ANALYSIS OF ENTROPY GENERATION MINIMIZATION (EGM) IN FLOW OF REE-EYRING NANOFLUID BETWEEN TWO COAXIALLY ROTATING DISKS

Muhammad Ijaz KHAN*1, Riaz MUHAMMAD2, Sumaira QAYYUM1, Niaz B. KHAN3, M. JAMEEL4
1Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan
2Mechanical Engineering Department College of Engineering, University of Bahrain, Bahrain
3School of Mechanical and Manufacturing Engineering, National University of Sciences and Technology, Islamabad, Pakistan
4Department of Civil Engineering, College of Engineering, King Khalid University, Abha, Saudi Arabia

*Corresponding author; E-mail: mikhan@math.qau.edu.pk

The present communication addresses MHD radiative nanomaterial flow of Ree-Eyring fluid between two coaxially rotating disks. Both disks are stretchable. Buongiorno model is used for nanofluids. Nanofluid aspects comprise random motion of particles (Brownian diffusion) and thermophoresis. MHD fluid is considered. Furthermore, dissipation, radiative heat flux and Ohmic heating effects are considered to model the energy equation. Total entropy rate is calculated through implementation of second thermodynamics law. Series solutions are developed through homotopy analysis method. Impacts of physical parameters on the velocity, temperature, entropy and concentration fields are discussed graphically. Skin friction coefficient and heat and mass transfer rates are numerically calculated through Tables 2-4. It is noticed that the velocity of liquid particles decreases versus higher estimations of magnetic parameter while it enhances via larger rotational parameter. Temperature field significantly increases in the presence of both Brownian diffusion and thermophoresis parameters.

Key words: stretchable rotating disks; Entropy generation; MHD Ree-Eyring nanofluid; Brownian and thermophoresis diffusion effects; Thermal radiation and Ohmic heating.

1. Introduction

Several valuable utilizations in biological, engineering, physical and other disciplines stimulated the scientific commune towards the dynamics of non-Newtonian liquids. Such natured liquids nowadays are acknowledged more apposite for technological and scientific exhibitions in comparison to Newtonian liquids. The printing ink, shampoos, emulsions, glues, paints, muds, condensed milk, melts, sugar solution, tomato paste, soaps are a few non-Newtonian type materials. The salient aspects of such materials are problematic to describe as a solo mathematical expression but numerous attempts have been reported by modern researchers to elaborate the rheological aspects of liquids comprising non-Newtonian nature. It is apparent that liquids with non-Newtonian nature are more convoluted and possess extremely non-linear nature. The researchers elaborated numerous models to express the convoluted natured non-Newtonian liquids. Here we have used the Ree-Eyring fluid to investigate the flow between two stretchable rotating disks. The idea about Ree-Eyring fluid was gave by the following researchers [1-3].

Inspired by the above literature survey of non-Newtonian fluid over a stretchable surface, the present research work aimed to explore the radiative nanomaterial flow of Ree-Eyring fluid subject to stretchable surface with entropy generation. Series solutions are developed via homotopy analysis method [24-30]. Furthermore, Heat and mass transfer rates and skin friction coefficient are numerically discussed and presented through tables.

2. Problem description

Here we have addressed MHD radiative flow of nanofluid of non-Newtonian material (Ree-Eyring fluid) between two coaxially stretchable disks. Buongiorno model is used in the mathematical modeling of nanofluid. MHD fluid is considered. Induced magnetic field is ignored due to small Reynolds number. Both of disks are stretched with different stretching rates $a_1$ and
and rotating with angular speeds $\Omega_1$ and $\Omega_2$ respectively. Nanofluid aspects comprise Brownian movement and thermophoresis diffusion of particles. Fig. 1 is sketched for the schematic flow diagram.

![Schematic flow diagram](image)

**Fig. 1: Schematic flow diagram**

The nonlinear flow expressions for the considered problem are:

\[
\begin{align*}
\frac{\partial u}{\partial r} + u \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} &= 0, \\
\rho \left( \frac{u}{r} \frac{\partial v}{\partial r} - \frac{v}{r} + w \frac{\partial u}{\partial z} \right) &= \left( \mu + \frac{1}{\beta c_1} \right) \left\{ 2 \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} \right\}, \\
\rho \left( \frac{u}{r} \frac{\partial w}{\partial r} + w \frac{\partial u}{\partial z} \right) &= \left( \mu + \frac{1}{\beta c_1} \right) \left\{ 2 \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2} \right\}, \\
\rho \left( \frac{u}{r} \frac{\partial w}{\partial r} + w \frac{\partial u}{\partial z} \right) + \frac{\partial p}{\partial r} &= \left( \mu + \frac{1}{\beta c_1} \right) \left\{ 2 \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2} \right\}.
\end{align*}
\]

\[
\begin{align*}
\rho c_p \left( \frac{u}{r} \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} \right) &= k \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right) + \left( \rho c_p \right) \left( \frac{\partial C}{\partial r} + \frac{\partial C}{\partial z} \right), \\
+ \left( \rho c_p \right)_s \left( \frac{\partial C}{\partial r} \right)_s \left( \frac{\partial C}{\partial r} \right)_s &= 0, \\
+ u \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} &= D_B \left( \frac{\partial^2 C}{\partial z^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial r^2} \right) + D_T \left( \frac{\partial^2 T}{\partial z^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right), \\
- k_r \left( C - C_t \right)^n \exp \left( \frac{-E_a}{kT} \right).
\end{align*}
\]

with
\[ u = ra_1, \quad v = r\Omega_1, \quad w = 0, \quad T = T_1, \quad C = C_1 \text{ at } z = 0, \]
\[ u = ra_2, \quad v = r\Omega_1, \quad T \to T_2, \quad C \to C_2 \text{ when } z \to h, \]

where \( u, v, w \) highlight velocity components, \( r, \theta, z \) cylindrical coordinate, \( \sigma \) electrical conductivity, \( \rho \) density, \( c_p \) specific heat, \( \nu \) kinematic viscosity, \((\rho c_p)\) heat capacity of nanofluid, \((\rho c_p)\) heat capacity of base fluid, \( k \) thermal conductivity, \( C_1 \) and \( C_2 \) the concentrations at lower and upper disks, \( T_1 \) temperature at lower disk, \( \beta \) and \( c_1 \) are material constants, \( B_0 \) is strength of magnetic field, \( k_r \), \( n \), \( E_n \), \( k = 8.61 \times 10^{-5} \text{eV} / \text{K} \) are chemical reaction rate coefficient, fitted rate constant, activation energy and Boltzmann constant, \( T_2 \) temperature at upper disk, \( D_r \) thermophoresis diffusion coefficient, \( D_b \) Brownian diffusion coefficient, \( k^* \) and \( \sigma^* \) denotes mean absorption coefficient and Stefan-Boltzmann constant respectively.

Considering
\[ u = r\Omega_1 f(\eta), \quad v = r\Omega_1 g(\eta), \quad w = -2h\Omega_1 f(\eta), \quad p = \rho\Omega_1 \nu \left( P + \frac{1}{2} \frac{v^2}{\eta^2} \right), \]
\[ \theta(\eta) = \frac{T-T_1}{T_2-T_1}, \quad \varphi(\eta) = \frac{c-C_1}{c_1-C_2}, \quad \eta = \frac{z}{h}. \]

We have the following system of equations
\[ (1+We) f^{''''} + \text{Re}(-f^{'''} + 2ff'' + g^2 - Mf') = 0, \]
\[ (1+We) g^{''''} + \text{Re}(-2f'g + 2fg' - Mg') = 0, \]
\[ \frac{1}{\text{Re}} \frac{\partial^2 \theta}{\partial \tau^2} + 2f \theta' + Nt \theta^2 + Nb \theta' \varphi' + \frac{\text{Re}}{\text{Pr}} \left( \left( \theta - 1 \right) + 1 \right)^3 \theta' \]
\[ + \frac{E_{\text{rad}}}{\text{Re} A^2} (1+We) \left[ 10f^2 + A^2f''^2 + A^2g^2 \right] + \text{Ec}^2 \left( f^2 + g^2 \right) = 0, \]
\[ \frac{1}{\text{Sc}} \phi^{''''} + 2f \phi' + \frac{1}{\text{Sc}} \left( \frac{Nt}{Nb} \theta^2 - k_1 (1+\alpha_1 \theta) \phi \text{Exp} \left[ \frac{E_1}{1+\alpha_1 \theta} \right] \right) = 0, \]

with
\[ f(0) = 0, \quad f'(0) = A_i, \quad f''(0) = A_2, \quad g(0) = 1, \quad g'(0) = \Omega, \quad \theta(0) = 1, \quad \theta'(0) = 0, \]
\[ \phi(0) = 1, \quad \phi'(0) = 0. \]

In the above expressions \( M \left( = \frac{\sigma^2 \mu^2}{\pi k} \right) \) denotes the magnetic parameter, \( \text{Re} \left( = \frac{\Omega k^2}{\nu} \right) \) the Reynolds number, \( \text{We} \left( = \frac{1}{\beta \nu k \Theta_1 h} \right) \) the Weissenberg number, \( \text{Rd} \left( = \frac{16 \nu T_1}{3 \mu^2} \right) \) the radiation parameter, \( \theta_w \left( = \frac{T_1}{T_2} \right) \) the temperature ratio parameter, \( \text{Sc} \left( = \frac{\nu}{T_0} \right) \) the Schmidt number, \( \text{Nb} \left( = \frac{c_p (c_1-c_2)}{\nu} \right) \) the Brownian motion parameter, \( \text{Pr} \left( = \frac{\nu n}{k} \right) \) the Prandtl number, \( \text{Ec} \left( = \frac{r^2 q^2}{c_p(T_2-T_1)} \right) \) the Eckert number, \( k_1 \left( = \frac{k^2}{\pi k} \right) \) the chemical reaction parameter, \( E_1 \left( = \frac{E}{x^2 T_2} \right) \) the
activation energy parameter, \( Nt \left( \frac{\varepsilon D_l (r_l - T)}{v_l^2} \right) \) the thermophoresis parameter, \( A_1 \left( \frac{\alpha_f}{\Theta} \right) \) and \( A_2 \left( \frac{\alpha_f}{\Theta} \right) \) the stretching parameters, \( \alpha_s \left( \frac{T_r - T}{r_r} \right) \) temperature ratio parameter, \( A \left( \frac{v^3}{h^2} \right) \) the dimensionless parameter, \( \Omega \left( \frac{\Omega}{\Theta} \right) \) the rotation parameter and \( \varepsilon \) the constant pressure.

3. Physical quantities

Mathematically, the skin friction coefficient, heat and mass transfer rates (Nusselt and Sherwood numbers) at both lower and upper disks are defined as

\[
C_{hi} \text{Re} = \frac{1}{A} \left[ f''(0) + \frac{1}{We} f''(0) \right],
\]
\[
C_{ri} \text{Re} = g'(0) + \frac{1}{We} g'(0),
\]
\[
Nu_{xi} = 1 + (1 + Rd) \theta'(0),
\]
\[
Nu_{x_2} = 1 + (1 + Rd) \theta'(1),
\]
\[
Sh_{xi} = -\phi'(0),
\]
\[
Sh_{x_2} = -\phi'(1),
\]

where \( \text{Re} \) denotes the Reynolds number.

4. Entropy generation equation

Mathematically the entropy generation in dimensional form is addressed as

\[
S_g = \frac{1}{T} \left[ 1 + \frac{16 \eta T \Theta}{3 \lambda_{h} \kappa} \right] \left[ \left( \frac{\partial T}{\partial r} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right] + \frac{\beta^2}{T} \left( u^2 + v^2 \right) + \frac{1}{T} \left[ \frac{2}{(\partial \phi)} + \frac{(\frac{\partial v}{\partial r} - \frac{\partial u}{\partial z})^2}{(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z})^2} \right] \right] + \frac{2}{C} \left[ \left( \frac{\partial C}{\partial r} \right)^2 + \left( \frac{\partial C}{\partial z} \right)^2 \right] + \frac{RD}{C} \left[ \left( \frac{\partial C}{\partial r} \right)^2 + \left( \frac{\partial C}{\partial z} \right)^2 \right]
\]

After applying transformations, one has

\[
N_G = \alpha_s \left( 1 + Rd \left( \phi \left( \frac{\theta_r}{\partial (\theta_r)} \right) + 1 \right) \right)^3 \theta'^2 + \frac{BR}{A} \left( 1 + \phi \left( \frac{\theta_r}{\partial (\theta_r)} + 1 \right) \right) \left( 10 f'^2 + A^2 f'^2 + A^2 g'^2 \right)
\]

and the Bejan number is addressed as

\[
Be = \frac{\text{Entropy due to mass and heat transfer}}{\text{Total entropy generator}}.
\]

\[
Be = \frac{\alpha_s \left( 1 + Rd \left( \phi \left( \frac{\theta_r}{\partial (\theta_r)} + 1 \right) \right)^3 \theta'^2 + \frac{BR}{A} \left( 1 + \phi \left( \frac{\theta_r}{\partial (\theta_r)} + 1 \right) \right) \left( 10 f'^2 + A^2 f'^2 + A^2 g'^2 \right)}{\alpha_s \left( 1 + Rd \left( \phi \left( \frac{\theta_r}{\partial (\theta_r)} + 1 \right) \right)^3 \theta'^2 + \frac{BR}{A} \left( 1 + \phi \left( \frac{\theta_r}{\partial (\theta_r)} + 1 \right) \right) \left( 10 f'^2 + A^2 f'^2 + A^2 g'^2 \right)} + \frac{MBr}{(\theta \left( \frac{\theta_r}{\partial (\theta_r)} + 1 \right) + 1)} \left( f'^2 + g'^2 \right)
\]
where $\alpha_1 \left( = \frac{T_1-T_2}{T'_1} \right)$, and $\alpha_2 \left( = \frac{C_1-C_2}{C'_1} \right)$ signifies the temperature and concentration difference parameter, $N_G \left( = \frac{s_r \beta b^2}{k(T_1-T_2)} \right)$ the entropy generation rate, $L \left( = \frac{\text{RD}(C_1-C_2)}{k} \right)$ the diffusion parameter, $Br \left( = \frac{\eta^2 \omega_1^2}{k(T_1-T_2)} \right)$ the Brinkman number.

5. HAM solution

Mathematically the initial guesses and linear operators are addressed as

$$
\begin{align*}
    f_0(\eta) &= \frac{1}{2} \left( 2A_1 \eta - A_1 \eta^2 + A_2 \eta^3 \right), \\
    g_0(\eta) &= 1 + \eta (\Omega - 1), \\
    \theta_0(\eta) &= 1 - \eta, \\
    \phi_0(\eta) &= 1 - \eta,
\end{align*}
$$

(19)

$$
\begin{align*}
    L_f &= \frac{\partial^3}{\partial \eta^3}, \\
    L_g &= \frac{\partial^2}{\partial \eta^2}, \\
    L_\theta &= \frac{\partial^2}{\partial \eta^2}, \\
    L_\phi &= \frac{\partial^2}{\partial \eta^2},
\end{align*}
$$

(20)

with

$$
\begin{align*}
    L_f &= \left[ z_1 + z_2 \eta + z_3 \eta^2 \right], \\
    L_g &= \left[ z_4 + z_5 \eta \right], \\
    L_\theta &= \left[ z_6 + z_7 \eta \right], \\
    L_\phi &= \left[ z_8 + z_9 \eta \right],
\end{align*}
$$

(21)

where $z_i \ (i = 1, 2, 3, \ldots, 9)$ denotes the arbitrary constants.

6. Convergence analysis

Here we have applied homotopy analysis method for series solutions. Series solutions depend on auxiliary parameters and play a significant role to control the convergence series solutions. We have plotted Fig. 2 for the appropriate ranges of auxiliary parameters. From Fig. 2, it is noticed that the admissible ranges for the momentum, energy and concentration equations are $-1.8 \leq h_f \leq -0.1, -1.6 \leq h_g \leq -0.2, -1.6 \leq h_\theta \leq -0.5$ and $-1.4 \leq h_\phi \leq -0.5$. Table 1 shows that 7th, 5th, 14th and 16th order of approximations are enough for convergence of velocities, temperature and concentration velocities respectively.
Table 1: Numerical analysis of velocity, temperature and concentration equations.

<table>
<thead>
<tr>
<th>Different order of approximations</th>
<th>$f''(0)$</th>
<th>$g'(0)$</th>
<th>$\theta'(0)$</th>
<th>$\phi'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.07408011</td>
<td>-0.6935546</td>
<td>2.553284</td>
<td>-4.592678</td>
</tr>
<tr>
<td>7</td>
<td>0.07408034</td>
<td>-0.6935546</td>
<td>2.552027</td>
<td>-4.588193</td>
</tr>
<tr>
<td>14</td>
<td>0.07408034</td>
<td>-0.6935546</td>
<td>2.552204</td>
<td>-4.588938</td>
</tr>
<tr>
<td>16</td>
<td>0.07408034</td>
<td>-0.6935546</td>
<td>2.552204</td>
<td>-4.588937</td>
</tr>
<tr>
<td>20</td>
<td>0.07408034</td>
<td>-0.6935546</td>
<td>2.552204</td>
<td>-4.588937</td>
</tr>
<tr>
<td>40</td>
<td>0.07408034</td>
<td>-0.6935546</td>
<td>2.552204</td>
<td>-4.588937</td>
</tr>
</tbody>
</table>

7. Discussion

In this section we discussed the physical results of involved parameters on fluid characteristics (flow, concentration, Nusselt number, Skin friction, temperature, entropy generation, Sherwood number, Bejan number) between two disks. Figs. (3–11) represent the impact of Hartmann number ($M$) on axial, radial and tangential velocity profiles ($f, f', g$) respectively. It is witnessed that velocities are decreasing functions of Hartmann number ($M$). For larger ($M = 0, 2, 4, 6, 8$) Lorentz force produces which is used to resist the motion of the fluid particles consequently velocity of the fluid decay. Figs. (6–8) display the behavior of ($f, f', g$) against Weissenberg number ($We$). It is noted that radial ($f'$) and axial ($f$) velocities decay for higher ($We = 0, 0.5, 1, 1.5, 2$) (see Fig. 6 and 7) and opposite influence is noticed for tangential velocity $g$ (see Fig. 8). Figs. (9 and 10) show that radial and axial velocities boost up for higher values of Reynolds number ($Re = 0, 0.2, 0.4, 0.6, 0.8$). Physically, when Reynolds number increases viscous effects become weaker due to which resistance between the fluid particles reduces hence velocity of the fluid increases. Fig. 11 shows that tangential velocity of the fluid enhances for rotational parameter ($\Omega = 0, 0.3, 0.6, 0.9, 1.2$).
Figs. (12–14) are shown for temperature impact against different parameters. Fig. 12 shows the impact of thermophoresis parameter ($Nt$) on temperature field $\theta$. With increasing parameter ($Nt=0,2,4,6,8$) temperature of the fluid enhances. Fig. 13 is depicted for the behavior of Brownian motion on thermal field. Here temperature is increased versus larger Brownian motion parameter. Impact of radiation parameter is portrayed in Fig. 14. As we rise the values of radiation parameter ($Rd=0,0.2,0.4,0.6,0.8$) temperature of the fluid enhances. Increasing values of ($Rd$) are responsible for decrease in mean absorption coefficient ($k^*$) due to which temperature enhances.

Figs. (15-18) show the effect of parameters which are involved in concentration equation. Trend of concentration field ($\phi$) against ($Sc$) is shown in Fig. 15. One can see that ($\phi$) is increasing function ($Sc=3,3.4,3.8,4.2,4.6$). Fig. 16 shows that concentration increases with increase in activation energy parameter ($E_t=0,3,6,9,12$). Figs. 17 and 18 are portrayed to see the behavior of concentration field ($\phi$) against ($Nt$) and ($Nb$) respectively. It is easily seen that concentration of the fluid rises for higher ($Nt=0,2,4,6,8$) while magnitude of ($\phi$) decays for larger ($Nb=0.5,1.5,2,2.5$).

Figs. (19 and 20) are portrayed for influence of Brinkman number ($Br$) on entropy generation and Bejan number. Opposite behavior of ($N_g$) and ($Be$) is seen for larger ($Br=0,0.2,0.4,0.6,0.8$). When we increase the value of ($Br$) viscosity of the fluid rises which produces resistance between fluid particles hence disorderedness in the system enhances. Due to increase in viscous effects heat and mass transfer irreversibility become less dominant due to which ($Be$) reduces (see Fig. 20). Figs. (21 and 22) are portrayed for behavior of ($N_g$) and ($Be$) against Weissenberg number ($We$). Opposite trends are noticed for higher values of ($We=0,0.2,0.4,0.6,0.8$). Figs. 23 and 24 are sketched for analysis of ($N_g$) and ($Be$) against chemical reaction parameter ($k_t$). It is noticed that entropy and Bejan number reduces near lower and upper disks while enhances between the disks for larger ($k_t=0,3,6,9,12$).

Tables (2-4) show the trend of skin friction, Sherwood number and Nusselt number near lower and upper disks via different parameters. It is seen that magnitude of drag force reduces in both radial and tangential directions for higher ($We$) while for larger ($M$) radial drag force decays and in tangential direction it shows increasing behavior (See Table 2). From Table 3 we concluded that Nusselt number at lower and upper disk rises for both radiation parameter ($Rd$) and Eckert number ($Ec$). Table 4 display that Sherwood number at lower and upper disk reduces
for higher estimation of activation parameter \((E_i)\) and chemical reaction parameter \((k_i)\).
Fig. 6: $f(\eta)$ versus $We$.

Fig. 7: $f'(\eta)$ versus $We$.

Fig. 8: $g(\eta)$ versus $We$.

Fig. 9: $f(\eta)$ versus $Re$.
Fig. 10: $f' (\eta)$ versus Re.

Fig. 11: $g(\eta)$ versus $\Omega$.

Fig. 12: $\theta(\eta)$ versus $Nt$.

Fig. 13: $\theta(\eta)$ versus Re.

$\Omega = 0, 0.3, 0.6, 0.9, 1.2$

$Nt = 0, 2, 4, 6, 8$

$Nb = 0, 2, 4, 6, 8$

$Rd = 0, 0.2, 0.4, 0.6, 0.8$
Fig. 14: $\theta(\eta)$ versus $Rd$.

Fig. 15: $\phi(\eta)$ versus $Sc$.

Fig. 16: $\phi(\eta)$ versus $E_1$.

Fig. 17: $\phi(\eta)$ versus $Nt$.

Fig. 18: $\phi(\eta)$ versus $Nb$. 
Fig. 19: $N_G(\eta)$ versus $Br$.

Fig. 20: $Be(\eta)$ versus $Br$.

Fig. 21: $N_G(\eta)$ versus $We$.

Fig. 22: $Be(\eta)$ versus $We$. 
Table 2: Analysis of skin friction coefficient at both upper and lower for different values of $We$ and $M$.

<table>
<thead>
<tr>
<th>$We$</th>
<th>$M$</th>
<th>$C_r_0 \text{Re}_r$</th>
<th>$C_g_0 \text{Re}_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>138.3225</td>
<td>-7.767466</td>
</tr>
<tr>
<td>0.2</td>
<td></td>
<td>74.16687</td>
<td>-4.233803</td>
</tr>
<tr>
<td>0.3</td>
<td></td>
<td>52.77987</td>
<td>-3.055909</td>
</tr>
<tr>
<td>0.1</td>
<td>0.6</td>
<td>137.1064</td>
<td>-7.805796</td>
</tr>
<tr>
<td>0.7</td>
<td></td>
<td>135.8889</td>
<td>-7.844075</td>
</tr>
<tr>
<td>0.8</td>
<td></td>
<td>134.6734</td>
<td>-7.882302</td>
</tr>
</tbody>
</table>

Table 3: Analysis of heat transfer rate at both upper and lower for different values of $Rd$ and $Ec$.

<table>
<thead>
<tr>
<th>$Rd$</th>
<th>$Ec$</th>
<th>$Nu_{r0}$</th>
<th>$Nu_{r1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.1</td>
<td>-1.965949</td>
<td>1.468251</td>
</tr>
<tr>
<td>0.2</td>
<td></td>
<td>-2.213118</td>
<td>2.090646</td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>-2.762508</td>
<td>3.482678</td>
</tr>
<tr>
<td>0.4</td>
<td>0.2</td>
<td>-4.427199</td>
<td>5.104134</td>
</tr>
<tr>
<td>0.3</td>
<td></td>
<td>-6.400475</td>
<td>7.890065</td>
</tr>
<tr>
<td>0.4</td>
<td></td>
<td>-8.379144</td>
<td>10.68386</td>
</tr>
</tbody>
</table>
Table 4: Analysis of Sherwood number at both upper and lower for different values of $E_1$ and $k_1$.

<table>
<thead>
<tr>
<th>$E_1$</th>
<th>$k_1$</th>
<th>$Sh_{x0}$</th>
<th>$Sh_{x1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.3</td>
<td>4.767886</td>
<td>-4.061725</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>4.766698</td>
<td>-4.059674</td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>4.765520</td>
<td>-4.057630</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>4.765650</td>
<td>-4.036756</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>4.763968</td>
<td>-4.012376</td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>4.762816</td>
<td>-3.988562</td>
</tr>
</tbody>
</table>

8. Conclusions

Here we have examined MHD radiative nanofluid flow of non-Newtonian fluid (Ree- Eyring fluid) between two coaxially stretchable disks. Numerous valuable applications in mechanical engineering, biological and physical sciences stimulated the scientific commune to the dynamics of non-Newtonian materials. Such natured fluids/materials nowadays are recognized more apposite for scientific and technological exhibitions in comparison to Newtonian fluids/materials. The emulsions, printing ink, paints, shampoos, glues, condensed milk, muds, sugar solution, melts, soaps and tomato paste are a few non-Newtonian types material. The main outcomes of the present analysis have been listed as:

- Radial and axial components of velocity decays versus higher estimations of Weissenberg number while tangential component of increases versus larger Weissenberg number.
- For increasing values of stretching parameter, the radial, axial and tangential components of velocity increases.
- Temperature field upsurges versus higher values of radiation, thermophoresis, Eckert number and Brownian diffusion parameters.
- For increasing values of Schmidt number the concentration of particles increases.
- Magnitude of surface drag force (skin friction coefficient) decreases at both lower and upper disks surface versus higher values of Weissenberg number.
- For higher estimations of magnetic parameter the magnetic of skin friction shows contrast behavior at lower and upper disks i.e., at lower disk the magnitude of skin friction decreases while it increases at upper disk surface.
- Magnitude of heat transfer rate increases at both disks surface in the presence of larger values of radiation parameter and Eckert number.
- For higher estimations of activation energy and chemical reaction parameters, the magnitude of Sherwood number decreases at both upper and lower disks surfaces.
References


