In a recent paper, Liversage and Trancossi have defined a new formulation of drag as a function of the dimensionless Bejan and Reynolds numbers. Further analysis of this hypothesis has permitted to obtain a new dimensionless formulation of the fundamental equations of fluid dynamics in their integral form. The resulting equations have been deeply discussed for the thermodynamic definition of Bejan number evidencing that the proposed formulation allows solving fluid dynamic problems in terms of entropy generation, allowing an effective optimization of design in terms of the Second law of thermodynamics. Some samples are discussed evidencing how the new formulation can support the generation of an optimized configuration of fluidic devices and that the optimized configurations allow minimizing the entropy generation.

Key words: fluid dynamic, conservation laws, Bejan number, drag, friction coefficient, entropy, exergy

**Introduction**

*All things are flowing. (Heraclitus)*

Liversage and Trancossi [1] have produced a new definition of the drag force in fluid dynamics problems. They have rewritten the traditional expression of drag force:

\[ D = \frac{1}{2} C_D A_f \rho u^2 \]  

(1)

by determining that the drag coefficient \( C_D \) is based on Reynolds number and Bejan number according to eq. (2):

\[ C_D = \frac{2 A_w}{A_f} \frac{A_w}{A_f} \frac{B_e L}{R e_L^2} \]  

(2)

where \( A_w \) is the wet area, \( A_f \) – the front area, \( R e_L \) – the Reynold Number related to fluid path length, and Bejan number has been adopted in the diffusive definition by Bhattacharjee and Grosshandler [2], as formulated by Mahmud and Fraser [3, 4] and generalized by Awad and Lage [5] and Awad [6, 7]:

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\[ \text{Be} = \frac{\Delta l^2}{\mu v} = \frac{\Delta l^2}{\rho v^2} = \frac{\rho \Delta l^2}{\mu^2} \]  

(3)

Trancossi and Sharma [8] have verified eq. (2) in the case of a low thickness high chamber wing profile with a good accuracy versus both experimental and CFD data. Trancossi and Pascoa [9] have produced an adequate formulation of the conservation laws in fluid dynamics, that takes advantage of new container terms, which are based on Bejan number, and a new derived term, which has been named Bejan number \( \xi \), which has the same dimensional formulation and is obtained:

\[ \text{Be} = \frac{l^2 \Delta p}{\mu v} = \frac{l^2 \Delta p}{\rho v^2} = \frac{l^2}{\rho v^2} (p_m - p_n) = \xi_m - \xi_n \]  

(4)

where \( m \) and \( n \) are a generic point of the domain

\[ \xi_m = \frac{l^2}{\rho v^2} p_m \]  

(5)

A more accurate analysis of the formulation in eq. (5), shows evidently that in fluid dynamics is equivalent to the Hagen number:

\[ \text{Hg} = \frac{\Delta P}{L \mu v} = \frac{\Delta P}{L \mu^2} \]  

(6)

Awad [10] have analyzed the Hagen number vs. the diffusive Bejan number. The first difference is that they have radically different physical meanings: Hagen number represents the dimensionless pressure gradient while the Bejan number represents the dimensionless pressure drop. In particular, they are coincident in the cases where the characteristic length, \( l \), is equal to the flow length, \( L \). The definition of Bejan energy relates to Hagen number by the following relation:

\[ \text{Hg} = \frac{L^3}{\rho v^2} \frac{dp}{dx} = L \frac{d}{dx} \left( \frac{L^2 p}{\rho v^2} \right) = L \frac{d \xi}{dx} \]  

(7)

Consequently, Bejan energy can be defined as the rate of change of the Bejan energy over the flow path. Trancossi and Pascoa have also expressed a still preliminary formulation of conservation laws as a function of Bejan number and Bejan energy.

Awad [11] and Klein et al. [12] have remarked that also the second definition of Bejan number, a second dimensionless number is defined Bejan number in scientific literature as the efficiency of convective heat exchange according to the second law of thermodynamics by Sciubba [13]. Efficiency according to First law is the ratio of useful output and used input.

\[ \eta = \frac{W_{\text{out}}}{E_{\text{in}}} \]  

(8)

Energy efficiency is a fundamental parameter of energy analysis, but it does not seem adequate for the effective optimization of a physical system. For design analysis and optimization, effectiveness seems much more adequate. The processes that involve heat exchange allows assessing the heat-exchange effectiveness:
\[ \eta = \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} \]  
(9)

A more general definition applies to any physical phenomenon and is obtained in terms of the ratio of the efficiencies according to the First law of thermodynamics:

\[ \eta = \frac{\eta_{\text{actual}}}{\eta_{\text{ideal}}} = \frac{W_{\text{out,actual}}}{W_{\text{out,ideal}}} \]  
(10)

Efficiency can also be defined according to the Second law of thermodynamics in terms of a ratio of entropy generations:

\[ \eta = \frac{S_{\text{desired}}}{S_{\text{total}}} = \frac{\dot{S}_{\text{desired}}}{\dot{S}_{\text{desired}} + \dot{S}_{\text{undesired}}} \]  
(11)

In convective heat exchange and fluid dynamic processes can be stated that irreversibility arises because of the combination of heat transfer phenomena and viscous effects of the fluid [14, 15]. Similar considerations allowed Bejan and Sciubba [16] to define the efficiency of any process involving fluid flow and convective heat exchange according to the Second law of thermodynamics. In the case of heat exchange phenomena, effectiveness is the ratio of entropy generations in a convective exchange fluid flow and the total entropy generation by both the heat exchanged and the pressure drop which is proportional to the necessary pumping or propulsive power:

\[ \eta_{\text{local}} = \frac{\dot{S}_{\text{gen,}\Delta T}}{\dot{S}_{\text{gen,}\Delta T} + \dot{S}_{\text{gen,}\Delta p}} \]  
(12)

In particular, Sciubba defined such a magnitude local Bejan number is the dimensionless ratio of the entropy created by heat transfer and the total entropy. Sciubba has also provided the integral formulation at the domain level, with volume V. It can be defined as the global Bejan number and has the following formulation:

\[ \overline{\text{Be}} = \frac{\dot{S}_{\text{gen,}\Delta T}}{\dot{S}_{\text{gen,}\Delta T} + \dot{S}_{\text{gen,}\Delta p}} = \frac{\int V \dot{S}_{\text{gen,}\Delta T} dV}{\int V \dot{S}_{\text{gen,}\Delta T} dV + \int V \dot{S}_{\text{gen,}\Delta p} dV} \]  
(13)

It can also be expressed:

\[ \overline{\text{Be}} = \frac{\dot{S}_{\text{gen,}\Delta T}}{\dot{S}_{\text{gen,}\Delta T} + \dot{S}_{\text{gen,}\Delta p}} = \frac{1}{1 + \text{Br}} \]  
(14)

where \( \text{Br} \) is Brinkman number, which is equal to:

\[ \text{Br} = \frac{\mu u^2}{k \Delta T} \]  
(15)

where \( u \) is the velocity of the fluid, and \( k \) is the conductivity. Consequently, it results:

\[ \overline{\text{Be}} = \frac{1}{1 + \text{Br}} = \frac{1}{1 + \frac{\mu u^2}{k \Delta T}} = \frac{k \Delta T}{k \Delta T + \mu u^2} \]  
(16)
It is evident that the Bejan number definition by Sciubba applies only to convective heat exchange. The diffusive definition by Bhattacharjee and Grosshandler is much more general because it can be applied to any fluid dynamic phenomenon, including convective heat exchange.

**Friction losses in fluids**

Herwig and Schmandt [17] have studied the possibility of more effective modeling of losses in terms of drag for external flows and head loss for internal flows in terms of entropy generation or exergy disruption rate.

Fluid flows are treated according to two different categories. External flows can be expressed in terms of drag force [18-21]:

\[ F_D = C_D A_f \frac{\rho}{2} u_v^2 \] (17)

where \( C_D \) is the drag coefficient, and \( A_f \) is front section area.

Two different formulations of drag are present in literature. The first one is a function of head loss [21-25] coefficient:

\[ \Delta P = K \frac{\rho}{2} u_{av}^2 \] (18)

where \( K \) is the head loss coefficient. In both cases, a second pressure loss formulation can be adopted. It can be expressed as a function of friction coefficient \( f \):

\[ \frac{dp}{dx} = -f \frac{\rho}{D} u_{av}^2 \rightarrow \Delta P = f \frac{L}{D} \frac{\rho}{2} u_{av}^2 \] (19)

where \( f \) is the friction factor, \( L \) – the flow path length, and \( D \) – the hydraulic diameter. Friction factor \( f = \tau/(\rho u^2/2) \) represents the friction effect of shear stress. It is not fully representative of friction phenomena. Drela [26] has observed that the friction factor represents only what happens on the surface being defined by eq. (19).

\[ D_f = \iint_{\gamma=0} \left[ \frac{\tau_{xy}}{\rho} \right] \, dx \, dz = \iint \rho \, u_{av}^3 \, f \, dx \, dz \] (20)

Head loss or dissipative terms consider all the losses in the boundary layer along its development and after the detachment.

\[ \Phi = \iint_{\gamma=0} \left[ \frac{\delta}{\rho} \frac{\partial u}{\partial y} \right] \, dx \, dz = \iint \rho \, u_{av}^3 \, C_D \, dx \, dz \] (21)

The meaning of the two terms can be explained at the domain level by fig. 1, in a case of external fluid dynamics, and fig. 2, in the case of internal fluid dynamics.

Consequently, this paper accounts only the losses according to the dissipative model, because they give an exhaustive answer to the problem of the determination of losses.

Any real process in the domain of fluid dynamics generates losses of mechanical energy that increases the internal energy of the fluid. Thus, the total energy is maintained. It means that energetic analysis in fluid dynamics deals with energy availability [27] and usefulness [28] and then with the second law of thermodynamics [29]. It can be expressed as the entropy generation or degradation of exergy or reduction of available work, and degradation of available energy in the flow field.
If \( \dot{S} \) it is the rate of entropy generation, the processes can be analyzed in terms of energy destruction rate:

\[
\dot{X}_{\text{loss}} = T_0 \dot{S}_{\text{gen}}
\]  

(22)

The dissipated mechanical power is equal to:

\[
\dot{E}_L = F_0 u_{\text{ref}} = \Delta P A u_{\text{ref}} = \dot{X}_{\text{loss}} = T_0 \dot{S}_{\text{gen,f}}
\]  

(23)

The case of an internal and an external fluid dynamic problem are reported in tab. 1.

**Bejan number and the Second law of thermodynamics**

Assuming \( u_{\text{ref}} \) the reference velocity for the specific problem, is evident that the general expression of pressure losses is:
Thus, the Bejan number related to losses can be expressed:

\[
\text{Be}_L = \frac{l^2}{\rho v^2} \Delta p = \frac{1}{\rho A_{u_{\text{ref}}}} \frac{l^2}{v^2} T \Delta S_{\text{gen}} = \frac{l^2}{mv^2} T \Delta S_{\text{gen}}
\]  

Equations (24a) and (24b) allows demonstrating that Bejan number related to fluid-dynamic problems is a thermodynamic related magnitude that refers to entropy generation and exergy dissipation rate.

### Integral equations of fluid dynamics

The integral equations of fluid dynamics can be expressed as follows.

**Conservation of mass**

The equation of conservation of mass is:

\[
\dot{m}_i = \dot{m}_e = \dot{m} \rightarrow \rho \dot{A}_i u_i = \rho \dot{A}_e u_e = \dot{m}
\]  

and being:

\[
\rho \dot{A}_i u_{i,x} = \dot{m}_{i,x}; \rho \dot{A}_i u_{i,y} = \dot{m}_{i,y}; \rho \dot{A}_e u_{e,x} = \dot{m}_{e,x}; \rho \dot{A}_e u_{e,y} = \dot{m}_{e,y}
\]

it results

\[
\dot{m}_i = \dot{m}_e = \dot{m} \rightarrow \dot{m}_i^2 = \dot{m}_e^2 \rightarrow \dot{m}_{i,x}^2 + \dot{m}_{i,y}^2 = \dot{m}_{e,x}^2 + \dot{m}_{e,y}^2
\]

where

\[
\dot{m}_i = \dot{m}_e = \dot{m} \rightarrow \rho \dot{A}_i u_i = \rho \dot{A}_e u_e = \dot{m}
\]
And, according to fig. 3.

\[
\begin{align*}
\dot{m}_{i,x} &= m_i \cos \phi_i \\
\dot{m}_{i,y} &= m_i \sin \phi_i \\
\dot{m}_{e,x} &= m_e \cos \phi_e \\
\dot{m}_{e,y} &= m_e \sin \phi_e
\end{align*}
\] (27)

**Conservation of momentum**

\[
\begin{align*}
p_{i,x} A_t - p_{e,x} A_e + R_x &= -\rho A_t u_{i,x} + \rho A_e u_{e,x} \\
p_{i,y} A_t - p_{e,y} A_e + R_y - W &= -\rho A_t u_{i,y} + \rho A_e u_{e,y}
\end{align*}
\] (28)

\[
\begin{align*}
p_{i,x} A_t - p_{e,x} A_e + R_x &= -\dot{m}_i u_{i,x} + \dot{m}_e u_{e,x} \\
p_{i,y} A_t - p_{e,y} A_e + R_y - W &= -\dot{m}_i u_{i,y} + \dot{m}_e u_{e,y}
\end{align*}
\] (29)

\[
\begin{align*}
p_A \cos \phi_t - p_e A_e \cos \phi_e + R_x &= -\rho A_t u_{i,x} + \rho A_e u_{e,x} = \\
&= -\rho A_t \cos \phi_i u_i^2 + \rho A_e \cos \phi_e u_e^2 \\
p_A \sin \phi_t - p_e A_e \sin \phi_e + R_y - W &= -\rho A_t u_{i,y} + \rho A_e u_{e,y} = \\
&= -\rho A_t \sin \phi_i u_i^2 + \rho A_e \sin \phi_e u_e^2
\end{align*}
\] (30)

\[
\begin{align*}
p_A \cos \phi_t - p_e A_e \cos \phi_e + R_x &= -\dot{m}_i u_{i,x} + \dot{m}_e u_{e,x} = \\
&= -\dot{m} \frac{\dot{m}}{\rho A_t} \cos \phi_i + \dot{m} \frac{\dot{m}}{\rho A_e} \cos \phi_e = -\frac{\dot{m}^2}{\rho A_t} \cos \phi_i + \frac{\dot{m}^2}{\rho A_e} \cos \phi_e \\
p_A \sin \phi_t - p_e A_e \sin \phi_e + R_y - W &= -\dot{m}_i u_{i,y} + \dot{m}_e u_{e,y} = \\
&= -\dot{m} \frac{\dot{m}}{\rho A_t} \cos \phi_i + \dot{m} \frac{\dot{m}}{\rho A_e} \cos \phi_e = -\frac{\dot{m}^2}{\rho A_t} \cos \phi_i + \frac{\dot{m}^2}{\rho A_e} \cos \phi_e
\end{align*}
\] (31)

If the considered system can be considered horizontal or at large curvature, eq. (30) becomes:
Conservation of energy

The general expression of the conservation in a fluid can be expressed in both differential and integral form. The integral form has been reported in eqs. (34) and (35).

\[
m \left( \frac{u_i^2}{2} + g z_i + \frac{p_i}{\rho_i} \right) + \dot{W}_{\text{in}} + \dot{Q}_{\text{in}} = \dot{m} \left( \frac{u_i^2}{2} + g z_i + \frac{p_i}{\rho_i} \right) + \dot{W}_{\text{out}} + \dot{Q}_{\text{out}} + m \frac{\Delta p}{\rho_{\text{av}}}
\]

(34)

where \( W_{\text{in}} \) is the work input, \( Q_{\text{in}} \) – the heat input, \( W_{\text{out}} \) – the work output, and \( Q_{\text{out}} \) – the heat output.

\[
\frac{I^2}{v^2} u_e^2 + \frac{I^2}{v^2} g z_e + \frac{I^2}{v^2} \frac{p_e}{\rho_e} + \frac{I^2}{v^2} \frac{W_{\text{in}}}{m} + \frac{I^2}{v^2} \frac{Q_{\text{in}}}{m} = \frac{I^2}{v^2} u_e^2 + \frac{I^2}{v^2} g z_e + \frac{I^2}{v^2} \frac{p_e}{\rho_e} + \frac{I^2}{v^2} \frac{W_{\text{out}}}{m} + \frac{I^2}{v^2} \frac{Q_{\text{out}}}{m} + \frac{I^2}{v^2} \frac{\Delta p}{\rho_{\text{av}}}
\]

(35)

If the fluid is compressible it can be assumed the ideal gas law, it results:

\[
\rho = \frac{p}{RT}
\]

and then

\[
 \frac{I^2}{v^2} u_e^2 + \frac{I^2}{v^2} g v + \frac{I^2}{v^2} RT + \frac{I^2}{v^2} \frac{W_{\text{in}}}{m} + \frac{I^2}{v^2} \frac{Q_{\text{in}}}{m} = \frac{I^2}{v^2} u_e^2 + \frac{I^2}{v^2} g v + \frac{I^2}{v^2} RT + \frac{I^2}{v^2} \frac{W_{\text{out}}}{m} + \frac{I^2}{v^2} \frac{Q_{\text{out}}}{m} + \frac{I^2}{v^2} \frac{\Delta p}{\rho_{\text{av}}}
\]

If \( \Delta p = p_i - p_e, \Delta u = u_i - u_e, \Delta z = z_i - z_e \), it becomes:

\[
\left( \frac{\Delta u^2}{2g} + \frac{\Delta z}{\rho_{\text{av}} g} + \frac{\Delta p}{\rho_{\text{av}} g} \right) + \frac{\Delta W}{mg} + \frac{\Delta Q}{mg} = \frac{\Delta p}{\rho_{\text{av}} g}
\]

(36)
A new expression of fluid dynamics laws

If we consider a 2-D flow in a 2-D closed domain, fig. 1, the steady-state of any fluid system \([30, 31]\) can be described by conservation laws for the control volume with inlets and outlets of mass, energy, work together with internal dissipations, fig. 1.

\[
\Delta p = \rho g \Delta y = \frac{\rho}{2} \Delta u^2
\]  

Equation (37) can be expressed:

\[
\left( \frac{\Delta u^2}{2} + \frac{l^2}{v^2} g \Delta y + \frac{l^2}{v^2} \Delta p \right) + \frac{l^2}{v^2} \frac{\Delta W}{\dot{m}} + \frac{l^2}{v^2} \frac{\Delta Q}{\dot{m}} = \frac{l^2}{\rho v^2} \rho \Delta \xi
\]

By considering the definition of Bejan Number by Mahmud and Fraser, which is expressed by equation (3) it is possible to define the following terms.

**Definition of Bejan numbers and Bejan energies**

**Pressure Bejan number**

\[
\text{Be}_p = \frac{l^2 \Delta p}{\rho v^2} = \frac{l^2}{v^2} \frac{p_i - p_e}{\rho_{av}} = \frac{l^2}{v^2} \left( \frac{p_i}{\rho} - \frac{p_e}{\rho_e} \right) = \xi_{p,i} - \xi_{p,e}
\]

or

\[
\text{Be}_p = \frac{l^2}{v^2} \frac{\Delta p}{\rho_{av}} = \frac{l^2}{v^2} \left( RT_i - RT_e \right) = \xi_{p,i} - \xi_{p,e}
\]

where \(\rho_{av} = (\rho_i + \rho_e)/2\). Hence, if \(\rho = \text{constant}\) it simplifies as:

\[
\text{Be}_p = \frac{l^2}{v^2} \frac{\Delta p}{\rho} = \frac{l^2}{v^2} \frac{p_i - p_e}{\rho} = \xi_{p,i} - \xi_{p,e}
\]

**Kinetic Bejan number**

\[
\text{Be}_K = \frac{l^2}{v^2} \frac{\Delta u^2}{2} = \frac{l^2}{v^2} \frac{\Delta p}{\rho_{av}} = \frac{l^2}{v^2} \left( \frac{u_i^2}{2} - \frac{u_e^2}{2} \right) = \xi_{K,i} - \xi_{K,e}
\]

\(\rho = \text{constant} \rightarrow \text{Be}_K = \frac{l^2}{\rho v^2} \left( \rho u_i^2 - \rho u_e^2 \right) = \xi_{K,i} - \xi_{K,e}\)

**Hydrostatic (potential) Bejan number**

\[
\text{Be}_H = \frac{l^2}{v^2} \Delta g = \frac{l^2}{v^2} \frac{\Delta p}{\rho_{av}} = \frac{l^2}{v^2} g (y_i - y_e) = \xi_{H,i} - \xi_{H,e}
\]

\(\rho = \text{constant} \rightarrow \text{Be}_H = \frac{l^2}{\rho v^2} \left[ \rho g (y_i - y_e) \right] = \xi_{H,i} - \xi_{H,e}\)
Work Bejan number

\[
\text{Be}_w = \frac{l^2}{v^2} \frac{\Delta W}{\dot{m}} = \frac{l^2}{v^2} \frac{\dot{W}_e - \dot{W}_i}{\dot{m}} = \frac{l^2}{v^2} \frac{\dot{W}_i - \dot{W}_e}{\dot{m}} = \xi_{W,i} - \xi_{W,e} \tag{45}
\]

Heat Bejan number

\[
\text{Be}_Q = \frac{l^2}{v^2} \frac{\Delta \dot{Q}}{\dot{m}} = \frac{l^2}{v^2} \frac{\dot{Q}_i - \dot{Q}_e}{\dot{m}} = \xi_{Q,i} - \xi_{Q,e} \tag{46}
\]

Loss Bejan number

\[
\text{Be}_L = \frac{l^2}{v^2} \frac{\Delta p_L}{\rho_{av} \dot{m}} = 0 - \frac{l^2}{v^2} \frac{\Delta p_L}{\rho_{av} \dot{m}} = 0 - \xi_L \tag{47}
\]

Reaction Bejan number

\[
\xi_R = \frac{l^2}{\rho_{av} v^2} \frac{R}{A_w} \tag{48}
\]

Weight Bejan number

\[
\xi_G = \frac{l^2}{\rho_{av} v^2} \frac{m}{A_w g} \tag{49}
\]

Conservation equations

It is then possible to determine the different equations of the conservation laws.

Conservation of mass

\[
\dot{m}_i = \dot{m}_e \rightarrow A_e \rho_i u_i = A_e \rho_e u_e \rightarrow A_e \frac{\rho_i u_i}{\mu} = A_e \frac{\rho_e u_e}{\mu} \rightarrow A_e \text{Re}_e = A_e \text{Re}_e
\]

\[
\dot{m}_i = \dot{m}_e \rightarrow A_e \rho_i u_i = A_e \rho_e u_e \rightarrow A_e \sqrt{\rho_1} \left( \frac{u_i}{\mu} \right)^2 = A_e \sqrt{\rho_e} \left( \frac{u_e}{\mu} \right)^2
\]

\[
A_e \sqrt{\rho_1} \sqrt{\xi_{k,i}} = A_e \sqrt{\rho_e} \sqrt{\xi_{k,e}} \tag{50}
\]

Relation between Bejan energy and Reynolds number

\[
\text{Re} = \sqrt{\frac{\rho}{\rho_1}} \sqrt{\xi_k} \rightarrow \xi_k = \frac{\text{Re}^2}{\rho} \tag{51}
\]
Conservation of momentum

\[
\frac{l^2}{\rho_0 v^2} \left( A_i \cos \phi_i p_i - A_e \cos \phi_e p_e \right) + \frac{l^2}{\rho_0 v^2} R_y = -A_i \cos \phi_i \frac{l^2}{\rho_0 v^2} \rho_0 u_i^2 + A_e \cos \phi_e \frac{l^2}{\rho_0 v^2} \rho_e u_e^2
\]

\[
= A_i \sin \phi_i \left( \xi_{p,i} - \xi_{K,i} \right) - A_e \sin \phi_e \left( \xi_{p,e} + \xi_{K,e} \right) + \frac{l^2}{\rho_0 v^2} R_y - \frac{l^2}{\rho_0 v^2} mg
\]

\[
= A_i \sin \phi_i \frac{l^2}{\rho_0 v^2} \rho_0 u_i^2 + A_e \sin \phi_e \frac{l^2}{\rho_0 v^2} \rho_e u_e^2
\]

Bernoulli theorem

Bernoulli theorem equation is obtained by the law of conservation of momentum expressed on a flow line and can be expressed:

\[
\frac{1}{2} \rho u^2 + \rho g z + p = \text{constant}
\]

Bernoulli theorem equation is expressed:

\[
\xi_{K,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,in} + \xi_{Q,in} = -A_i \xi_{K,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,in} + \xi_{Q,in} = -A_i \xi_{K,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,in} + \xi_{Q,in}
\]

(52)

Conservation of energy

Conservation of energy equation is expressed:

\[
\xi_{K,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,in} + \xi_{Q,in} = -A_i \xi_{K,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,in} + \xi_{Q,in} + \xi_{K_H+P} = -A_i \xi_{K,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,in} + \xi_{Q,in} + \xi_{K_H+P}
\]

(53)

(54a)

Assuming that Bejan energies could be summed it is possible to write a condensed form:

\[
\xi_{K_H+P} = \xi_{K_H+P}
\]

(54b)

or in function of Bejan number:

\[
(\text{Be}_K + \text{Be}_H + \text{Be}_P) = (\text{Be}_{K_H+P}) = 0
\]

(55)

Conservation of energy

Conservation of energy equation is expressed:

\[
\xi_{K,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,in} + \xi_{Q,in} = -A_i \xi_{K,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,in} + \xi_{Q,in} + \xi_{K_H+P} = -A_i \xi_{K,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,in} + \xi_{Q,in} + \xi_{K_H+P}
\]

(56)

(57)

\[
(\text{Be}_K + \text{Be}_H + \text{Be}_P) + \text{Be}_w + \text{Be}_Q = \text{Be}_L
\]

(58)
The Second law of thermodynamics and fluid laws

Being expressed in terms of the pressure difference (3), Bejan number caused by fluid dynamic losses has been referred to both exergy dissipation and entropy generation (23). Natterer and Camberos [35] and Arntz et al. [36, 37] evidenced the correspondence between exergy states and fluid dynamic energy states. As it will be easy to show the importance of the use of Bejan number and/or Bejan energy in the above equations is that they can be directly related to second law of thermodynamics.

Second law expression of Bejan number and Bejan energy

It is consequently possible to express all the terms in the conservation equations in thermodynamic terms according to second law in terms of both entropy created or exergy destroyed:

\[
\text{Be}_L = \frac{l^2}{\rho v^2} \Delta p = \frac{1}{\rho A_w u_{\text{ref}}} \frac{l^2}{v^2} T \Delta \dot{S}_{\text{gen}} = \frac{l^2}{m v^2} T \Delta \dot{S}_{\text{gen}}
\]

\[
\text{Be}_L = \frac{l^2}{\rho v^2} \Delta p = \frac{1}{\rho A_w u_{\text{ref}}} \frac{l^2}{v^2} \Delta \dot{X}_{\text{loss}} = \frac{l^2}{m v^2} \Delta \dot{X}_{\text{loss}}
\]

Pressure Bejan number

\[
\text{Be}_p = \frac{l^2}{v^2} \frac{\Delta p}{\rho_{av}} = \frac{l^2}{m v^2} T \Delta \dot{S}_p = \frac{l^2}{m v^2} \Delta \dot{X}_p
\]  

(59)

or

\[
\text{Be}_p = \xi_{p_j} - \xi_{p_s} = \frac{l^2}{m v^2} \dot{X}_{p_j} - \frac{l^2}{m v^2} \dot{X}_{p_s} = \frac{l^2}{m v^2} (\dot{X}_{p_j} - \dot{X}_{p_s})
\]  

(60)

Kinetic Bejan number

Being exergetic state correspondent to kinetic energy:

\[
\dot{X}_K = \frac{1}{2} \rho u^2
\]

it results:

\[
\text{Be}_K = \frac{l^2}{v^2} \frac{\Delta p_K}{\rho} = \frac{l^2}{v^2} \frac{1}{\rho} \frac{T}{2} \rho (u_i^2 - u_e^2) = \frac{l^2}{m v^2} T \Delta \dot{S}_{K,\text{gen}} = \frac{l^2}{m v^2} \Delta \dot{X}_{K,\text{loss}}
\]  

(61)

where Be\(_K\)

\[
\text{Be}_K = \xi_{K,i} - \xi_{K,e} = \frac{l^2}{m v^2} \dot{X}_{K,i} - \frac{l^2}{m v^2} \dot{X}_{K,e} = \frac{l^2}{m v^2} (\dot{X}_{K,i} - \dot{X}_{K,e})
\]  

(62)

Hydrostatic (potential) Bejan number

It is:

\[
\text{Be}_H = \frac{l^2}{v^2} g \Delta y
\]

(63)
and consequently:

$$\text{Be}_{HL} = \frac{\xi_H}{\xi_{H,e}} = \frac{l^2 \dot{X}_{H,i} - l^2 \dot{X}_{H,e}}{m v^2}$$

(64)

**Work Bejan number**

It is evidently related to second law:

$$\text{Be}_W = \frac{l^2}{v^2} \frac{\Delta W}{m} = \frac{l^2}{m v^2} (\dot{W}_{in} - \dot{W}_{out}) = \frac{l^2}{m v^2} \Delta \dot{S}_W = \frac{l^2}{m v^2} \Delta \dot{X}_W$$

(65)

Hence, assuming that,

$$\dot{X}_K = \frac{1}{2} \rho u^2,$$

It results:

$$\text{Be}_W = \xi_{W,in} - \xi_{W,out} = \frac{l^2}{m v^2} (\dot{W}_{in} - \dot{W}_{out}) = \frac{l^2}{m v^2} \dot{X}_{W,in} - \frac{l^2}{m v^2} \dot{X}_{W,out}$$

(66)

**Heat Bejan number**

It is also related to second law:

$$\text{Be}_Q = \frac{l^2}{v^2} \frac{\Delta Q}{m} = \frac{l^2}{m v^2} (\dot{Q}_{in} - \dot{Q}_{out}) = \frac{l^2}{m v^2} \Delta \dot{S}_Q = \frac{l^2}{m v^2} \Delta \dot{X}_Q$$

(67)

and results:

$$\text{Be}_W = \xi_{Q,in} - \xi_{Q,out} = \frac{l^2}{m v^2} (\dot{Q}_{in} - \dot{Q}_{out}) = \frac{l^2}{m v^2} \dot{X}_{Q,in} - \frac{l^2}{m v^2} \dot{X}_{Q,out}$$

**Loss Bejan number**

$$\text{Be}_L = \frac{l^2}{v^2} \frac{\Delta P_L}{\rho_{av}} = 0 - \frac{l^2}{v^2} \frac{\Delta P_L}{\rho_{av}} = \frac{l^2}{m v^2} T \Delta \dot{S}_{gen} = \frac{l^2}{m v^2} \Delta \dot{X}_{loss}$$

(68)

**Reaction Bejan number**

$$\xi_R = \frac{l^2}{\rho_{av} v^2} \frac{R}{A_w}$$

(69)

**Weight Bejan number**

$$\xi_G = \frac{l^2}{\rho_{av} v^2} \frac{m}{g}$$

(70)

**Second law expression of conservation laws**

It is then possible to determine the different equations of the conservation law of thermodynamics.
Conservation of mass

\[ A_e \sqrt{\rho_e} \sqrt{\frac{l^2 S_{K,j}}{m v^2}} = A_e \sqrt{\rho_e} \sqrt{\frac{l^2 \dot{X}_{K,j}}{m v^2}} = A_e \sqrt{\rho_e} \sqrt{\frac{l^2 \dot{S}_{K,e}}{m v^2}} = A_e \sqrt{\rho_e} \sqrt{\frac{l^2 \dot{X}_{K,e}}{m v^2}} \]  \hspace{1cm} (71)

which is

\[ A_e \sqrt{\frac{S_{K,j}}{A_{u_j}}} = A_e \sqrt{\frac{\dot{X}_{K,j}}{A_{u_j}}} = A_e \sqrt{\frac{\dot{S}_{K,e}}{A_{u_e}}} = A_e \sqrt{\frac{\dot{X}_{K,e}}{A_{u_e}}} \]  \hspace{1cm} (72)

Conservation of momentum

\[
\begin{align*}
A_e \left[ \cos \phi \frac{l^2}{m v^2} (\dot{X}_{p,j} - \dot{X}_{K,j}) - A_e \cos \phi \frac{l^2}{m v^2} (\dot{X}_{p,e} + \dot{X}_{K,e}) \right] &= -A_w \xi_{R,x} \\
A_e \left[ \sin \phi \frac{l^2}{m v^2} (\dot{X}_{p,j} + \dot{X}_{K,j}) - A_e \sin \phi \frac{l^2}{m v^2} (\dot{X}_{p,e} + \dot{X}_{K,e}) \right] &= -A_w \xi_{R,y} + A_w \xi_{G}
\end{align*}
\]  \hspace{1cm} (73)

Bernoulli theorem

\[
\frac{l^2 \dot{X}_{K,j}}{m v^2} + \frac{l^2 \dot{X}_{H,j}}{m v^2} + \frac{l^2 \dot{X}_{P,j}}{m v^2} = \frac{l^2 \dot{X}_{K,e}}{m v^2} + \frac{l^2 \dot{X}_{H,e}}{m v^2} + \frac{l^2 \dot{X}_{P,e}}{m v^2}
\]  \hspace{1cm} (74)

and become:

\[
\xi_{K+H+P,j} = \frac{l^2}{m v^2} (\dot{X}_{K,j} + \dot{X}_{H,j} + \dot{X}_{P,j}) = \xi_{K+H+P,e} = \frac{l^2}{m v^2} (\dot{X}_{K,e} + \dot{X}_{H,e} + \dot{X}_{P,e})
\]  \hspace{1cm} (75)

or

\[
Be_{K+H+P,j} = \frac{l^2}{m v^2} \left[ (\dot{X}_{K,j} - \dot{X}_{K,e}) + (\dot{X}_{H,j} - \dot{X}_{H,e}) + (\dot{X}_{P,j} - \dot{X}_{P,e}) \right]
\]  \hspace{1cm} (76)

Conservation of energy

Conservation of energy equation can be expressed:

\[
\xi_{k,j} + \xi_{z,j} + \xi_{p,j} + \xi_{w,in} + \xi_{Q,in} = \xi_{k,e} + \xi_{z,e} + \xi_{p,e} + \xi_{w,out} + \xi_{Q,out} + Be_L
\]  \hspace{1cm} (77)

\[
\frac{l^2}{m v^2} (\dot{X}_{K,j} + \dot{X}_{H,j} + \dot{X}_{P,j} + \dot{X}_{w,in} + \dot{X}_{Q,in}) =
\]

\[
= \frac{l^2}{m v^2} (\dot{X}_{K,e} + \dot{X}_{H,e} + \dot{X}_{P,e} + \dot{X}_{w,out} + \dot{X}_{Q,out}) + \frac{l^2}{m v^2} \dot{X}_L
\]  \hspace{1cm} (78)

\[
(\dot{X}_{K,j} + \dot{X}_{H,j} + \dot{X}_{P,j}) + \dot{X}_{w,in} + \dot{X}_{Q,in} = (\dot{X}_{K,e} + \dot{X}_{H,e} + \dot{X}_{P,e}) + \dot{X}_{w,out} + \dot{X}_{Q,out} + \dot{X}_L
\]  \hspace{1cm} (79)

Analysis of a specific sample case

The effects of compressibility can be evaluated by considering the ideal gas law:
If a certain length, $l$, of the pipe is considered, the loss of pressure can be expressed as $\text{Be}_L$ and is:

$$\text{Be}_L = \frac{l^2}{\rho v^2} \Delta p_{\text{loss}} = \frac{1}{\rho A_m u_{\text{ref}}} \frac{l^2}{v^2} T \Delta \dot{S}_{\text{gen}} = \frac{l^2}{\dot{m} v^2} T \Delta \dot{S}_{\text{gen}}$$  \hspace{1cm} (81)

It is then possible to evaluate:

$$\zeta_{P,j} - \zeta_{P,e} = \text{Be}_L \rightarrow \frac{l^2 \dot{X}_{P,j}}{\dot{m} v^2} - \frac{l^2 \dot{X}_{P,e}}{\dot{m} v^2} = \frac{1}{\rho A_m u v^2} T \Delta \dot{S}_{\text{gen}} = \text{Be}_L$$  \hspace{1cm} (82)

from which it result:

$$\frac{l^2}{\rho v^2} (P_1 - P_e) = \frac{1}{\dot{m}} \rho T \Delta \dot{S}_{\text{gen}} \rightarrow (P_1 - P_e) = \frac{1}{\dot{m}} \rho T \Delta \dot{S}_{\text{gen}}$$  \hspace{1cm} (83)

and then:

$$P_1 - P_e = \frac{1}{\dot{m}} \rho T \Delta \dot{S}_{\text{gen}} = \frac{1}{\dot{m} R} \Delta \dot{S}_{\text{gen}} \rightarrow \Delta \dot{S}_{\text{gen}} = \dot{m} R \frac{P_1 - P_e}{P}$$

An accurate verification of the proposed equations allows to consider some samples, fig. 4.

Figure 4. Two samples of flow in ducts

**Throttle flow**

An ideal gas is considered. It can be considered the ideal gas law $pV = RT$ and the constancy of entropy $h_1 = h_2$. By applying the previous method, it can be easily obtained the entropy generation during the flux along a horizontal pipe without friction between two sections 1 and 2.

If the total loss of entropy is $\Delta \rho$

$$\Delta S_{1-2} = -R \int_1^2 \frac{dp}{p} = -R \ln \frac{p_2}{p_1} = R \ln \left(1 - \frac{\Delta \rho}{p_1}\right)$$  \hspace{1cm} (84)

$$\dot{S}_{\text{gen}} = \dot{m} R \frac{\Delta \rho}{p_1}$$  \hspace{1cm} (85)

Moreover, the result which has been obtained in eq. (22) is verified according to scientific literature.
Flow with friction

It can be possible to analyze the steady and adiabatic flow of an ideal gas through the segment of a pipe. In this case, the same result of eq. (22) can be expressed:

$$S_{gen} = mR \ln \frac{\Delta p}{p_1} \rightarrow S_{gen} = mR \frac{\Delta p}{p_1}$$

The work lost because of irreversibility is:

$$\dot{W}_{lost} = m \left[ (h_1 - T_0 s_1) - (h_2 - T_0 s_2) \right] = mT_0A\Delta x_{1-2} = TS_{gen}$$

$$\dot{W}_{lost} = TS_{gen} = mR T_0 \frac{\Delta p}{p_1}$$

It can be remarked that the decrease in exergy is proportional to the pressure drop as well as the mass flow rate.

Conclusions

In a recent paper, Liversage and Trancossi have defined a new formulation of drag as a function of the dimensionless Bejan and Reynolds numbers. Further analysis of this hypothesis has permitted to obtain a new dimensionless formulation of the fundamental equations of fluid dynamics in their integral form. This activity shows that fluid mechanics problems can be modeled by mean of Bejan number, which allows correlating the fluid dynamic problems to entropy generation.

In particular, it demonstrates that fluid dynamic phenomena can be described by new sets of dimensionless equations that account both first and second law of thermodynamics. This direct correlation which currently under further investigation, allows describing fluid phenomena as function of dimensionless exergy dissipation rate or entropy generation rate.

The resulting equations have been deeply discussed for the thermodynamic definition of Bejan number evidencing that the proposed formulation allows solving fluid dynamic problems in terms of entropy generation, allowing an effective optimization of design in terms of the second law of thermodynamics.

A final sample has allowed verifying that in simple cases results coincides with the ones obtained by traditional methods. Further activity is planned in the direction of defining an effective method for solving fluid dynamic problems in the domain of second law of thermodynamics.

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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>area, [m$^2$]</td>
<td></td>
</tr>
<tr>
<td>$A_f$</td>
<td>front Area, [m$^2$]</td>
<td></td>
</tr>
<tr>
<td>$A_w$</td>
<td>wet Area, [m$^2$]</td>
<td></td>
</tr>
<tr>
<td>$Be$</td>
<td>Bejan number</td>
<td></td>
</tr>
<tr>
<td>$Be_H$</td>
<td>hydrostatic Bejan number</td>
<td></td>
</tr>
<tr>
<td>$Be_K$</td>
<td>kinetic Bejan number</td>
<td></td>
</tr>
<tr>
<td>$Be_{CL}$</td>
<td>Bejan number caused by losses</td>
<td></td>
</tr>
<tr>
<td>$Be_{P}$</td>
<td>pressure Bejan number</td>
<td></td>
</tr>
<tr>
<td>$Be_{Q}$</td>
<td>Bejan number by Heat transfer</td>
<td></td>
</tr>
<tr>
<td>$Be_{W}$</td>
<td>Bejan number caused by Work</td>
<td></td>
</tr>
<tr>
<td>$C_D$</td>
<td>drag coefficient</td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration of gravity, [ms$^{-2}$]</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>pressure loss coefficient</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>heat conduction coefficient, [Wm$^{-2}$K$^{-1}$]</td>
<td></td>
</tr>
<tr>
<td>$l$</td>
<td>characteristic length, [m]</td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>mass flow, [kgs$^{-1}$]</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>pressure, [Pa]</td>
<td></td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>pressure difference [Pa]</td>
<td></td>
</tr>
<tr>
<td>$\Delta p_H$</td>
<td>hydrostatic pressure [Pa]</td>
<td></td>
</tr>
<tr>
<td>$\Delta p_L$</td>
<td>pressure losses [Pa]</td>
<td></td>
</tr>
</tbody>
</table>
\[ \dot{Q} \] – heat flux, [W]
\[ \dot{Q}_{in} \] – inlet heat transferred, [J]
\[ \dot{Q}_{out} \] – outlet heat transferred, [J]
\[ \text{Re} \] – Reynolds number, \([- \text{pol} / \mu = \text{ud}/v] \)
\[ \mathcal{S} \] – integral entropy rate, [WK\(^{-1}\)]
\[ \dot{s} \] – local entropy rate, [Wm\(^{-2}\)K\(^{-1}\)]
\[ \dot{s}_{\text{gen,AP}} \] – local entropy rate by dissipation, [Wm\(^{-2}\)K\(^{-1}\)]
\[ \dot{s}_{\text{gen,AT}} \] – local entropy generation by heat exchange, [Wm\(^{-2}\)K\(^{-1}\)]
\[ T \] – temperature, [K]
\[ u \] – velocity, [ms\(^{-1}\)]
\[ u_e \] – outlet velocity, [ms\(^{-1}\)]
\[ u_i \] – inlet velocity, [ms\(^{-1}\)]
\[ W_{\text{in}} \] – power input, [W]
\[ W_{\text{out}} \] – power output, [W]
\[ W_{\text{out}} \] – work output, [J]
\[ W_P \] – pressure power, [W]
\[ X \] – exergy dissipation flux, [W]
\[ X_H \] – hydrostatic exergy dissipation flux, [W]
\[ X_K \] – kinetic exergy dissipation flux, [W]
\[ X_{loss} \] – exergy dissipation flux by loss, [W]
\[ X_P \] – pressure exergy dissipation flux, [W]
\[ X_{Q} \] – exergy dissipation flux by heat, [W]
\[ X_W \] – exergy dissipation flux by work, [W]

**Greek symbols**
- \( \dot{e} \) – heat-exchange effectiveness
- \( \eta \) – efficiency
- \( \mu \) – dynamic viscosity [Pa s]
- \( \nu \) – kinematic viscosity [m\(^2\)/s]
- \( \rho \) – density, [kgm\(^{-3}\)]
- \( \xi_H \) – hydrostatic Bejan energy
- \( \xi_K \) – kinetic Bejan energy
- \( \xi_P \) – Bejan energy caused by losses
- \( \xi_{Q} \) – pressure Bejan Energy
- \( \xi_{W} \) – Bejan energy by Heat transfer
- \( \xi_{W} \) – Bejan Energy caused by Work

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