NUMERICAL STUDY OF FORCED CONVECTION HEAT TRANSFER OVER THREE CYLINDERS IN STAGGERED ARRANGEMENT IMMERSED IN POROUS MEDIA

by

Habib-Ollah SAYEHVANDa, Mehrdad YARISAKENEb, and Amir BASIRIPARSA

aDepartment of Mechanical Engineering, Bu-Ali Sina University, Hamedan, Iran
bYoung Researches and Elite Club, Hamedan Branch, Islamic Azad University, Hamedan, Iran

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Staggered arrangement is one of the common configurations in heat exchangers that make better mixing of flow and heat transfer augmentation than other arrangements. In this paper forced convection heat transfer over three isothermal circular cylinders in staggered configuration in isotropic packed bed was investigated. In this work laminar 2-D incompressible steady-state equations of momentum and energy were solved numerically by finite volume method. Simulation was done in three Reynolds numbers of 80, 120, and 200. The results indicate that, using porous medium the Nusselt number enhanced considerably for any of cylinders and it presents thin temperature contours for them. Also is shown that by increasing Reynolds number, the heat transfer increased in both channel but the growth rate of it in porous media is larger. In addition, results of simulation in porous channel show that with increasing Peclet number, heat transfer increased logarithmically.

Key words: forced convection heat transfer, staggered arrangement, porous media, laminar flow

Introduction

The circular cylinder is one of the most important elementary components used in tube bundles in heat exchangers, cooling systems for nuclear power, evaporators of power plants, air conditioners and so on. So many studies were done about fluid-flow and heat transfer around it. Researches of the fluid-flow about simple configurations of two and three cylinders help our understanding of the flows around more complex configurations in tube banks. In general, three kinds of geometrical configuration are categorized for two or more circular cylinders: side-by-side, tandem, and staggered arrangements with respect to the direction of the upstream flow.

For the purpose of examination the heat transfer from these arrangements, studying the flow pattern around them is very helpful. Chen [1, 2] and Zdravkovich [3, 4] conducted a general study of papers about flow interaction between two cylinders in various arrangements. The configuration of cylinders and spacing between them are important factors in flow field identification. Good investigations about these parameters in each of three arrangements were performed by Zdravkovich [5] and Sumner et al. [6].

Because of special configuration of staggered arrangement, it is better than two others in mixing the fluid-flow and consequently heat transfer to or from them. Using staggered arrangement generally provides higher pressure drop and higher overall heat transfer coefficient than

*Corresponding author, e-mail: hsayeh@basu.ac.ir, hsayehvand@yahoo.com
in-line arrangement. It obviously indicates that using staggered arrangement increases operating
cost whereas it lowers the capital cost [7]. Therefore, the use of such a structure is preferable.

Four parameters affect heat transfer from tube bundle: Reynolds number, thermal con-
ditions, pitch ratios, and configuration of the cylinders [8]. The longitudinal pitch to diameter
ratio and transverse pitch to diameter ratio are determinant factors in the 2-D flow around
staggered circular cylinders [6, 9, 10]. Some of the most important researches about flow around
staggered configuration of two cylinders can be found in works [11-21].

Investigations about three cylinders, unlike the studies around two cylinders, had
done rarely. In this field, a few works that can be said are [22-26].

Buyruk et al. [27] have investigated experimentally the effect of blockage ratios on
the local Nusselt number within a single tube row at Reynolds number between 7960 and
47770. Also Kostić and Oka [28] observed that the mean Nusselt numbers of two tandem cyl-
dinders, depending on whether the distance is greater or smaller than critical distance between
them, varies with Reynolds numbers by different exponents.

At low Reynolds number, the temperature field in the wake of heated cylinders was
obtained by Eckert [29] using Mach-Zehnder interferometer technique. Buyruk et al. [30] us-
ing a stream function-vorticity formulation by CFD technique and also experimentally studied
the variation of local heat transfer and dependence of Nusselt number to blockage ratio for
Reynolds number 120 and 390 on a single cylinder. Also, Buyruk [8] predicted the heat trans-
fer characteristics in tube banks with in-line, tandem and staggered arrangement at Reynolds
numbers 80, 120, and 200.

The scope of this work is the study of local and average heat transfer characteristics of
staggered arrangement of cylinders within porous media. This study was done by simulation of
fluid-flow. Advantage of this, is that, avoided from modeling, which is usual procedure in po-
rous media problems, and basic equations of flow and energy solved. Actually, porous medium
contains voids and pores that may naturally formed (e. g., rocks, sponges, woods) and would
have irregular shapes and sizes throughout the solid matrix or be in fabricated kind with var-
iou pore shapes such as square, spherical, cubical, elliptical, etc. Thermal conduction can be
considered for particles but, here, our goal is investigation of mixing effect of packed bed. In-
creased contact surface area and flow mixing and provided enough time to conduction, because
of the porosity of porous media, causes obtaining a high-heat-flux removal. This technique has
found many applications in convective heat transfer in engineering systems. Indeed, by using
porous media, thermal boundary-layer around cylinders, as a result of changing hydrodynamic
conditions, changed.

Problem definition and formulation

Geometry model and boundary conditions

In this investigation, porous medium is considered to be
composed of many periodic arrays of tiny circular cylinders, which
the physical model of it, is according to fig. 1. Figure 2 displays
representative elementary volume (REV) that saturated with con-
tinuous fluid-flow. Working fluids are air and water with Prandtl
number of 0.7 and 6.97, respectively. Reynolds numbers 80, 120,
and 200 selected to studying. The numerical method that is used
in this study is finite-volume method with SIMPLE algorithm for
calculation of the flow field equations [31]. In this problem, a con-
vergence criteria for all of the basic equations is considered, $10^{-5}$. 
The computational domain is shown in Fig. 3. According to [32], to neglect the channeling effect of channel walls on the heat transfer from the cylinder surfaces, the ratio of height to width should be small. The top and bottom boundaries of the domain were taken as the slippery wall with the same temperature of incoming flow. Velocity inlet and outflow boundary conditions considered at the entrance and exit of domain respectively. Longitudinal and transverse centre to centre spacing between cylinders equals $2 \times D$, i.e., $(S_i/D) \times (S_j/D) = 2 \times 2$. Cylinder to particle diameter ratio $D/d_p$ and porosity was chosen 10 and 0.5, respectively, [31]. It should be noted that the porosity of the domain is measured as $\varepsilon = [1 - (\pi/4)(D/M)^2]$.

**Governing equations**

The non-dimensional governing equations for steady laminar 2-D incompressible flow of Newtonian fluid with constant properties can be expressed in eqs. (1)-(4). The boundary conditions of problem and definition of local and mean Nusselt numbers are brought in relations (5) and (9)-(11), respectively, where $n$ in eq. (9) and (10) represents the unit normal vector on $s$, the surface of the cylinders. It must be mentioned, in this work, $T_0 = 300$ K and $T_h = 400$ K were considered.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$ (1)

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{1}{\text{Re}} \left[ \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right]$$ (2)

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \frac{1}{\text{Re}} \left[ \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right]$$ (3)

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{\text{RePr}} \left[ \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right]$$ (4)

$$U = 1, \quad V = 0, \quad T = T_0, \quad (x = 0, 0 < y < H)$$ (5)

$$\frac{\partial U}{\partial X} = \frac{\partial T}{\partial X} = 0, \quad (x = L, 0 < y < H)$$ (6)
\[ V = 0, \quad T = T_0, \quad (0 < x < L, \quad y = 0 \quad \text{and} \quad H) \quad (7) \]
\[ U = 0, \quad V = 0, \quad T = T_k, \quad \text{at cylinders surfaces} \quad (8) \]
\[ U = 0, \quad V = 0, \quad \frac{\partial T}{\partial n} = 0, \quad \text{at particles surfaces} \quad (9) \]
\[ \text{Nu}_\theta = \frac{h_c D}{k}, \quad -k \left( \frac{\partial T_f}{\partial n} \right)_{\text{cylinder surface}} = h_c (T_k - T_0) \quad (10) \]
\[ \overline{\text{Nu}} = \frac{1}{2\pi} \int_0^{\pi/2} \text{Nu}_\theta d\theta \quad (11) \]

**Grid independency**

Table 1 shows the variation of mean fluid Nusselt number of circular cylinders vs. element numbers of three sets of test grids. It can be found that grid refinement from G2 to G3 has the difference less than 2% in Nusselt number results of both cylinders. Hence, the grid G3 is seemed to be adequately refined to capture the heat transfer phenomena properly.

<table>
<thead>
<tr>
<th>Element number</th>
<th>Re = 80</th>
<th>Re = 120</th>
<th>Re = 200</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First cylinder</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1</td>
<td>261142</td>
<td>10.6420</td>
<td>14.6820</td>
</tr>
<tr>
<td>G2</td>
<td>549122</td>
<td>11.3249</td>
<td>15.5672</td>
</tr>
<tr>
<td>G3</td>
<td>885976</td>
<td>11.1763</td>
<td>15.3386</td>
</tr>
<tr>
<td><strong>Second cylinder</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1</td>
<td>261142</td>
<td>9.2164</td>
<td>13.1309</td>
</tr>
<tr>
<td>G2</td>
<td>549122</td>
<td>9.7794</td>
<td>13.9591</td>
</tr>
<tr>
<td>G3</td>
<td>885976</td>
<td>9.5900</td>
<td>13.6832</td>
</tr>
</tbody>
</table>

**Verification**

In order to validate our numerical procedure, we simulated fluid-flow and heat transfer around three circular cylinders which was configured in staggered arrangement without considering porous media, i.e., empty channel. We compared numerical calculations with the results of Buyruk [8], who predicted local heat transfer on tandem, inline, and staggered geometries in cross-flow of air at Reynolds numbers 80, 120, and 200 and \( S_t \times S_t = 2 \).

Figure 4 shows the computational domain that has applied to verification. It can be seen from the fluid local Nusselt number distribution in figs. 5 and 6 that results of our method is in good agreement with that of Buyruk [8].
Results and discussion

The mean Nusselt number has calculated at Reynolds numbers 80, 120, and 200 for all of the cylinders. Because of flow field symmetry and that two upstream cylinders are isotherm, heat transfer characteristics of upstream cylinders are very close to each other and considered same. So in the following discussion, just data of one of them displayed.

According to fig. 7, it is observed that thermal boundary-layers around cylinders in empty channel are thicker than that in porous media. In both medium, thermal boundary-layer about downstream cylinder is thicker than that of upstream one because it is placed in the domain was affected by the thermal field of first column cylinders and, therefore, experiences lower heat transfer in comparison to first cylinders. As well as thermal boundary-layer becomes thinner, temperature gradient in vicinity of cylinder surface approach higher values and consequently heat transfer from it will increase. It can be seen that, in both medium, by increasing Reynolds number, the thickness of thermal boundary-layer about cylinders decreased and fluid-flow with upstream temperature goes more and more into the area of between three cylinders.

Figure 7 shows that stretched thermal-layer about cylinders appear in the porous media, rear of the cylinders, does not exist in the other media. This developed domain with approximately identical temperature throughout itself, make high gradient region spreading into the flow field resulting in better heat transfer from cylinders. This is because of that, porosity results in mixing the flow and having enough time to heat conduction between fluid particles. It is also observed that thermal boundary-layer around cylinders, in both media becomes thinner with increasing Reynolds number which consequently enhances heat transfer.
Figure 8 exhibits mean Nusselt number of first and second cylinders at various Reynolds numbers. It is seen that, in both media, by increasing Reynolds number, the total convection heat transfer from any of cylinders increased. It is observed that the dependence of mean Nusselt number coefficient on Reynolds number differs depending on whether the porous media does exist or not. In porous channel, approximately for both up and down cylinders, \( \text{Nu} \sim \text{Re}^{0.8} \) whereas in empty channel \( \text{Nu} \sim \text{Re}^{0.4} \). It indicates that porous media application, by increasing Reynolds number, causes more increasing in mean Nusselt number than empty channel. In other words, porosity makes better thermal performance in higher Reynolds number. For better comparison, percentage of the heat transfer enhancement (\( \%\text{HTE} \)), according relation (12), resulting from packing the empty channel by cylindrical porous materials, is defined.

\[
\%\text{HTE} = \left( \frac{\text{Nu}_{f, \text{porous}} - \text{Nu}_{f, \text{empty}}}{\text{Nu}_{f, \text{empty}}} \right) \times 100
\]  

(12)

It is observed from fig. 8 that the application of porous media can considerably enhance heat transfer from upstream and downstream cylinders. In details, \( \%\text{HTE} \), for up and down cylinders is between \( \%94-176 \). Other interesting topics are that \( \%\text{HTE} \) difference between up and down cylinders, at different Reynolds numbers, remains constant and growth rate of \( \%\text{HTE} \) decreased by increasing Reynolds number for all of cylinders. Here, another index, according relation (13), defined which called heat transfer reduction percentage (\( \%\text{HTR} \)). It is the indicator of studying the effect of Reynolds number on the difference between Nusselt numbers of up and down cylinders in both empty and porous channels. In figure 9, it is seen that Nusselt number decreasing from up cylinder to down cylinder in porous media is greater than that of without porosity media. This phenomenon has importance in heat exchanger operation and efficiency.

\[
\%\text{HTR} = \left( \frac{\text{Nu}_{\text{up cylinder}} - \text{Nu}_{\text{down cylinder}}}{\text{Nu}_{\text{up cylinder}}} \right) \times 100
\]  

(13)

Figure 11 shows pressure drop in flow direction in both channels. However, it is clear that empty channel has lower pressure drop compared to porous channel because of increasing contact surface resulting from porosity of media. Also, it is acceptable that by increasing Reynolds number and consequently intensifying fluid velocity, pressure loss that defined according to relation (14), would be increased. Difference between pressure loss values of porous channel and empty channel, is changed, of order 10000 to 1000 with Reynolds number increasing, fig. 10.

\[
\Delta p = |p_{\text{inlet}} - p_{\text{outlet}}|
\]  

(14)

Another parameter affecting heat transfer in porous channel is Prandtl number that was investigated in fig. 12. For examination this factor, simulation was repeated at Re = 80, 120, and 200 with water as working fluid which has \( \text{Pr} = 6.97 \) and results presented in terms of Peclet number. Results indicate that by increasing Peclet number, Nusselt number is increased.
and also difference between heat flux from up and down cylinders, increased. It is reasonable, because with enlargement of Peclet number, capacity of heat transport by convection in comparison to heat conduction increased.

![Figure 9. Heat transfer enhancement comparison between porous channel with empty channel at various Reynolds number](image1)

![Figure 10. Heat transfer reduction comparison between porous channel with empty channel at various Reynolds number](image2)

![Figure 11. Pressure loss vs. Reynolds number in porous and empty channels](image3)

![Figure 12. Mean Nusselt number vs. Peclet number in porous and empty channels](image4)

**Conclusions**

In this investigation, numerical computations for laminar flow around three isothermal circular cylinders in staggered configuration have been carried out and heat transfer characteristics of them in empty channel and porous channel were compared. The main goal of this research is considering the effect of porosity on the enhancement of Nusselt number for tubes at Reynolds number 80, 120, and 200 with considering basic equations of fluid-flow and energy without the use of modeling. Numerical method that used is based on the finite volume method. Some of the main conclusions are as follows.

- The mean Nusselt number of upstream cylinder is higher than that of downstream cylinder in both channels.
- Using porous media, heat transfer from both first and second cylinders enhanced considerably, however increasing Nusselt number for first cylinder is larger than second one.
- In both medium, by increasing Reynolds number, the mean Nusselt number increased but the growth rate of it in porous medium is higher.
- Nusselt number difference between upstream and downstream cylinders in porous medium is higher than that of in without porosity media but by increasing Reynolds number this parameter decreased in all of them.
About pressure drop along channel, it can be said that porous channel experiences larger pressure loss in comparison with empty channel, however in both of them as well as Reynolds number increased, pressure drops to lower value.

Porosity causes that thermal boundary-layer around any of cylinders would be less thick than that of in without porosity media and consequently thermal interaction between downstream cylinders with upstream one would be slighter, and

By increasing Peclet number, transport phenomena of fluid enlarged and consequently heat transfer increased.

**Nomenclature**

- $D$ – cylinder diameter, [m]
- $d_p$ – particle diameter, [m]
- $H$ – channel height, [m]
- $h$ – mean convection heat transfer coefficient, [W m$^{-2}$ K$^{-1}$]
- $h_o$ – local convection heat transfer coefficient, [W m$^{-2}$ K$^{-1}$]
- $k_f$ – fluid thermal conductivity, [W m$^{-1}$ K$^{-1}$]
- $L$ – channel length, [m]
- $M$ – horizontal distance between two particles, [m]
- $N_u$ – mean Nusselt number, ($= hD/k$)
- $N_{u_0}$ – local Nusselt number, ($= h_0D/k$)
- $P$ – dimensionless fluid pressure, ($= p/(p_0U_0^2)$)
- $Pc$ – Peclet number, ($= U_0D/a_0$)
- $Pr$ – Prandtl number, ($= \nu/\alpha$)
- $p$ – fluid pressure, [N m$^{-2}$]
- $Re$ – Reynolds number, ($= U_0D/\nu$)
- $S_0$ – longitudinal pitch, [m]
- $S$ – transversal pitch, [m]
- $T$ – temperature, [K]
- $T_f$ – fluid temperature, [K]
- $T_w$ – cylinder wall temperature, [K]
- $T_0$ – inlet fluid temperature, [K]
- $U$ – dimensionless horizontal component of fluid velocity, ($= u/U_0$)
- $U_0$ – horizontal component of inlet fluid velocity
- $V$ – dimensionless vertical component of fluid velocity, ($= v/U_0$)
- $W$ – vertical component of fluid velocity [m$^{-1}$]
- $X$ – dimensionless horizontal co-ordinates, ($= x/D$)
- $Y$ – dimensionless vertical co-ordinates, ($= y/D$)
- $Y$ – vertical coordinates, [m]

**Greek symbols**

- $\alpha_f$ – fluid thermal diffusivities, [m$^{2}$s$^{-1}$]
- $\epsilon$ – porosity
- $\nu_f$ – fluid kinematic viscosity, [m$^{2}$s$^{-1}$]
- $\theta$ – peripheral angle, [°], $\theta = \theta$ at stagnation point

**Subscripts**

- $f$ – fluid

**References**


