center Fluid mechanics laboratory (FML) (fig. 1). The tunnel test section is 10 ft. (3.048 m) long, 4 ft. (1219.2 mm) wide and 32 in. (app. 813 mm) high, and is transparent on two sides. A freestream turbulence level is 0.15%. Temperature, relative humidity and atmospheric pressure are measured while the tunnel is operational. A description of the FML 32" × 48" wind tunnel is provided in [16].

The NASA CRM model geometry and experimental results are available online in [15]. For these experiments the trip dots 0.0114" or 0.2896 mm in diameter at 2.54 mm spacing were applied to the wing at 10% of the chord away from the leading edge. This was performed after flow visualization experiments to induce transition and reduce the possibility of separation and sensitivity to the incoming flow. Figure 1(a) shows a diagram of the semi-span CRM model in the test section. Figure 1(b) displays the position of the cameras (outside the measurement area), while CRM and calibration target are inside the test section.

![Figure 1](a.png)  ![Figure 1](b.png)

**Figure 1.** (a) A 3% scaled semi-span Common Research Model (CRM) in wind tunnel: 1-left and 2-right CMOS cameras and 3-CRM, (b) 1 and 2-CMOS cameras, 3-CRM and 4-target

The CRM was mounted on the wind tunnel wall at the angles of attack (α): 0°, 2°, and 4°. The CRM dimensions and position in FML wind tunnel are specified in [11], as well as in fig. 2(a). The HSS PIV system calibration set-up is shown in figs. 1(b) and 2(a). Some details, concerning the region of interest (ROI), are presented in fig. 2(b). Measurements were taken in a vertical cross-stream plane three tip-chords downstream of the wing tip trailing edge, i. e. 9.6" or 243.84 mm, which corresponds to the origin of the coordinate system, where the z-axis is directed downstream with the fluid flow.

The height of the wing tip trailing edge from the test section bottom was checked for each angle of attack. It is assumed that only this dimension changes. The height varied from 427.1 mm for $\alpha = 0^\circ$, fig. 2(b), 411.2 mm for $\alpha = 2^\circ$ and 393.7 mm for $\alpha = 4^\circ$. 

An optimal HSS PIV arrangement was designed using test-planning visualization software, figs. 1(a) and 2(b) [17]. The HSS PIV experimental set-up is presented in fig. 1. A double-pulsed Nd:YLF laser (Quantronix Darwin-Duo) with 527 nm and average power greater than 100 W at 3 kHz illuminated the flow.

![Figure 2. CRM at test section; (a) HSS PIV calibration setup for angle of attack 2°: 1 – CRM, 2 – laser sheet inlet window, 3 – calibration target and 4 – window for CMOS cameras, (b) view from behind – position of the wing tip trailing edge point for angle of attack 0° and ROI (All dimensions are in millimeters)](image)

The operating frequency during experiments was 2 kHz with pulse spacing of the order of 10 μs. The laser sheet was shaped using spherical and cylindrical lenses and was directed to the ROI by a set of mirrors through a window (fig. 2(a)). The laser sheet thickness in the ROI was approximately 1 mm.

Two high-speed 12-bit CMOS cameras, Phantom v641s, were oriented to a nearly forward scatter arrangement from the opposite sides of the laser sheet (fig. 1). The cameras operated at 4 kHz with window size 1600 × 700 pixels. Two-axis Scheimpflug focusing was required to achieve perfect sharpness of the laser light sheet plane while using a large aperture in the lens, fig. 1(b). The obtained dimensions of ROI were $x = 220$ mm and $y = 90$ mm. Ten runs were obtained for each angle of attack and 2088 image pairs per camera for each run were recorded. Each run spans 1.044 s. The flow was seeded with 0.7 μm to 1 μm oil droplets that were injected just upstream of the tunnel inlet [11].

**Experimental results and analysis**

The wind tunnel speed was approximately the same for all measurements near $V_{∞} = 50$ m/s, or 49.9 m/s for $\alpha = 0°$ and $\alpha = 2°$, while 50.15 m/s for $\alpha = 4°$. A chord Reynolds number, calculated on the basis of the wing tip chord length (3.2", i. e. 81.28 mm) is $Re_C \approx 2.68 \cdot 10^5$.

Calibration, data acquisition and processing were performed using DaVis software by La Vision. Time-averaged velocity components ($V_i$, where $i = x, y$, and $z$) and averaged vorticity ($\Omega_i$, where $i = x, y$, and $z$) were determined on the basis of the measured instantaneous velocity fields. In addition, velocity fluctuations ($v_i$, where $i = x, y$, and $z$) and stresses were calculated on the basis of the measured velocity fields in DaVis software. Invariant maps and turbulent kinetic energy were determined by in-house code at the Faculty of Mechanical Engineering, University of Belgrade [11, 18, 19].
All three velocity components were reconstructed in the ROI in the following way. The stereo cross-correlation was applied using multi-pass iterations starting with the size of the interrogation window $96 \times 96$ pixels in one pass and 50% overlap down to three passes with a final window size of $32 \times 32$ pixels and 75% overlap. Median filtering was applied and smoothing at a window size of $3 \times 3$. All three velocities were accepted if stereo reconstruction error was less than 5 pixels. Missing vectors were interpolated. Exactly, 17 661 points with the step of 1.111 mm in the x-axis direction and 1.115 mm in the y-direction were generated in each data run [11, 14].

Velocity fields

The ensemble averaged total velocity fields ($V_e$) for all angles of attack are presented in [11]. Figure 3 presents the total velocity field for one run out of ten for each angle of attack.

Figure 3. Total velocity fields for angle of attack ($\alpha$); (a) 0°, (b) 2°, and (c) 4°. Note: Cross denotes the vortex core position (for color image see journal website)

One data set includes 2088 image pairs for each camera, which generates three-component velocity fields ($V_x$, $V_y$, and $V_z$). The ensemble averaged velocities ($V_{x,e}$, $V_{y,e}$, and $V_{z,e}$) are formed on the basis of ten completely independent, i.e., stochastic data sets (20 880 image pairs), as well as the ensemble averaged total velocity fields ($V_e$).
On the basis of averaged total velocity field and ensemble averaged total velocity field, it was shown that flow for all angles of attack could be treated as statistically stationary [11].

The vortex core is identified as the total velocity minimum in the vortex core region. Its position is \( x = -1.25123 \, \text{mm} \) and \( y = 13.8945 \, \text{mm} \) for \( \alpha = 0^\circ \) (fig. 3(a)). It is obvious that the wing tip vortex circulates counterclockwise as viewed from downstream. It is the direction opposite to that for a wing creating a positive lift [11]. The total velocity reached at this point is \( V = 47.86 \, \text{m/s} \), and although this is not the lowest velocity in the entire measured area, it is the lowest in the region of the vortex core. For \( \alpha = 2^\circ \) the vortex core has the co-ordinates \( x = -6.80899 \, \text{mm}, \ y = 11.6714 \, \text{mm} \), fig. 3(b), while for \( \alpha = 4^\circ \) they are \( x = -12.3667 \, \text{mm}, \ y = -2.77881 \, \text{mm} \) (fig. 3(c)). The achieved total velocity in the vortex core is \( V = 47.0561 \, \text{m/s} \) for \( \alpha = 2^\circ \), while it is \( V = 45.8836 \, \text{m/s} \) for \( \alpha = 4^\circ \). The second value is the total velocity minimum in the entire measurement area. These points are not aligned with the wing tip profile end point. The direction of circulation at higher angles of attack is reversed and consistent with a lifting wing.

Vortex sheets are obvious in fig. 3. The total velocity minima are reached for \( \alpha = 0^\circ \) and \( 2^\circ \) inside the vortex sheet, while for \( \alpha = 4^\circ \) the velocity minimum is exactly in the vortex core center.

It was shown in [11] that both cross-stream velocities \( (V_x \text{ and } V_y) \) have small values compared to the dominant axial velocity \( (V_z) \) for all angles of attack and data runs, so they are not presented here.

The average of ten vorticity fields \( (\Omega_{ij}) \), obtained from ten averaged vorticity fields, for all three angles of attack are calculated and presented in [11]. It was shown that the maximum streamwise vorticities are obtained in the same points, where total velocity minima occur and the highest streamwise vorticity is reached for \( \alpha = 4^\circ \). Vortex sheets are identified for all three angles of attack with higher vorticities [11].

**Investigations by using invariant maps**

The state of turbulence is analyzed by the amount of anisotropy in the flow [20, 21]. Components of the non-dimensional anisotropy tensor, as a measure of turbulence, are introduced as:

\[
a_{ij} = \frac{1}{2k} v_i v_j - \frac{1}{3} \delta_{ij},
\]

where \( k = (1/2)(v_x^2 + v_y^2 + v_z^2) \) is turbulence kinetic energy and \( \delta_{ij} \) is Kronecker delta symbol.

In the case of isotropic turbulence all components of tensor vanish \( (a_{ij} = 0) \). Other states of turbulence anisotropy are analyzed on the basis of \( a_{ij} \) values. Paper [22] discusses possible states of turbulence, i.e. trajectories of joint variation of invariants for incompressible turbulent channel flow. It is shown in [23] that "... large degrees of turbulent anisotropy occur in the region that is dominated by the oscillating vortex core." The invariant theory for the HSS PIV study of the turbulent swirl flow behind the axial fan in a pipe is applied in [18, 19]. Scalar functions of the components of the anisotropy tensor present the first, second and third invariant of the second order tensors in the following way:

\[
I_a = 0, \quad II_a = -\frac{1}{2} a_{ij} a_{ij}, \quad III_a = \frac{1}{3} a_{ij} a_{ik} a_{jk}.
\]
Experimentally determined scalar invariants $I_{II}$ and $I_{III}$ are presented in $I_{III}(-II_{a})$ plane, i.e., in anisotropy invariant maps (fig. 4) for all three angles of attack. It seems that the turbulence states tend to isotropy for $\alpha=2^\circ$ and $\alpha=4^\circ$, which is not the case for $\alpha=0^\circ$.

![Figure 4. Anisotropy invariant maps for angles of attack (\(\alpha\))](image)

All possible states of turbulence anisotropy belong to the curvilinear triangle area (012), called the Lumley triangle or anisotropy invariant map. Frontiers of the invariant map, which describe the border states of turbulence, are defined as follows:

\[
\begin{align*}
201: \quad & -II_{a} = \frac{3}{4} \left(4|I_{III_{a}}|\right)^{2/3} + \frac{1}{9} \\
21: \quad & -II_{a} = 3I_{III_{a}} + \frac{1}{9}
\end{align*}
\]  

Here 201 denotes axisymmetric turbulence states (two components are equal in magnitude), 20 stands for axisymmetric contraction (one component of the velocity fluctuations is smaller than the other two), 01 for axisymmetric expansion (one component dominates over the other two) and 21 for two-component turbulence (one component of the velocity fluctuations is negligible compared to the other two). Lumley triangle vertices, fig. 4(a) correspond to turbulence boundary conditions: 0 ($I_{II} = I_{III} = 0$) - three-component isotropic turbulence, point 1 describes boundary layer of one-component isotropic turbulence and 2 corresponds to two-
component isotropic turbulence. These two invariants are calculated for one data set for each angle of attack, for all 10,075 measured points. All these points belong to the domain of the invariant map and describe various turbulence states that are physically possible. Most of the points, for all angles, are distributed in the right part of the invariant map ($\mathrm{III}_a > 0$) and along the line $\mathrm{O}1$ which characterizes the turbulence state of the axisymmetric expansion. This region is characterized by one fluctuating component dominating the other two. It is evident (fig. 4) that in the case of $\alpha = 0^\circ$ and $\alpha = 2^\circ$ points don't reach the three-component isotropic turbulence state, but this is achieved for $\alpha = 4^\circ$. The left part of the map ($\mathrm{III}_a < 0$) denotes axisymmetric turbulence, where one component is smaller than the other two. Lower number of points lay in this region for all angles.

Figure 5 shows invariant maps along one vertical direction through the vortex core centers for each angle of attack. All points belong to the Lumley triangle, but flow behavior in these points is different. All 59 points have the same $y$-coordinate. There is one additional point for $\alpha = 2^\circ$ and two points for $\alpha = 4^\circ$. Turbulence states are analyzed by the amount of anisotropy in the flow along the vertical directions in the vortex core $x$-positions. It is obvious that isotropic turbulence doesn't occur in these directions (fig. 5). High degrees of turbulent anisotropy occur in the entire region dominated by the wing tip vortex.

![Figure 5](image-url)

Figure 5. Anisotropy invariant maps for angles of attack ($\alpha$) and specified directions; (a) $0^\circ$ and $x = -1.25123 \text{ mm}$, (b) $2^\circ$ and $x = -6.80899 \text{ mm}$, and (c) $4^\circ$ and $x = -12.3667 \text{ mm}$
If the vortex core is observed as one point in the fluid flow region, invariants are calculated for all three angles. Values $\text{III}_a = -0.00108$ and $-\text{II}_a = 0.03917$ (fig. 5(a)) are obtained for $\alpha = 0^\circ$ in the vortex core, whereas $\text{III}_a = 0.00293$ and $-\text{II}_a = 0.06943$ (fig. 5(b)) for $\alpha = 20^\circ$, and for $\alpha = 4^\circ$ the values are $\text{III}_a = 0.01368$ and $-\text{II}_a = 0.11895$ (fig. 5(c)).

This resolves a variety of anisotropy behavior of the vortex cores for all three angles. Only in the case of $\alpha = 0^\circ$ the vortex core is closer to the 20 frontier, i.e. axisymmetric contraction, while for other two angles it is closer to axisymmetric expansion.

All three cases reveal that generated vortex cores are highly anisotropic. Most of the points for $\alpha = 0^\circ$ and $\alpha = 4^\circ$ occur in the region of the axisymmetric expansion (01), characterized by one fluctuating component that dominates over the other two. It is opposite for $\alpha = 2^\circ$, where the majority of points are positioned closer to the line 20 that represents axisymmetric contraction.

**Positions of total velocity minimum**

It was shown in [11] that the vortex core was where the cross-stream velocities changed sign and where the gradients were largest. This provides the same result as the total velocity minimum criterion for defining the vortex core used, as shown in [18, 19].

The criterion of the velocity minimum is applied in studying the wing tip vortex core, as well as the vortex sheet. These points are presented in fig. 6. They resolve the vortex sheets. It is obvious that the points are predominantly located on the vortex sheet (fig. 6). Not all the points are unique. In the case for $\alpha = 0^\circ$ only 580 points are visible, while for $\alpha = 2^\circ$ 689 points. Most of them are not unique and this is shown in histograms in fig. 7. The number of repetitions varies for different angles and positions.

In the case for $\alpha = 0^\circ$ the number of unique points is 211, that is, 36.4% of all visible points, for $\alpha = 2^\circ$ their number is 288, that is, 41.8% and for $\alpha = 4^\circ$ it is 518, what makes up 58.9%. It is obvious that the highest number of repetitions is achieved along the vortex sheet. The highest repetition number of 185 is achieved for $\alpha = 4^\circ$ in app. vortex core center. The highest repetition number for other two angles is only 20 for $\alpha = 0^\circ$ and 33 for $\alpha = 2^\circ$. So, it means that the number of unique points, as well as the repetition number increases as the angle of attack increases and reaches maxima for $\alpha = 4^\circ$.

![Figure 6. Positions of the velocity minimum for the angles of attack; (a) 0° and (b) 2°](image-url)
Figure 7. Distributions of the velocity minima repetitions in the measurement plane for the angles of attack; (a) 0°, (b) 2°, and (c) 4°

Conclusions

This paper studies the wing tip vortex on a 3% scaled semi-span CRM applying the HSS PIV measurement method. Experimental results were obtained in the measurement plane three cords downstream from the end point of the wing tip profile for all three angles of attack ($\alpha = 0^\circ$, 2°, and 4°). Highly turbulent flows are generated. It is shown that the wing tip vortex circulates counterclockwise as viewed from downstream for $\alpha = 0^\circ$. The generated wing tip vortices for $\alpha = 2^\circ$ and $\alpha = 4^\circ$ create positive lift.

Paper [11] indicates that the velocity intensities do not change significantly as a function of the angle of attack flow and the flow could be treated for all three angles of attack.
as a statistically stationary flow. This is also proved here. It is shown that the positions of the vortex core centers for the studied cases correspond to the ones for the ensemble averaged total velocity fields.

All possible states of turbulence anisotropy of the studied flow for all three angles of attack are investigated in the anisotropy invariant map. Almost all points are grouped in the right part of the Lumley triangle, close to the line 01 that describes the turbulence state of axisymmetric expansion. A high number of points for \( \alpha = 2^\circ \) and \( \alpha = 4^\circ \) are in the vicinity of the three-component isotropic turbulence (state 0), whereas for \( \alpha = 0^\circ \) none of the points approach this state.

The total velocity minima criterion defined vortex sheets for \( \alpha = 0^\circ \) and \( \alpha = 2^\circ \), but the vortex core for \( \alpha = 4^\circ \). The vortex core center is determined as the point of the highest repetition number for \( \alpha = 4^\circ \).

Additional HSS PIV measurements in the planes one and two tip-chords downstream of the wing tip are necessary for documenting the streamwise evolution of the wing tip vortex.

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Nomenclature

\[ a_{ij} \quad \text{components of the anisotropy tensor of Reynolds stresses, [-]} \]
\[ \text{Re} \quad \text{Reynolds number, [-]} \]
\[ V \quad \text{total velocity, [ms}^{-1}] \]
\[ V_e \quad \text{ensemble averaged total velocity, [ms}^{-1}] \]
\[ V_{i,e} \quad \text{ensemble averaged total velocity component, [ms}^{-1}] \]

Greek symbols

\[ \alpha \quad \text{angle of attack, [º]} \]
\[ \delta_{ij} \quad \text{Kronecker delta symbol, [-]} \]
\[ \rho \quad \text{fluid density, [kgm}^{-3}] \]

References


