Fiber reinforced composites are increasingly being introduced into a variety of structures in aeronautical, mechanical and marine engineering applications where high ratios of stiffness and strength to weight are of paramount importance. Filament-wound, fiber-reinforced polymer-matrix composite laminate tubes have been used for a wide range of new engineering applications, owing to their high specific stiffness, strength and superior corrosion resistance. A systematic study, based on analysis and experiments, has been conducted to investigate the mechanisms and failure mechanics of filament-wound composite laminate tubes under internal pressure [1]. Filament wound composite tubes made of GRP have many potential advantages over pipes made of conventional materials, such as their resistance to corrosion, high strength, light weight and good thermal insulation properties. Continuous filaments are an economical and excellent form of fiber reinforcement and can be oriented to match the direction of stress loaded in a structure.

Rousseau et al. [2] performed parametric studies about the influence of winding patterns on the damage behavior of filament wound structures. Beakou et al. [3] used the classical laminated theory in order to analyze the influence of variable scattering on the optimum winding angle of cylindrical composites. Kabir [4] performed a finite element analysis of composite pressure vessels having a load sharing metallic liner with a 3-D laminated shell element of the commercial FEM code, NISA-II. There has been a growing interest in the application of filament wound fiber-reinforced cylindrical composite structures in the manufacturing of filament-wound composite tubes. Composites offer many advantages over metals due to their considerably higher strength-to-weight ratio. Advantages of composite materials and their applications can be found in references [5–8].

Owing to their anisotropic nature, fiber reinforced composite material properties can be tailored by varying the laminate fiber orientations. This is beneficial as the stiffness or strength of a structure can be maximized. Alternatively, the weight or cost can be minimized. Thin-walled filament-wound E-glass fiber-reinforced polyester tubes were tested under internal pressure to determine their residual burst strength. The finite element method, based on Mindlin plate and shell theory, is used in this application in conjunction with initial failure criterions in order to obtain the failure load of layered composite tube under internal pressure.

The main objective of this work is a comparison of calculation-acquired pressure values, which causes the initial failure of tube models, and experimentally obtained values for hydraulic burst pressure of filament-wound composite tubes, i.e. the verification of computation procedure for initial failure analysis of composite tube.

**FAILURE ANALYSIS AND INITIAL FAILURE CRITERIA**

To determine initial failure of layered composite structures under internal pressure, various initial failure criterions can be used. In this work, three initial failure criterions are considered: a) maximum strain criterion, b) maximum stress criterion and c) Tsai-Wu criterion. Strength analysis of fiber reinforced composite structures until the initial failure is significantly more complex than the calculation of structures of isotropic materials. In strength theory, for isotropic materials, the load function is equalized with a single parameter (for example, tensile strength of the material). In strength theories for anisotropic materials, the load function owns more than one strength parameter. Theories of failure of the composite structures are load functions and appropriate properties of material strength. The complexity of failure analysis for composite structures originates from the fact that material consists of thin orthotropic layers (lamina), and each layer consists of reinforcing agent (fibers) and impregnation agent (resin).

In most theories of failure for composite multi-layered materials, or laminate, squared load functions...
are used. Goldenblant and Kopnov have suggested that the strength function \( F \) can be expressed as a tensor-polynomial approximation [9]:

\[
F = (F_{ij})^2 + (F_{ijkl})^2 + (F_{ijklmn})^2 - 1 \tag{1}
\]

where \( F_{ij}, F_{ijkl}, F_{ijklmn} \) are strength tensors of 2, 4, and 6\(^{th} \) progression, \( \alpha, \beta \) and \( \gamma \) are materials constants, and \( \sigma_{ij}, \sigma_{ijkl} \) are load tensors of 2, 4, and 6\(^{th} \) progression.

For determination of the initial failure, different criteria are used, and the two most significant failure criteria for laminate are resin failure criteria and fiber failure criteria. It is considered that the resin failure is the most complex in laminate failure. The most commonly used initial failure criteria for fiber reinforced composite materials are based on tensor-polynomial formulation.

For the case of \( \alpha = \beta = \gamma = 1 \), the tensor-polynomial approximation includes the formation of a polynomial as a scalar function of the load components, which can be written in the most basic form as:

\[
F_{ij} \sigma_{ij} + F_{ijkl} \sigma_{ijkl} + \ldots = \ldots, i, j, k = 1, 2, ..., 6 \tag{2}
\]

where \( \sigma_{ij}, \sigma_{ijkl} \) are load tensor components, and \( F_{ij}, F_{ijkl} \) are components for tensors of strength failure of unidirectional material, which were 2, 4, and 6\(^{th} \) progression.

Minimal progression of the tensor-polynomial function, Eq. (2), depends on material anisotropy. It has been detected that for materials which own an orthotropic symmetry the tensor-polynomial function can be reduced into the second progression. For computational convenience each criterion was defined in truncated tensor polynomial format, i.e. in the form:

\[
F_{ij} \sigma_{ij} + F_{ijkl} \sigma_{ijkl} = 1, ... , i, j, k, l = 1, 2, ..., 6 \tag{3}
\]

Normal and shear components of strength tensors of the second progression \( \{F_0\} \) as well as all components of tensors differences of strength of the first progression \( \{F_i\} \), in most failure theories of anisotropic materials, are defined in the following method:

\[
F_i = X_i - X_c^{-1}, \quad F_j = X_c^{-1} - Y_c^{-1} \tag{4}
\]

\[
F_i = Y_i - Y_c^{-1}, \quad F_j = Y_c^{-1} - Y_c^{-1} \tag{5}
\]

where \( X \) is tensile strength in the direction of fibers, \( X_c \) is compression strength in the direction of fibers, \( Y \) is tensile strength transversal to the direction of fibers, and \( Y_c \) is compression strength transversal to the direction of fibers. The coefficients \( F_i \) and \( F_j \) in Eq. (3) are functions of the unidirectional lamina strengths and are presented below for each of initial failure criteria.

**Maximum strain criterion**

From several criteria for the resin failure, the criteria of greatest deformation was chosen and applied. By using the appropriate relations, the general form for the initial failure criteria for the greatest deformation has the following values for components \( F_i \) and \( F_j \):

\[
F_1 = (X_i - X_c^{-1}) - \nu TL (Y_i - Y_c^{-1}) \tag{6}
\]

\[
F_2 = -\nu TL E_t E_t^{-1} (X_i - X_c^{-1}) + (Y_i - Y_c^{-1}) \tag{7}
\]

\[
F_{11} = (X_i X_c)^{-1} + \nu TL (X_i - X_c^{-1}) (Y_i - Y_c^{-1}) + \nu TL (Y_i Y_c)^{-1} \tag{9}
\]

\[
F_{12} = -\nu TL E_t E_t^{-1} (X_i X_c)^{-1} - 1/2 (1 + \nu TL E_t E_t^{-1}) (X_i - X_c^{-1}) (Y_i - Y_c^{-1}) \tag{10}
\]

where \( \nu TL \) is Poisson’s coefficient at tension and compression, \( E_t \) is the elastic modulus in the direction of fibers under tension and compression, and \( E_t \) is the elastic modulus transversal to the fibers under tension and compression.

**Maximum stress criterion**

Using relation (3), the Tsai-Wu criterion for lamina with orthotropic properties can be expressed in the following form:

\[
F_{i1} = X_i X_c^{-1} - 1 \tag{11}
\]

\[
F_{i2} = Y_i Y_c^{-1} - 1 \tag{12}
\]

\[
F_{i1} = (X_i X_c)^{-1} \tag{13}
\]

\[
F_{i2} = -1/2 (X_i - X_c^{-1}) (Y_i - Y_c^{-1}) - 1/2 (1 + \nu TL E_t E_t^{-1}) (X_i - X_c^{-1}) (Y_i - Y_c^{-1}) \tag{14}
\]

**Tsai-Wu criterion**

Using relation (3), the Tsai-Wu criterion for lamina with orthotropic properties can be expressed in the following form:

\[
F_{i1} = X_i X_c^{-1} - 1 \tag{11}
\]

\[
F_{i2} = Y_i Y_c^{-1} - 1 \tag{12}
\]

\[
F_{i1} = (X_i X_c)^{-1} \tag{13}
\]

\[
F_{i2} = -1/2 (X_i - X_c^{-1}) (Y_i - Y_c^{-1}) - 1/2 (1 + \nu TL E_t E_t^{-1}) (X_i - X_c^{-1}) (Y_i - Y_c^{-1}) \tag{14}
\]

where \( \sigma_1 \) and \( \sigma_2 \) are normal stress components and \( \tau_1, \tau_2 \) are shear stress components.
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shells for determination of load conditions of layered composite materials with orthotropic properties and failure criteria based on the greatest deformation for the initial failure, and every finite element consists of the calculated number of layers of the appropriate tube. The loading levels, i.e., pressure levels inside the tube models for which the initial failure occurs inside the individual layers are calculated from the working load, which is obtained from FEM analysis, failure strength of the materials and abovementioned initial failure criteria.

The first information obtained from the calculation of hydraulic pressure that causes the initial failure of tube model is the coefficient of initial failure (Failure Index – F.I.). This coefficient represents the relation between the failure strength of composite material and the working loads inside the tubes as a result of action of the internal hydraulic pressure. In case such information is unavailable, any other estimated value is taken into account. After determining the coefficient of initial failure, the relation between the hydraulic pressure, which is taken into account, and the abovementioned coefficient, are calculated and by this method the calculated pressure of the initial tube failure is obtained. It practically represents the pressure that causes the initial failure, i.e. the first burst of any layer inside the tube.

If for a value of hydraulic pressure that acts within the tube one takes the value of the hydraulic pressure that causes the burst of the tube, and if the calculation for coefficient of initial failure gets a value of 1.0, the calculation corresponds perfectly with the experiment. If the calculated value for the coefficient of initial failure is different from 1.0, then it represents the difference between the calculation and experiment.

**Comparisons computation with experimental results**

Four types of filament-wound composite tubes were analyzed to verify the computation procedure. The geometries of these tubes are given in Table 1. The computation procedure is based on combining FEM for the stress analysis of composite tubes and initial failure criteria. Detailed descriptions of producing these composite tubes and experimental tests are described in references [8,14]. 400 mm-long composite tube specimens were cut from the tube by machining, and only a layer of pure resin was removed from the outer surface, so that the final layer of the glass fiber remained undamaged. Four groups of tube samples, as well as the winding structure (from the inside toward the outside), internal diameter, outer diameter and wall thickness are presented in Table 1. Figure 1 shows a composite tube from group A with glued two-axis strain gauges.

**Initial failure analysis: comparisons predicted with experimental results**

The composite tube is tested under internal pressure up to failure. Experimental values of failure loads are given in Table 2. In this case failure load is defined as hydraulic burst pressure. A detailed description of experimental tests is given in reference [8].

Material properties of composite tubes are:
- tensile strength in the direction of fibers: $X_t = 694.7$ MPa
- compression strength in the direction of fibers: $X_c = 409.3$ MPa
- tensile strength transversal to the direction of fibers: $Y_t = 9.72$ MPa

<table>
<thead>
<tr>
<th>Group markings</th>
<th>Winding structure</th>
<th>Internal diameter, mm</th>
<th>Outer diameter, mm</th>
<th>Wall thickness, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$[90^0(±61^0)]_n$</td>
<td>64.20</td>
<td>67.60</td>
<td>1.70</td>
</tr>
<tr>
<td>B</td>
<td>$[90^0(±45^0)]_n$</td>
<td>64.20</td>
<td>67.60</td>
<td>1.70</td>
</tr>
<tr>
<td>C</td>
<td>$[90^0(±61^0)^2]$</td>
<td>64.20</td>
<td>71.10</td>
<td>3.45</td>
</tr>
<tr>
<td>D</td>
<td>$[90^0(±45^0)^2]$</td>
<td>64.20</td>
<td>71.10</td>
<td>3.45</td>
</tr>
</tbody>
</table>

**Figure 1. Composite tube from group A with glued two-axis strain gauges.**
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– compression strength transversal to the direction of fibers: \( Y_c = 11.05 \text{ MPa} \)
– shear strength: \( XLT = 11.06 \text{ MPa} \)
– elastic modulus under tensile in the direction of fibers: \( EL, t = 10.95 \text{ GPa} \)
– elastic modulus under compression, in the direction of fibers: \( EL,c = 1.171 \text{ MPa} \)
– elastic modulus under tensile, transversal to the direction of fibers: \( ET,t = 3.51 \text{ GPa} \)
– elastic modulus under compression, transversal to the direction of fibers: \( ET,c = 767.1 \text{ MPa} \)
– Poisson’s coefficient under tensile in the direction of fibers: \( \nu_{Lt} = 0.296 \)
– Poisson’s coefficient under tensile, transversal to the direction of fibers: \( \nu_{Tt} = 0.13 \)
– Poisson’s coefficient under compression, in the direction of fibers: \( \nu_{Lc} = 0.312 \)
– Poisson’s coefficient under compression, transversal to the direction of fibers: \( \nu_{Tc} = 0.181 \).

To determine stresses in layers of composite tubes the finite element method (FEM) is used. Composite tubes are modeled with 4-node laminated shells [13]. The finite element models of these tubes are given in Figure 2. Models of composite tube made from finite elements are shown in Figure 1, and the distribution of coefficient of initial failure (Failure Index – F.I.) is shown in the Figure 3. In this case maximum strain criterion is used.

The analysis of load conditions, applying FEM, was conducted by using certain values of hydraulic pressure. For example, a 23.0 MPa hydraulic pressure was used for the calculation of group A tube samples. This pressure was chosen based on experimentally calculated middle arithmetic value of the hydraulic burst pressure of group A tube sample, which was 22.34 MPa. The choice can be made completely randomly, but the fact is that the experimental value of burst pressure was already known. The calculated coefficient value of the Failure Index (F.I.) for group A tubes was: F.I. = 0.876.

By combining the existing data for strength and the coefficient of initial failure, according to the calculated analysis, the calculated pressure of initial failure for group A tubes is: \( 23.0/0.876 = 26.2 \text{ MPa} \). Present computation results also reveal that the initial failure occurs in the internal layer under an angle of 90°, at all four groups of tubes (See Table 4).

Markings of the four groups of layered composite tube, chosen hydraulic pressure, coefficient of initial failure and computed internal hydraulic pressures up to initial failure are given in Table 3.

For A and C group composite tubes, which have middle layers winded under an angle of 61°, computed coefficients of initial failure (Failure Index) have values lower than 1.0. This data shows that the stacking se-

Table 2. Experimental values for hydraulic burst pressure of tube samples [8]

<table>
<thead>
<tr>
<th>Group markings</th>
<th>Hydraulic burst pressure, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Single values (( X_i ))</td>
</tr>
<tr>
<td>A/1</td>
<td>23.08</td>
</tr>
<tr>
<td>A/2</td>
<td>22.50</td>
</tr>
<tr>
<td>A/3</td>
<td>23.49</td>
</tr>
<tr>
<td>A/4</td>
<td>21.56</td>
</tr>
<tr>
<td>A/5</td>
<td>21.10</td>
</tr>
<tr>
<td>A/6</td>
<td>22.31</td>
</tr>
<tr>
<td>B/1</td>
<td>19.04</td>
</tr>
<tr>
<td>B/2</td>
<td>17.83</td>
</tr>
<tr>
<td>B/3</td>
<td>17.25</td>
</tr>
<tr>
<td>C/1</td>
<td>50.23</td>
</tr>
<tr>
<td>C/2</td>
<td>48.04</td>
</tr>
<tr>
<td>D/1</td>
<td>36.01</td>
</tr>
<tr>
<td>D/2</td>
<td>41.56</td>
</tr>
<tr>
<td>D/3</td>
<td>38.02</td>
</tr>
</tbody>
</table>

Figure 2. Finite element model of layered composite tube.
Compute the hydraulic pressure of initial failure for the tube model that belongs to the group B is 17.15 MPa (Table 3), while the experimentally determined hydraulic burst pressure is 18.0 MPa (Table 2), and the difference between them is about 5%. The results are similar with the tubes from group D, because its calculated hydraulic pressure of initial failure for the model tubes is 33.78 MPa (Table 3), and the experimentally determined hydraulic burst pressure is 38.5 MPa (Table 2), the difference between them is about 12%. However, it is estimated that this deviation can be tolerated because the calculation, in this case, determines the level of load that produces the initial failure of one of the layers, while experiments determine the effective burst, therefore the matching of calculated and experimentally determined values for hydraulic burst pressure is acceptable even with these tubes.

**Deflections of tube: comparisons predicted with experimental results**

Computational radial displacements are compared here with experimental layered composite tubes under internal pressure. Specimens were exposed to closed-end internal pressure tests using the instrument design shown in Figure 4.
Comparisons of computation values of displacements using 4-node layered shell finite elements with experimental results are given in Table 4.

Good agreement between computational radial displacements with experimental results is obtained, Table 4. The difference between computational and experimental results is within 10%.

**CONCLUSIONS**

This paper presents a computation procedure for initial failure analysis of layered composite tubes under internal pressure. The computation procedure is based on combining the finite element method for stress analysis and initial failure criteria. The finite element analysis used in this work is based on refined higher order shear deformation theory of the laminated plates [13]. This theory allows parabolic description of the transverse shear stresses, and therefore the shear correction factors of the usual shear deformation theory are not required in the present theory. Computational results are compared with experimental results [8,14]. Failure criteria used with finite element analysis qualitatively predict the load level and the location of local failures in the laminates that correspond to experimental results. Good agreement between computational and experimental results is obtained, meaning that the computation procedure based on combining 4-node layered shell finite elements and initial failure criteria can be used in practical initial failure analysis of composite tubes under internal pressure.

**REFERENCES**


IZVOD

ANALIZA ČVRSTOĆE KOMPOZITNE CEVI IZRAĐENE TEHNOLOGIJOM MOKROG NAMOTAVANJA

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Predmet rada je analiza čvrstoće kompozitne cevi izrađene tehnologijom mokrog namotavanja od materijala poliesterska smola/stakleno vlakno opterećeni unutrašnjim nadpritiskom. Primarna pažnja ovog istraživanja je da se uspostavi pouzdana proračunska procedura za analizu naponskog stanja i koeficijenata inicijalnog loma cevi od višeslojnih kompozitnih materijala. Za tu svrhu korišćen je metod konačnih elemenata (MKE) u sprezi sa odgovarajućim kriterijumima loma. Za analizu naponskih stanja u višeslojnoj kompozitnoj cevi sa ortotropnim karakteristikama pojedinih slojeva pod dejstvom unutrašnjeg pritiska korišćen je softverski paket MSC/NASTRAN. U okviru analize naponskih stanja primenom MKE korišćeni su višeslojni 4-čvorni konačni elementi ljuski. Korišćeni konačni elementi u ovoj analizi su razvijeni na bazi teorije smicanja, u čijoj su formulaciji uključeni efekti transverzalnog smicanja kakvi su adekvatni za modeliranje višeslojnih kompozitnih ljuski. Radi verifikacije proračunske procedure za analizu naponskih stanja primenom MKE izvršena su poređenja sa eksperimentom. U okviru eksperimenta izvršena su merenja naponskog stanja koristeći merne trake. Merne trake su postavljene na spoljnoj površini kompozitne cevi. Dobijeno je dobro slaganje rezultata proračuna čvrstoće sa eksperimentom. Dobro slaganje rezultata proračuna čvrstoće, zasnovanih na korišćenju MKE u sprezi sa odgovarajućim kriterijumima loma, potvrđuje valjanost razmatrane proračunske procedure, tako da se ista može koristiti u praktičnim analizama čvrstoće cevi od višeslojnih kompozitnih materijala.

KLJUČNE REČI: Kompozitne strukture • Kompozitne cevi • Inicijalni kriterijumi loma • Metod konačnih elemenata

Key words: Composite structures • Filament-wound composite tubes • Initial failure criteria • Finite element method