HEAT TRANSFER AND CROSS-DIFFUSION DUE TO A SPHERE OF CONSTANT THERMAL ENERGY EMBEDDED IN A POROUS MEDIUM

by

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Original scientific paper
https://doi.org/10.2298/TSCI17S2503M

Heat transfer and cross-diffusion due to a sphere of constant thermal energy and concentration embedded in unbounded homogeneous porous medium in a regime where the temperature gradient produces mass flux is analytically studied using Darcy flow model. Analytical solution is obtained with regular perturbation analysis in the limit of small Rayleigh number. Due to cross-diffusion, solute front initially shows stronger convection than thermal front, but ultimately reaches steady-state at approximately the same time as that of thermal front. Quantity of heat necessary to maintain the steady-state is found to be least near the rear stagnation point and the mean Nusselt number is found to be unaffected by cross-diffusion. Nusselt number variation for different cone angles and Soret number is studied and it is found that higher improvement is achieved when cone angle is changed from 80° to 100°.

Key words: thermal energy, thermo-diffusion, Darcy flow, porous medium, sphere

Introduction

Heat transfer due to the presence of heat sources in saturated porous media has been the subject of study in recent years in view of its wide ranging applications to a variety of fields such as in the storage of thermal energy, and in the management of nuclear waste to mention but a few. However, in most practical situations, species concentration gradients greatly affect the flow and as a result, they play a decisive role in the development of thermal fields, such as migration of moisture in fibrous insulations, spreading of pollutants in water saturated soil and in underground disposal of nuclear waste. When the temperature gradient is steep, cross-coupling between thermal and solutal diffusions are no longer negligible and presumably for this reason, Soret effect is being widely considered for isotope separation and presumably for this reason, Soret effect is being widely considered for isotope separation and in mixture of gases of light and medium molecular weights [1]. In fact, the importance of thermo-diffusion, though considered as a second-order phenomenon, is becoming more widely accepted in a variety of fields that includes mineral enrichment of geothermal sources, hydrocarbon separation in petrology and magma differentiation in geosciences, besides others [2].

Analytical studies on double-diffusive convection in a horizontal fluid-saturated porous layer, heated and salted from below in the presence of Soret and Dufour effects, using linear and non-linear stability analyses is done by Malashetty and Biradar [3]. In natural con-
Vanadium oxide nanoparticles (VONPs) have been investigated for their potential as a high-performance anode material in lithium-ion batteries. The unique properties of VONPs, such as high electrical conductivity, good Li+ diffusion, and chemical stability, make them an attractive candidate for improving the energy density and power output of lithium-ion batteries. In this study, we have synthesized VONPs using a hydrothermal method and then evaluated their electrochemical performance in lithium-ion batteries.

The synthesis of VONPs was performed in a 250-mL autoclave under hydrothermal conditions. The reaction mixture consisted of vanadium(V) chloride (0.05 M), oxalic acid (0.1 M), and deionized water (100 mL). The autoclave was sealed and heated to 180°C for 12 hours. After cooling to room temperature, the precipitate was collected by centrifugation, washed with deionized water, and dried at 60°C for 12 hours.

The electrochemical performance of VONPs was evaluated using lithium electrodes in a half-cell configuration. The electrodes were prepared by mixing VONPs, lithium metal, and polytetrafluoroethylene (PTFE) in a weight ratio of 8:1:1. The slurry was then coated on copper foil and dried at 60°C for 12 hours. The half-cells were assembled in an argon-filled glovebox, and the lithium metal disc was used as the counter electrode. The electrolyte was 1 M lithium hexafluorophosphate (LiPF6) in a 1:1:1 mixture of ethylene carbonate (EC), dimethyl carbonate (DMC), and diethyl carbonate (DEC).

The cyclic voltammetry (CV) and galvanostatic charge-discharge tests were performed using a battery tester (Arbin BT2000). The CV measurements were conducted in the potential range of 0.01 to 3.0 V at a scan rate of 0.1 mV s⁻¹. The galvanostatic charge-discharge tests were performed at a current density of 100 mA g⁻¹. The electrochemical impedance spectra (EIS) were measured using an AC voltage of 10 mV at open circuit voltage.

The results showed that the synthesized VONPs exhibited high specific capacitance and good stability. The specific capacitance of the VONPs was 350 F g⁻¹ at a current density of 100 mA g⁻¹, which was higher than that of the pure V2O5 (250 F g⁻¹). The specific capacitance retention after 1000 cycles was 90%, indicating good cycle stability. The EIS results showed that the synthesized VONPs had a lower resistance compared to the pure V2O5, which further contributed to the high electrochemical performance.

In conclusion, VONPs synthesized using a hydrothermal method were found to be a promising anode material for lithium-ion batteries. The high specific capacitance and good stability suggest that VONPs can be used in high-power lithium-ion batteries. Further studies are needed to optimize the synthesis conditions and evaluate the full-cell performance of VONPs in lithium-ion batteries.
The medium is isotropic, meaning that transport properties do not depend on the direction of the experiment.

At any point in the porous medium, the solid matrix is in thermal equilibrium with the fluid filling the pores.

The local Reynolds number based on averaged velocity and $k^{1/2}$ does not exceed the range 1-10, meaning that the Darcy law applies in its original form.

Thus, with the Darcy flow model, we have for the conservation of mass, momentum, energy, and species concentration with thermo-diffusion and in the absence of dispersion.

$$\nabla \bar{q} = 0$$ (2)

$$\bar{q} = \left( \frac{K}{\mu} \right) (\nabla P + \rho g \bar{n})$$ (3)

$$\sigma \frac{\partial T}{\partial t} + (\bar{q} \nabla)T = \alpha \Delta T$$ (4)

$$\varepsilon \frac{\partial C}{\partial t} + (\bar{q} \nabla)C = \Delta(D_m C + D_C T)$$ (5)

In writing the above equations we have assumed that the medium and the saturating fluid are in thermal equilibrium, the thermo-diffusion coefficient is not excessive and that all physical quantities are constant except in the buoyancy term [7]. Taking note of spherical symmetry in the angular direction $\theta$ and making use of eq. (2), we define a stream function $\psi$ such that:

$$\bar{q} = \left( \frac{1}{r^2 \sin \varphi} \frac{\partial \psi}{\partial \varphi}, -\frac{1}{r \sin \varphi} \frac{\partial \psi}{\partial r}, 0 \right)$$ (6)

and introduce the non-dimensional quantities:

$$R = \frac{r}{L}, \tau = \frac{at}{L^2}, \Psi = \psi aL, \Theta = \left( \frac{k}{L} \right)(T - T_w), C^* = \left( \frac{D_m}{L} \right)(C - C_w)$$ (7)

to obtain for the conservation of momentum, energy and species concentration:

$$\frac{1}{R^2} \left( \frac{\partial}{\partial \varphi} \left( \frac{1}{\sin \varphi} \frac{\partial \Psi}{\partial \varphi} \right) + \frac{\partial^2 \Psi}{\partial R^2} \right) =$$

$$= Ra \left[ \cos \varphi \frac{\partial \Theta}{\partial \varphi} + R \sin \varphi \frac{\partial \Theta}{\partial R} \right] + N \left[ \cos \varphi \frac{\partial C}{\partial \varphi} + R \sin \varphi \frac{\partial C}{\partial R} \right]$$ (8)

$$\frac{\partial \Theta}{\partial \tau} + \left( \frac{1}{R^2 \sin \varphi} \right) \frac{\partial (\Psi, \Theta)}{\partial (\varphi, R)} = \frac{1}{R^2} \left[ \frac{\partial}{\partial R} \left( R^2 \frac{\partial \Theta}{\partial R} \right) + \frac{1}{\sin \varphi} \frac{\partial}{\partial \varphi} \left( \sin \varphi \frac{\partial \Theta}{\partial \varphi} \right) \right]$$ (9)

$$A^2 \frac{\partial C}{\partial \tau} + Le \left( \frac{1}{R^2 \sin \varphi} \right) \frac{\partial (\Psi, C)}{\partial (\varphi, R)} = \frac{1}{R^2} \left[ \frac{\partial}{\partial R} \left( R^2 \frac{\partial C}{\partial R} \right) + \frac{1}{\sin \varphi} \frac{\partial}{\partial \varphi} \left( \sin \varphi \frac{\partial C}{\partial \varphi} \right) \right]$$

$$+ \frac{\lambda}{R^2} \left[ \frac{\partial}{\partial R} \left( R^2 \frac{\partial \Theta}{\partial R} \right) + \frac{1}{\sin \varphi} \frac{\partial}{\partial \varphi} \left( \sin \varphi \frac{\partial \Theta}{\partial \varphi} \right) \right]$$ (10)
where, \( A^2 = \left[(\varphi/\sigma) \text{Le}\right] \) and the Soret number, \( \lambda \), is related to the Soret coefficient by:

\[
\lambda = D_{CT} \left( \frac{Q}{k_m} \right)
\]

(11a)

and

\[
\frac{\partial(p, q)}{\partial(x, y)} = \frac{\partial p}{\partial x} \frac{\partial q}{\partial y} - \frac{\partial p}{\partial y} \frac{\partial q}{\partial x}
\]

(11b)

is the Jacobian. The asterisk in \( C \) is dropped for convenience. Though it is customary to use the radius of the sphere as the characteristic length scale, we have so chosen \( L \) that the solution for the analogous problem that treats a concentrated point source could be deduced algebraically from the results of the present study using an appropriate analytical technique [8].

The initial and boundary conditions in non-dimensional formulation are:

\[
U, V, \Theta, C = 0 \text{ when } \tau = 0
\]

(12a)

\[
U, V, \Theta, C \to 0 \text{ as } R \to \infty
\]

(12b)

\[
\frac{\partial U}{\partial \varphi} = V = \frac{\partial \Theta}{\partial \varphi} = \frac{\partial C}{\partial \varphi} = 0 \text{ at } \varphi = 0, \pi \text{ when } R = R_0
\]

(12c)

where, \( R_0 = a/L \) and \( (U, V) = (L/\alpha)(u, v) \). For the heat and concentration balance we have:

\[
\lim_{R \to R_0} \left( \frac{\partial \Theta}{\partial R} \right) = -1, \quad \lim_{R \to R_0} \left( \frac{\partial C}{\partial R} \right) = -1
\]

(13)

**Method of solution**

We seek a perturbation solution by assuming power series expansions of the form:

\[
(\Psi, \Theta, C)(R, \varphi, \tau; R_0) = \sum_{n=0}^{\infty} Ra^n (\Psi_n, \Theta_n, C_n)(R, \varphi, \tau; R_0)
\]

(14)

and substitute eq. (14) into eqs. (8)-(10) to collect terms of equal powers in \( Ra \) for the determination of the various order solutions. As the zero-order solution corresponds to a state of thermal conduction, there is no fluid motion and hence without any loss one may take \( \Psi_0 = 0 \). The functions \( \Theta_0 \) and \( C_0 \) are then found from the solutions of the equations:

\[
\frac{\partial \Theta_0}{\partial \tau} = \frac{1}{R^2} \left[ \frac{\partial}{\partial R} \left( R^2 \frac{\partial \Theta_0}{\partial R} \right) \right]
\]

(15a)

\[
A^2 \frac{\partial C_0}{\partial \tau} = \frac{1}{R^2} \left[ \frac{\partial}{\partial R} \left( R^2 \frac{\partial C_0}{\partial R} \right) \right] + \frac{\lambda}{R^2} \left[ \frac{\partial}{\partial R} \left( R^2 \frac{\partial \Theta_0}{\partial R} \right) \right]
\]

(15b)

in which by setting

\[
\Theta_0 = R^{-1} G(R, \tau), \quad C_0 = R^{-1} H(R, \tau)
\]

(16)

one obtains after taking the Laplace transform:
\[ \ell[G(R, \tau)] = R_0^2 \left[ \frac{1}{s(1 + R_0 \sqrt{s})} \exp[-(R - R_0) \sqrt{s}] \right] \]  
(16a)

\[ \ell[H(R, \tau)] = R_0^2 \left[ \frac{(1 + \omega)}{s(1 + AR_0 \sqrt{s})} \exp[-A(R - R_0) \sqrt{s}] - \frac{\omega}{s(1 + R_0 \sqrt{s})} \exp[-(R - R_0) \sqrt{s}] \right] \]  
(16b)

where \( \omega = \lambda/(1 - A^2) \).

Using the table of Laplace transforms by Campbell and Foster [9], we have after inversion:

\[ \Theta_0 = F(\eta, \eta_0) \]  
(17a)

\[ C_0 = [(1 + \omega)F(A\eta, A\eta_0) - \omega F(\eta, \eta_0)], \quad (A \neq 1) \]  
(17b)

\[ = F(\eta, \eta_0) + \frac{\lambda R_0}{4\eta_0^2} \left( \frac{\eta_0}{\eta} \right) \frac{2}{\sqrt{\pi}} \eta_0 \exp[-(\eta - \eta_0)^2] + \exp \left( \frac{\eta}{\eta_0} - 1 + \frac{1}{4\eta_0^2} \right) \text{erfc} \left( \eta - \eta_0 + \frac{1}{2\eta_0} \right) \], \quad (A = 1) \]  
(17c)

and

\[ F(\eta, \eta_0) = R_0 \left( \frac{\eta_0}{\eta} \right) \left[ \text{erfc}(\eta - \eta_0) - \exp \left( \frac{\eta}{\eta_0} - 1 + \frac{1}{4\eta_0^2} \right) \text{erfc} \left( \eta - \eta_0 + \frac{1}{2\eta_0} \right) \right] \]  
(18)

\text{erfc}(\cdot) being the complementary error function.

The first convective correction to the flow field is now found from the solution of the equation:

\[ \frac{1}{R^2} \frac{\partial}{\partial \phi} \left( \frac{1}{\sin \phi} \frac{\partial \Psi_1}{\partial \phi} \right) + \frac{1}{\sin \phi} \left( \frac{\partial^2 \Psi_1}{\partial R^2} \right) = R \sin \phi \left[ \left( \frac{\partial \Theta_0}{\partial R} \right) + N \left( \frac{\partial C_0}{\partial R} \right) \right] \]  
(19)

in which the separation of the variables is achieved by setting:

\[ \Psi_1 = 2\sqrt{\pi} R_0^2 \sin \phi f(\eta, \eta_0) \]  
(20)

This then yields for the determination of \( f(\eta, \eta_0) \):

\[ \eta^2 \frac{d^2 f}{d \eta^2} - 2f = (1 - N\omega) \chi(\eta, \eta_0) + \left( \frac{N}{A^2} \right) (1 + \omega) \chi(A\eta, A\eta_0) \]  
(21)

where

\[ \chi(\eta, \eta_0) = -\eta^2 \left[ \left( \frac{\eta_0}{\eta} \right) \text{erfc}(\eta - \eta_0) + \left( \frac{1 - \eta_0}{\eta} \right) \exp \left( \frac{\eta}{\eta_0} - 1 + \frac{1}{4\eta_0^2} \right) \text{erfc} \left( \eta - \eta_0 + \frac{1}{2\eta_0} \right) \right] \]  
(22)

Solution of eq. (21) is found to be:
\[ f(\eta, \eta_0) = (1 - N\omega)\xi(\eta, \eta_0) + \left(\frac{N}{A}\right)(1 + \omega)\xi(A\eta, A\eta_0), \quad (A \neq 1) \]  

\[ = (1 + N)\xi(\eta, \eta_0) + \frac{1}{2} N\lambda\Omega(\eta, \eta_0), \quad (A = 1) \]

where

\[ \xi(\eta, \eta_0) = -\eta_0 \left(1 - \frac{\eta_0}{\eta}\right) \exp\left(\frac{\eta}{\eta_0} - 1 + \frac{1}{4\eta_0^2}\right) \text{erfc}\left(\eta - \eta_0 + \frac{1}{2\eta_0}\right) + \left(\frac{1}{2\eta}\right) \right] \exp[-(\eta - \eta_0)^2] \]  

\[ \Omega(\eta, \eta_0) = -\eta_0 \left[\frac{1}{2} \left(\frac{\eta_0}{\eta}\right) - \frac{\eta_0}{\eta}\right] \exp\left(\frac{\eta}{\eta_0} - 1 + \frac{1}{4\eta_0^2}\right) \text{erfc}\left(\eta - \eta_0 + \frac{1}{2\eta_0}\right) - \eta_0 \left(\frac{\eta_0}{\eta}\right) \exp[-(\eta - \eta_0)^2] \]

\[ \text{Discussion} \]

While \( \omega \) determines the Soret effect, the parameter \( A \), being the ratio of thermal penetration length \( O[(\alpha t/\sigma)^{1/2}] \) to that of species penetration \( O[(D_m t/\epsilon)^{1/2}] \), determines the impact of the species concentration gradients upon the thermally driven flow. In order to demonstrate the effect of cross-diffusion on heat transfer, contours of the transient streamlines \( \Psi/\text{Ra} = \text{constant} \) are drawn at a common time \( \tau = 0.5 \) (fig. 3) for two different values of \( \omega \) choosing in illustration \( R_0 = 1, A = 0.4, N = -0.4. \) The choice of \( A \) is justified, since in the case of diffusion of methanol and hydrogen in water saturated glass beads, \( A \approx 0.36. \) Likewise when the two buoyancy mechanisms are opposed, \( N \) is negative and for simplicity we have so chosen \( N \) that \( N/A = -1. \)

Obviously, in the early stages of flow development, counter rotating cells are formed with those in the colder front rotating anti-clockwise giving rise to a downward flow and that the colder front showing stronger convection than the thermal front. Consequently, the volume of the region, in which the thermal effect of the sphere is felt, is significantly reduced, the reduction in volume being about 13.9% when \( \lambda = 0.252, \) that is, when \( \omega = 0.3 \) and the stagnation points of

![Figure 3. Transient streamlines \( \Psi/\text{Ra} = \text{constant} \) for various values of \( \omega \) (\( R_0 = 1, A = 0.4, N = -0.4, \tau = 0.5 \)); (a) \( \omega = 0 \), (b) \( \omega = 0.3 \)](image-url)
the flow field are found to be at \( R = 1.96, 1.89 \) when \( \omega = 0, 0.3 \), respectively. From the maps of streamlines drawn in fig. 4 one may observe that as time increases the geometry of the flow pattern changes. Initially the thermal front moves faster than the solute front but ultimately they both reach the steady-state at approximately the same time. Consequently, for very large times, the dual counter rotating cells break down into a single cell. The temperature and species concentration decrease outward from the sphere in all directions with symmetry about the horizontal plane through the origin. Most of these results are in conformity with the observations put forth by Nejad et al. [5]. As our study is primarily concerned with a single component chemical species in a single phase fluid and as migration of fluid particles is from the hotter region to the colder one, the Soret parameter assumes only positive values.

As the algebra is overwhelming, the higher-order convective corrections thermal and concentration fields could not be determined. However, the error due to the non-inclusion of these terms is not of great physical significance [10]. One may observe that in the limit \( N \to 0 \), we recover the results of Sano and Okihara [11], of course, with a different non-dimensionalization scheme.

**Steady-state**

Since the method of solution is straight forward, we present only the final results. In what follows, \( \zeta \) represents \( R/R_0 \). Thus, with the same notations as before we have:

\[
\Psi = \text{Ra} \left( \frac{R_0^3}{2} \right) (1 + N) (\zeta^{-1} - \zeta^{-1}) \sin^2 \varphi
\]

\[
+ \text{Ra} \left( \frac{R_0^5}{2} \right) (1 + N)[1 + N(Le - \Lambda)] \left[ \frac{1}{3} \zeta^{-1} - \frac{5}{8} \zeta^{-1} - \frac{5}{24} \zeta^{-2} \right] \sin^2 \varphi \cos \varphi + O(\text{Ra}^{-3}) \quad (24)
\]

\[
\Theta = R_0 \zeta^{-1} + \text{Ra} \left( \frac{R_0^3}{2} \right) (1 + N) \left[ \zeta^{-1} - \frac{5}{4} \zeta^{-2} + \frac{1}{2} \zeta^{-3} \right] \cos \varphi +
\]

\[
+ \text{Ra}^2 \left( \frac{R_0^5}{4} \right) (1 + N)[(1 + N)(Y_1 + Y_2 \cos 2\varphi) + [1 + N(Le - \Lambda)](Z_1 + Z_2 \cos 2\varphi)] + O(\text{Ra}^{-3}) \quad (25)
\]
\[
C = R_0^{\alpha^{-1}} + Ra \left( \frac{R_0^{\beta}}{2} \right) (1 + N)(Le - \lambda) \left( \alpha^{-1} - \frac{5}{4} \alpha^{-2} + \frac{1}{2} \alpha^{-3} \right) \cos \varphi + \\
+ Ra \left( \frac{R_0^{\gamma}}{4} \right) (1 + N)(1 + N)[Le(Le - \lambda) - \lambda](\alpha_{1} + \alpha_{2} \cos 2\varphi) + \\
+(1 + N)[Le(Le - \lambda) - \lambda](\alpha_{1} + \alpha_{2} \cos 2\varphi) + \\
+(Le - \lambda)(1 + N(Le - \lambda))(\beta_{1} + \beta_{2} \cos 2\varphi) + O(Ra^{-1})
\]

where

\[
\alpha = \frac{7}{12} \lambda^{5-1} + \frac{5}{32} \lambda^{5-2} + \frac{377}{1008} \lambda^{5-3} + \frac{1}{12} \lambda^{5-4} \ln R - \frac{5}{16} \lambda^{5-5} + \frac{27}{280} \lambda^{5-6}
\]

\[
\beta = \frac{25}{32} \lambda^{5-2} + \frac{265}{336} \lambda^{5-3} + \frac{1}{4} \lambda^{5-4} \ln R - \frac{5}{16} \lambda^{5-5} + \frac{5}{56} \lambda^{5-6}
\]

\[
\gamma = \frac{1}{18} \lambda^{5-1} - \frac{5}{32} \lambda^{5-2} + \frac{157}{2160} \lambda^{5-3} + \frac{1}{10} \lambda^{5-4} \ln R - \frac{5}{144} \lambda^{5-5} + \frac{5}{48} \lambda^{5-6}
\]

In the absence of cross-diffusion, one may recover the results of Lai and Kulacki [12] with \( R_0 = 1 \). It is observable from fig. 5, that more closer the streamline to the vertical is, the more wid apart it is from the vertical in the lower half space with the phenomenon getting more pronounced with increasing \( \lambda \). This then implies that the momentum transfer is retarded in the colder front than on the thermal front. Likewise, profiles of the steady-state isotherms and lines of constant concentration (fig. 6) show that larger the value of \( \lambda \) is, greater is the shift of the warm and high concentration region below the sphere. As an essential characteristic of heat transfer, we evaluate the Nusselt number given by:

![Figure 5. The effect of the Soret parameter on the steady-state streamlines \( \Psi R a = 0.4 \) (\( R_0 = 1 \), \( Le = 2.0, N = -0.4, Ra = 3 \)); numbers on the graph represent values of \( \lambda \)](image1)

![Figure 6. Profiles of isotherms \( \Phi = 0.3 \) (left side) and lines of constant concentration \( C = 0.3 \) (right side) (\( R_0 = 1, Le = 2.0, N = -0.4, Ra = 3 \)); NUMBERS on the graphs represent values of \( \lambda \)](image2)
Nu = \lim_\limits_{r \to a} \left[ \frac{a - Q}{k (T - T_a)} \right] 
\tag{28}

which in non-dimensional form reduces to:

\[
Nu = \lim_\limits_{R \to R_0} \left( \frac{R_0}{\Theta} \right) 
\tag{28a}
\]

With a three-term approximation of \( \Theta \), we have:

\[
Nu = 1 - \frac{R_0^2}{8} (1 + N) \cos \varphi Ra + \frac{R_0^4}{40320} (1 + N)^2 \left[ (3028 - 30 \cos 2\varphi) - 203 \frac{1 + N(Le - \lambda)}{1 + N} \left( \frac{1}{3} + \cos 2\varphi \right) \right] Ra^2 
\tag{29}
\]

and the mean Nusselt number averaged over the surface of the sphere is found to be:

\[
Nu_{\text{ave}} = \frac{1}{2} \int_0^\pi Nu \sin \varphi d\varphi = 1 + \frac{217}{2880} R_0^4 (1 + N)^2 Ra^2 
\tag{30}
\]

It is observable from tab. 1 that \( Nu \) is a minimum when \( \varphi = 0 \) and as \( \varphi \) increases, \( Nu \) increases monotonically so that the quantity of heat necessary for the maintenance of the steady-state is least near the rear stagnation point than elsewhere, and the phenomenon is more pronounced with increasing values of \( \lambda \). However, one may observe from the same table that there is a dip in the value of \( Nu \) as \( \lambda \) assumes values greater than one. We find that the mean Nusselt number is independent of the Soret effect. Figure 7 shows the variation of Nusselt number for different cone angles and for different soret number. It is observed that, from 0 to 40° and from 140 to 180° cone angle, for different Soret number, a significant variation in Nusselt number is seen. But in between 40 to 140° cone angle, for all Soret numbers, there is no much different in Nusselt number. A sharp increase in Nusselt number is experienced between 40 to 140° cone angles. Higher Nusselt number is observed with 0.1 soret number.

<table>
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<th>( \pi ) ( / ) 9</th>
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Figure 7. Variation of Nusselt number for different Soret number

Table 1. Values of 100 (\( Nu - 1 \)) for increasing values of \( \lambda \) (\( N = -0.05 \), Ra = 1, Le = 2.0)
Conclusion

Assuming the Darcy flow model, phenomenon of thermal convection and the effect of cross-diffusion due to a heated sphere embedded instantaneously in a porous medium is investigated analytically. From the analysis we deduce that cross-diffusion is found to inhibit heat transfer. Further, due to cross-diffusion, there is a reduction in the volume of the region in which the thermal effect is felt and that there is retardation in the rate of momentum transfer in the colder front than on the thermal front. There is a monotonic decrease in the heat transfer rate as Soret number increases and the quantity of heat necessary to maintain the steady-state is found to be least near the rear stagnation point than elsewhere. The mean Nusselt number remains unaffected by cross-diffusion.

Nomenclature

A — non-dimensional parameter, \(\equiv \left(\frac{\varepsilon}{\sigma} \alpha D_{m}\right)^{1/2}\)

\(a\) — radius of the sphere, [m]

\(C\) — the species concentration

\(C^*\) — species concentration, [-]

\(D_{CT}\) — Soret coefficient

\(D_m\) — solutal diffusivity

\(k\) — thermal conductivity of the fluid-porous matrix, [kgms\(^{-3}\)K\(^{-1}\)]

\(\ell\) — Laplace transform

\(L\) — characteristic length scale

\(Le\) — Lewis number, \(= \alpha D_{m}\)

\(m\) — species generation rate, [kgs\(^{-1}\)]

\(N\) — buoyancy ratio

\(\equiv \left[\beta_{C} m k (\beta Q D_{m})\right]\)

\(N_u\) — Nusselt number

\(\bar{N}_u\) — mean Nusselt number

\(\bar{n}\) — unit vector in the vertical direction

\(Q\) — thermal energy of the sphere, [W]

\(R\) — non-dimensional radial co-ordinate

\(R_a\) — thermal Rayleigh number,

\(= \left(\frac{\beta g K}{(\alpha K)}\right) Q L\)

\(R_0\) — non-dimensional radius of the sphere, \(= a/L\)

\(r\) — radial co-ordinate, [m]

\(T\) — temperature, [K]

\(t\) — time, [s]

\(u\) — radial velocity

\(V\) — non-dimensional form of the transverse velocity

\(v\) — transverse velocity, [ms\(^{-1}\)]

\(Y_{1,2}\) — functions of \(\zeta\) (eq. 27a-b)

\(Z_{1,2}\) — functions of \(\zeta\) (eq. 27c-d)

\(\alpha\) — effective thermal diffusivity of the fluid-porous matrix

\(\beta\) — thermal expansion coefficient

\(\beta_c\) — solutal expansion coefficient

\(\varepsilon\) — porosity of the porous matrix

\(\eta\) — similarity variable, \(= R/2\sqrt{T}\)

\(\eta_0\) — similarity variable, \(= R_0/2\sqrt{T}\)

\(\Theta\) — non-dimensional temperature

\(\lambda\) — Soret number, \(= D_{CT}(Q/km)\)

\(\mu\) — coefficient of viscosity of the fluid

\(\nu\) — kinematic viscosity of the fluid

\(\xi\) — function of \((\eta, \eta_0)\), eqn. (23c)

\(\rho\) — fluid density

\(\sigma\) — heat capacity ratio,

\(\varepsilon + (1 - \varepsilon)(\chi \rho_c)/(\chi \rho_f)\)

\(\tau\) — non-dimensional time

\(\varphi\) — cone angle

\(\chi\) — function of \((\eta, \eta_0)\), eq. (22)

\(\psi\) — non-dimensional stream function

\(\Omega\) — function of \((\eta, \eta_0)\), eq. (23d)

\(\omega\) — non-dimensional parameter,

\(\lambda/(1 - \omega^2)\)

\(\phi\) — fluid phase

\(s\) — solid phase

\(\infty\) — reference state

\(n\) — \(n^{th}\) term \((n = 0, 1, 2, ..)\)

References


