Failure to adequately equipping the wellhead with appropriate well control equipment may result in a high pressure in the gas drilling when it reaches a high-yield gas layer. This would require the execution of a well killing operation to facilitate follow-up drilling and well completion. An empty well of gas-phase brings implementation of a gas-liquid two-phase transient flow, filling with killing mud. The operation may also entail coupling of the transient seepage from the formation with the gas-liquid two-phase transient flow through the wellbore. In this study, a mathematical model and a numerical method for coupling the transient gas flow through the formation with the gas-liquid two-phase transient flow in the wellbore were developed. The numerical simulation results showed that successful killing of an empty gas-drilled well required proper coordination of several key parameters among which the wellhead backpressure is particularly crucial to the operation. The findings of this study promise to facilitate the process design and parameter optimization for the killing of empty gas-drilled wells.

Key words: gas drilling, well control, gas-liquid two-phase transient flow, formation seepage, well killing

Introduction

When a well killing operation is initiated in an empty gas-drilled well, an empty hole (wellbore) of gas phase is initially formed. A high-density killing fluid is then injected into the wellbore, resulting in the gradual formation of a fluid column [1]. The flow within the wellbore annulus eventually evolves into a gas-liquid two-phase transient flow, with a gradual increase of the bottom-hole flowing pressure (BHFP) and a gradual decrease in the gas production from the formation. The injection of the killing fluid induces a coupling effect between the gas-liquid two-phase flow through the wellbore annulus and the gas seepage through the formation [2, 3]. However, a well-established theory and complete numerical simulation method for this effect have not been developed so far. Existing theories on gas-liquid two-phase flow in a wellbore mainly comprise steady-state theories developed with the
assumption of a constant volume of the gas-liquid mixture. Many factors vary dynamically over the duration of a well killing operation [4-6]. In this study, we developed a method for computing the coupling between the transient formation gas flow and the gas-liquid two-phase transient wellbore flow during the killing of an empty gas-drilled well. The proposed method can be used to determine the optimal parameters for successful killing of an empty gas-drilled well.

**Transient formation gas production rate in changing intermediate conditions**

**Transient distribution of formation pressure**

Distribution of formation pore pressure near borehole wall under Darcy seepage

Darcy’s PDE of transient radial seepage is expressed [7]:

\[
\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{1}{\rho \mu C_i} \frac{\partial p}{\partial t}, \quad \eta = \frac{K}{\rho \mu C_i} \tag{1}
\]

where \( p \) [MPa] is the formation pore pressure, \( \phi \) – the porosity, \( \mu \) [mPa·s] – the fluid viscosity, \( C_i \) [MPa\(^{-1}\)] – the fluid compression coefficient, and \( K \) [mD] – the permeability. The initial condition and the outer and inner boundary conditions are \( p|_{t=0} = p_0 \), \( p|_{r\rightarrow r_e} = p_0 \), and \( p|_{r=r_w} = p_w \), respectively [8], where \( p_w \) [MPa] is the annular pressure and \( p_0 \) [MPa] – the original formation pressure. Equation (1) can be discretized using Taylor’s series expansion:

\[
\frac{\hat{p}_j^{i+1} - \hat{p}_j^i}{\Delta t} = \frac{\hat{p}_j^{i+1} - 2\hat{p}_j^{i+1} + \hat{p}_j^i}{\Delta r^2}, \quad \frac{\hat{p}_j^{i+1} + \hat{p}_j^i}{\Delta t} = \hat{p}_j^i - \hat{p}_j^{i-1}, \quad r = r_w + i\Delta r \tag{2}
\]

The distribution of the pore pressure at any time and location (\( j, i \)) is obtained:

\[
p_j^i = \left[ \frac{\Delta t}{\Delta r^2} + \frac{\Delta t}{\Delta r(r_w + i\Delta r)} \right] p_j^{i+1} + \left[ 1 - \frac{2\eta_j^i \Delta t}{\Delta r^2} + \frac{\eta_j^i \Delta t}{\Delta r(r_w + i\Delta r)} \right] p_j^{i-1} + \eta_j^i \left( \frac{K_j^i}{\phi_j^i \mu C_i} \right) \tag{3}
\]

**Distribution of pore pressure near borehole wall under high-velocity non-Darcy seepage**

This distribution is expressed [9]:

\[
A \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + B \left( \frac{\partial p}{\partial r} \right)^2 = \frac{\mu \phi}{K} \frac{\partial p}{\partial t} \tag{4}
\]

where \( A = (p \sigma Z)/(Z - pZ'(p)) \) and \( B = \sigma + (p \sigma'(p))/(Z - pZ'(p)) \). The initial condition and the outer and inner boundary conditions are \( p|_{r=0} = p_0 \), \( p|_{r=r_e} = p_0 \), and \( p|_{r=r_w} = p_w \), respectively. The same method applied to the Darcy seepage can be used for the differential discretization of the equation to obtain the differential pressure distribution:
where \( A = p_j^i \sigma Z / [Z - p_j^i Z(p_j^i)] \), \( B = \sigma + p_j^i \sigma \sigma(p_j^i) / [Z - p_j^i Z(p_j^i)] \), and \( \sigma \) is Darcy’s correction factor.

**Calculation of gas production rate**

In the condition of Darcy seepage, the gas production rate is given by:

\[
Q_{gs}(t) = \frac{2\pi K h [p_{r_w + \phi}(t)^2 - p_w(t)^2]}{T \mu Z \ln \left( \frac{r_w + dr}{r_w} \right)}
\]

(6)

In the condition of non-Darcy seepage, the production rate is given by:

\[
Q_{gs}(t) = \frac{-A \pm \sqrt{A^2 + 4B \left[ p_{r_w + \phi}(t)^2 - p_{of}(t)^2 \right]}}{2B}
\]

(7)

where

\[
A = \frac{0.4043 \mu Z T h}{k h} \ln \left( 0.472 + \frac{dr + r_w}{r_w} \right) \text{ and } B = \frac{2.947 \times 10^{-21} \beta_{gy} Z T h}{h_{gy}^2} \left( \frac{1}{r_w} - \frac{1}{dr + r_w} \right)
\]

\( p_{r_w + \phi}(t) \) [MPa] is the pressure generated at the differential element boundary [10], \( p_w(t) \) [MPa] – the BHFP, \( Q_{gy}(t) \) [m³d⁻¹] – the gas production rate in standard conditions, \( A \) and \( B \) – the Darcy and non-Darcy seepage coefficients, respectively, \( K \) [mD] – the formation permeability, \( \mu_g \) [Pa·s] – the gas viscosity, \( T \) [K] – the formation temperature, \( h \) [m] – the thickness of the penetrated gas layer, \( r_w \) and \( dr \) [m] – the well diameter and differential element boundary thickness, respectively, and \( \gamma_g \) [–] – the relative density of the gas.

**Variation of gas-liquid two-phase transient flow during killing of empty gas-drilled well**

**Control equation for gas-liquid two-phase transient flow in wellbore**

The following equation applies to a gas-producing layer [11]:

\[
\frac{\partial}{\partial t} (\rho_g E_g A) + \frac{\partial}{\partial z} (\rho_g v_g E_g A) = q_g
\]

(8)

where \( \rho_g \) [kgm⁻³] is the gas density, \( E_g \) [%] – the gas content ratio, \( A \) [m²] – the cross-sectional area of the flow, \( t \) [s] – the time, \( v_g \) [ms⁻¹] – the gas-phase flow rate, \( z \) [m] – the vertical distance, and \( q_g \) [m³s⁻¹] – the gas production rate of the formation.

In the case of a non-gas-producing layer, the following equation applies:
The fluid-phase continuity equation can be expressed:

$$\frac{\partial}{\partial t} (\rho_g E_g A) + \frac{\partial}{\partial z} (\rho_g v_g E_g A) = 0$$

The momentum equation is:

$$\frac{\partial}{\partial t} (\rho_m E_m A) + \frac{\partial}{\partial z} (\rho_m v_m E_m A) = 0$$

Calculation of height of fluid surface

At the initial time of the injection of killing fluid into the bottom hole, the initial gas content is obtained and the height of the fluid column in the first second is calculated. Then the flow patterns are classified, and the pressure gradient and BHFP are determined to obtain the initial BHFP and the corresponding gas production rate. The gas contents for the various classified flow patterns can be used to calculate the gas content for every step size before superimposition. The calculated gas content divided by the number of steps gives the average fluid content in the entire wellbore. Finally, the volume of the gas-liquid two-phase mixture is obtained. The height of the moving fluid surface can be determined according to the average area of the wellbore annulus [12].

For every 1 m step, the volume of the gas-liquid two-phase mixture can be numerically determined using:

$$V_t(t) = \frac{Q_t}{\sum_{i=0}^{h_i(t-1)} [1 - E_g(i)]}$$

where $Q_t$ [Ls$^{-1}$] is the displacement of the killing fluid, $t$ [min] – the killing time beginning from the moment the killing fluid enters the wellbore, $V_t(t)$ [m$^3$] – the volume of the killing fluid injected into the wellbore annulus, $E_g(i)$ [%] – the gas content at the $i^{th}$ height node of the fluid column, and $h_i$ [m] – the height of the moving fluid surface.

The height of the moving fluid surface is calculated using:

$$h_f(t) = \frac{\sum_{k=0}^{n} A(k) h(k)}{A(k+1)} + h(k-1) + h(k-2) \cdots + h(1)$$

where $n$ is the number of changes in the cross-sectional area of the wellbore annulus between the bottom hole and the moving fluid surface, starting with 0 at the bottom hole, $k$ – the $k^{th}$ change in the cross-sectional area of the wellbore annulus, counted from the bottom hole, $A$ [m$^2$] – the area of the annulus, and $h(k)$ [m] – the wellbore length at the $k^{th}$ change in the cross-sectional area.

Obtaining dynamic pressure profile of wellbore annulus

A method for evaluating the gas-liquid two-phase transient flow during the killing of an empty gas-drilled well was developed in this study, fig. 1(a). Following is a description of the procedure:
Time meshing: Considering each time step to be 1 second, the time iterations are commenced when the killing fluid enters the bottom hole, and continued until the moving surface of the killing fluid reaches the wellhead.

Wellbore meshing: Beginning from the bottom hole, the wellbore is partitioned using 1 m steps until the wellhead is reached. The partitioning of the wellbore grid is then integrated with the wellbore structure and bottom hole assemblies (BHA).

Fluid column meshing: The height of the fluid column formed by the gas-liquid mixture varies with time. Beginning from the bottom hole, partitioning is performed using 1-m steps until the surface of the gas-liquid mixture is reached.

The flowchart for the formation-wellbore coupling process is shown in fig. 1(b) [13].

Numerical example

Formation parameters: The thickness of the production layer is 2 m, the formation pressure is 60 MPa, and the permeability of the production layer is 5 mD.

Construction parameters: The wellhead back pressure is 10 MPa, the pump displacement is 60 L/s, the density of the killing fluid is 1.4 g/cm³, and the viscosity of the killing fluid is 30 mPa·s.
The casing dimensions and depth parameters of its insertion are listed in tab. 1, while the BHA parameters are listed in tab. 2.

### Table 1. Casing dimensions and insertion depth parameters

<table>
<thead>
<tr>
<th>Drilling sequence</th>
<th>Well section [m]</th>
<th>Drill bit dimension [mm]</th>
<th>Casing dimension [mm]</th>
<th>Casing insertion depth in well section [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Third drilling stage</td>
<td>4692</td>
<td>311.2</td>
<td>244.5</td>
<td>0-2600</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>250.83</td>
<td>2600-4690</td>
</tr>
<tr>
<td>Fourth drilling stage</td>
<td>5000</td>
<td>215.9</td>
<td>177.8</td>
<td>4478-4998</td>
</tr>
</tbody>
</table>

### Table 2. The BHA parameters

<table>
<thead>
<tr>
<th>Name of drilling tool × specifications/model</th>
<th>Outer diameter [mm]</th>
<th>Inner diameter [mm]</th>
<th>Length [m]</th>
<th>Cumulative length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drilling pipe × S135I</td>
<td>127</td>
<td>108.6</td>
<td>4691.2</td>
<td>5000</td>
</tr>
<tr>
<td>Helical drill collar</td>
<td>177.8</td>
<td>71.4</td>
<td>9</td>
<td>30.3</td>
</tr>
<tr>
<td>Stabilizer</td>
<td>212</td>
<td>—</td>
<td>2</td>
<td>21.3</td>
</tr>
<tr>
<td>Helical drill collar</td>
<td>177.8</td>
<td>71.4</td>
<td>18</td>
<td>19.3</td>
</tr>
<tr>
<td>Float valve</td>
<td>177.8</td>
<td>—</td>
<td>0.5</td>
<td>1.3</td>
</tr>
<tr>
<td>Kelly joint 630 × NC56</td>
<td>177.8</td>
<td>—</td>
<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>Drill bit</td>
<td>215.9</td>
<td>—</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

### Numerical simulation results

The variation of the gas production rate with the killing time is shown in fig. 2(a). The decrease in the internal gas production was minimal during the first 10 minute, but became noticeable thereafter until 30 minute of the operation. The variation of the BHFP is shown in fig. 2(b). The BHFP was initially 18.2 MPa and limitedly increased during the well killing operation to 31.8 MPa, far less than the formation pressure of 60 MPa. As can be observed from fig. 2(c), the height of the fluid column continuously increased with the injection of the killing fluid.

It can be seen that, when the proposed construction parameters were employed, the flow pattern within the wellbore annulus was that of an annular mist flow. When the killing
fluid reached the wellhead, the BHFP was still lower than the formation pressure and the forma-
tion gas production rate remained high. The well killing was thus unsuccessful in the pre-
scribed conditions.

**Effect of varying wellhead back pressure**

With increasing killing time, the gas production rate continued to decrease until it was
0 at 42 minutes, fig. 3(a). The 5-MPa increase in the wellhead backpressure led to a
11.5 MPa increase of BHFP, fig. 3(b). The height of the moving fluid surface gradually in-
creased from 0 with the injection of the killing fluid, fig. 3(c). The height of the moving fluid
surface eventually increased steadily but drastically dropped at 42 minutes for a short moment.

Figure 3. Variation of the gas production rate (a), the BHFP (b), the height of moving fluid surface (c),
and with the killing time and distribution of the flow patterns (d) for a wellhead backpressure of 15 MPa

The BHFP and the formation pressure were balanced at 42 minutes, as shown in fig.
3(d). After 42 minutes, the formation no longer produced gas, resulting in the flow within the
wellbore annulus becoming similar to a pure fluid flow. This flow pattern was maintained un-
til the killing fluid reached the wellhead, indicating a successful killing of the well.

**Effect of varying pump displacement**

The parameter changes caused a gradual decrease of the gas production rate as the
killing fluid entered the wellbore annulus, fig. 4(a). Figure 4(b) shows that there was signifi-
cant fluctuation of the BHFP between 40 and 52.6 minutes and the BHFP became equal to the
formation pressure at 72 minutes. In fig. 4(c), the height of the moving fluid surface increased
continuously with the injection of the killing fluid into the wellbore annulus. As shown in fig.
4(d), at least four different flow patterns occurred with respect to the variation of the afore-
mentioned construction parameters.

Figure 4. Variation of the gas production rate (a), the BHFP (b), the height of the moving fluid surface
(c), and with time and distribution of the flow patterns (d) for a wellhead back pressure of 15 MPa and
pump displacement of 35 L/s
The previous analysis reveals that successful well killing requires an increase of the wellhead backpressure to 15 MPa, and a decrease of the pump displacement from 60 L/s to 35 L/s. This shows that the wellhead backpressure is critical to a successful well killing operation.

**Effect of varying density of killing fluid**

A good correlation was observed between the gas production rate, fig. 5(a) and the BHFP, fig. 5(b). During the period when the BHFP significantly increased, there was a large increase in the formation gas production rate. Further, fig. 5(c) shows that the height of the moving fluid surface rapidly increases during the initial period of the well killing. Eventually, as the formation pressure stabilized, the height of the moving fluid surface significantly dropped.

![Figure 5. Variation of the gas production rate (a), the BHFP (b), the height of the moving fluid surface (c), and with time and distribution of the flow patterns (d) for a wellhead back pressure of 15 MPa and killing fluid density of 1.25 g/cm³](image)

The distribution of the flow patterns within the wellbore is shown in fig. 5(d). As can be seen from the figure, the agitated flow occupied the entire wellbore annulus at the beginning of the well killing operation. With progressive injection of the killing fluid, the fluid column increased in length, while the gas production rate from the formation decreased.

![Figure 6. Combinations of the wellhead back pressure and density of the killing fluid for a displacement of (a) 40 L/s, (b) 50 L/s, and (c) 60 L/s](image)
Optimal combination of key construction parameters

The proposed numerical calculation method was applied to a section of Well X in the fourth drilling stage. The obtained results are shown in fig. 6.

For a given killing fluid density, the wellhead back pressure required is inversely proportional to the displacement. For a constant displacement, it is necessary to significantly increase the density of the killing fluid when the wellhead back pressure is decreased. If the wellhead back pressure drops, the well killing operation may not succeed even if the killing fluid is in a high density.

Conclusion

Gas drilling and well killing involve the coupling of the transient seepage from the formation and the gas-liquid two-phase transient flow within the wellbore annulus, as well as the gradual establishment of a killing fluid column. This paper proposed a method for killing an empty gas-drilled well through the coupling of the transient seepage from the formation and the gas-liquid two-phase transient flow within the wellbore annulus. The proposed method can be used to determine the key dynamic parameters of the killing operation. To ensure successful well killing, it is necessary to determine the optimal combination of the key construction parameters, among which the wellhead back pressure is particularly critical to successful well killing.

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