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## IMPROVED COMPUTATION METHOD IN RESIDUAL LIFE ESTIMATION OF STRUCTURAL COMPONENTS

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**Abstract.** *This work considers the numerical computation methods and procedures for the fatigue crack growth predicting of cracked notched structural components. Computation method is based on fatigue life prediction using the strain energy density approach. Based on the strain energy density (SED) theory, a fatigue crack growth model is developed to predict the lifetime of fatigue crack growth for single or mixed mode cracks.*

*The model is based on an equation expressed in terms of low cycle fatigue parameters. Attention is focused on crack growth analysis of structural components under variable amplitude loads. Crack growth is largely influenced by the effect of the plastic zone at the front of the crack. To obtain efficient computation model plasticity-induced crack closure phenomenon is considered during fatigue crack growth. The use of the strain energy density method is efficient for fatigue crack growth prediction under cyclic loading in damaged structural components. Strain energy density method is easy for engineering applications since it does not require any additional determination of fatigue parameters (those would need to be separately determined for fatigue crack propagation phase), and low cyclic fatigue parameters are used instead.*

*Accurate determination of fatigue crack closure has been a complex task for years. The influence of this phenomenon can be considered by means of experimental and numerical methods. Both of these models are considered. Finite element analysis (FEA) has been shown to be a powerful and useful tool<sup>1,6</sup> to analyze crack growth and crack closure effects. Computation results are compared with available experimental results.*

## 1. INTRODUCTION

Fatigue crack closure is a phenomenon that consists of the contact between fracture surfaces during a portion of the load cycle. This contact affects the local stress and plastic deformation fields near the crack tip, and thus the micro mechanisms responsible for fatigue propagation (cyclic plastic deformation, oxidation, creep, etc.). Plasticity-induced crack closure is an observed phenomenon during fatigue crack growth.

The constant search to improve aircraft safety has led, over recent years, to the increasingly widespread application of „damage tolerance” concepts. Reliable fatigue life prediction is very important for safe design and maintenance of structural components subjected to cyclic loading<sup>1</sup>. In general, fatigue process consists of three stages: initiation and early crack propagation, subsequent crack growth and final fracture. Due to the fact that if occurs, failure leads to catastrophe, crack growth stage must be carefully studied and analyzed. Each crack growth model for life prediction must be based on a suitable failure criterion. For crack growth analysis, as failure criteria could be used: plastic/total strain ahead of crack<sup>3</sup>, the magnitude of crack tip opening<sup>4,5</sup> and the energy criteria<sup>6,7</sup>. Since crack closure effect is included in fatigue crack growth analysis, the concept of crack opening/closure was used in this paper.

The aim of this paper is to analyze the effect of plasticity-induced crack closure (PICC) using finite element method and determination of new corrective factors for the effective stress intensity factors. Moreover, with crack growth analysis desire was to assess how new corrective factors can to improve crack growth life prediction to failure of structural component.

Due to the fact that the formulated procedure for fatigue crack prediction includes analysis level of external loading as well as the effect of plasticity-induced crack closure we can say that it is adequate as an engineering application.

## 2. CRACK GROWTH PREDICTING

In this paper two numerical simulation approaches to crack propagation and, accordingly, evaluation of residual life for structural elements with initial damages are presented. First approach is based on conventional laws of crack propagation, such as Paris` law of crack propagation<sup>8</sup>. The other approach is based on the strain energy density method.

## 3. CONVENTIONAL CRACK PROPAGATION MODEL

When analyzing crack growth prediction, the usual starting point is relation in which the fatigue crack growth rate is expressed as a function of the stress intensity factor, i.e., a well known and widely used Paris law<sup>8</sup> :

$$\frac{da}{dN} = C(\Delta K)^m, \quad (1)$$

where:  $da/dN$  is crack growth rate,  $C$  and  $m$  – coefficient and exponent dependent upon the materials, respectively. However, with this law, it is not possible to make allowance for the interactions found in real-life spectra.

Equation defined by Paris, even though commonly used in engineering practice, still has some deficiencies. Basic deficiency is the fact that it does not include alternating load/stress and mean load/stress. During their service life structural components could be subject to both of those loads. The mean load effect on fatigue crack growth rate is commonly introduced through the stress ratio  $R$ . Since the mean load effect is not included in Paris's equation it was necessary to either modify Paris' equation or develop new concepts. The crack closure concept is one of those concepts where the stress ratio is analyzed. In general, all crack closure concepts<sup>9,10</sup> are based on the Elber's observation<sup>10,11</sup> which reveals the premature contact of the crack faces during the unloading portion of the loading cycle while some tensile load is still applied. Elber was the first researcher who introduced the effective stress intensity factor range instead of stress intensity factor range  $\Delta K$ , i.e.:

$$\frac{da}{dN} = C(\Delta K_{eff})^m \quad (2)$$

where the effective stress intensity factor range is the function of stress ratio as well as stress intensity factor:

$$\Delta K_{eff} = (0.5 + 0.4R)\Delta K. \quad (3)$$

After Elber, Schjive<sup>8</sup> analyzed the same relation (2) and he found that effective stress intensity factor range could be expressed as:

$$\Delta K_{eff} = (0.55 + 0.33R + 0.12R^2)\Delta K \quad (4)$$

Previously mentioned Elber's and Schjive's approaches could be improved or modified by introducing the effect of plasticity-induced crack closure. As a consequence of introduction of the effect of plasticity-induced crack closure, it is necessary to correct the effective stress intensity factor.

To include the effects of the stress ratio  $R$  the conventional Forman's crack growth model<sup>16</sup> is used. In region III rapid and unstable crack growth occurs, so Forman at al. Proposed equation for region III as well as for region II<sup>17</sup>:

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1-R)K_C - \Delta K} \quad (5)$$

where  $K_C$  is the fracture toughness. Forman's equation has been developed to model of unstable crack growth domain (III). To include PICC effects  $\Delta K_{eff}$  need to use in equation (5).

#### 4. CRACK PROPAGATION MODEL BASED ON THE STRAIN ENERGY DENSITY METHOD

While predicting life of a structural element with initial damage it's necessary to establish the functional dependency between the crack propagation gradient  $da/dN$  and the stress intensity factor  $K_I$ .

The severest damage accumulation occurs in the process zone<sup>18,20</sup>, therefore it's necessary to define and calculate the energy which causes damage in the process zone.

For the zone around the tip of the crack (process zone) it's possible to define the energy generated through plastic strain  $\omega_p$  in a cycle using length unit as a function of stress intensity factor range  $\Delta K_I$ :

$$\omega_p = \left( \frac{1 - n'}{1 + n'} \right) \frac{\Delta K_I^2}{E I_{n'}} \psi \quad (6)$$

where:  $n'$  - cyclic strain hardening exponent,  $E$  - Young's modulus of elasticity,  $I_{n'}$ ,  $\psi$  - constants which depend on the cyclic strain hardening exponent  $n'$ . For most metals the value of  $n'$  usually varies between 0,10 and 0,25, with an average value close to 0,15. Since the dependency for energy generated due to plastic strain  $\omega_p$  as a function of  $\Delta K_I$  is established, it's necessary to establish the dependency between the crack propagation gradient  $da/dN$  and  $\omega_p$ . While establishing the dependency a fact that the crack propagates if energy which generates due to plastic strain during the cycle reaches the energy absorbed during the same cycle  $W_c$  must be taken into account:

$$\frac{da}{dN} = \frac{\omega_p}{W_c} \quad (7)$$

In equation (7) energy absorbed during the cycle  $W_c$  can be defined if stress - strain relation, or the material behaviour equation, is known. Adequate relation for material behaviour which includes both elastic and plastic behaviour is known as Ramberg - Osgood equation<sup>21</sup>:

$$e_a = \frac{S_a}{E} + \left( \frac{S_a}{k'} \right)^{1/n'} \quad (8)$$

where:  $e_a$  - strain amplitude,  $S_a$  - stress amplitude and  $k'$  - cyclic strength coefficient. If the material behavior equation is presented by equation (8), energy absorbed during the cycle  $W_c$  represents the area below the curve in S-e coordinate system, or:

$$W_c = \frac{4}{1+n'} \sigma_f' \varepsilon_f' \quad (9)$$

where:  $\sigma_f'$  - fatigue strength exponent,  $\varepsilon_f'$  - fatigue ductility coefficient. Finally, if equations (6) and (8) get placed in equation (7), functional dependency between crack propagation gradient and stress intensity factor gets established. Subsequently, that dependency can be integrated from initial crack length  $a_i$  to final crack length  $a_c$  in order to obtain the relation which could be used for the prediction of life of structural elements which contain initial damage:

$$N = \frac{(1 - n') \psi}{4 E I_{n'} \sigma_f' \varepsilon_f'} \int_{a_i}^{a_c} (\Delta K_I - \Delta K_{th})^2 \quad (10)$$

where  $\Delta K_{th}$  is range of threshold stress intensity factor.  $\Delta K_{th}$  is a material constant but it is sensitive to stress ratio  $R = S_{min}/S_{max}$ . A relation between  $\Delta K_{th}$  and  $R$  is given below based on experimental results [19]

$$\Delta K_{th} = \Delta K_{th0} (1 - R)^y \quad (11)$$

where  $\Delta K_{th0}$  is the range of threshold stress intensity factor for the stress ratio  $R=0$ , and  $\gamma$  is a material constant which varies from 0 to 1 [12,13]. For most of materials  $\gamma$  comes out to be 0.71 [19]. Equation (10) presents the law of crack propagation based on strain energy density method. It's obvious that in this dependency cyclic characteristics of material from low-cycle fatigue domain are being used instead of dynamic parameters from more conventional laws for crack propagation by Paris, Forman and others. Main advantage of this Strain Energy Density (SED) approach, as shown in eq. (10), is the use of same cyclic material characteristics being used for initial and residual fatigue life predictions [19-21].

### 5. THE STRESS INTENSITY FACTOR

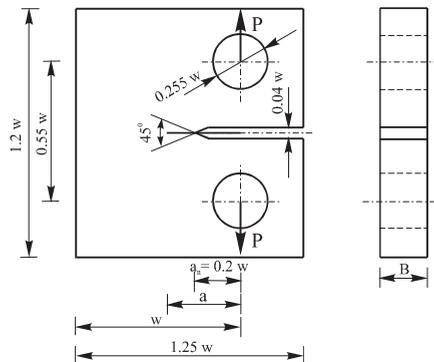
It is well known that stress intensity factors play a major role in crack growth analysis. Actually, with stress intensity factors, geometry of structural component and the type of loading are introduced. The stress intensity factor can be determined using analytical and/or numerical approaches.

In analytical approach, the stress intensity factor range could be determined as a function:

$$\Delta K = f(P, a, w, \dots) \tag{12}$$

where: P is load/force, a – crack length and w – width of specimen. For example, when dealing with CT specimen, relation for stress intensity factor range can be written as:

$$\Delta K = \frac{\Delta P}{B\sqrt{w}} \left( \frac{2 + \frac{a}{w}}{\left(1 - \frac{a}{w}\right)^{3/2}} \left( 0.886 + 4.64 \left(\frac{a}{w}\right) - 13.32 \left(\frac{a}{w}\right)^2 + 14.72 \left(\frac{a}{w}\right)^3 - 5.6 \left(\frac{a}{w}\right)^4 \right) \right) \tag{13}$$



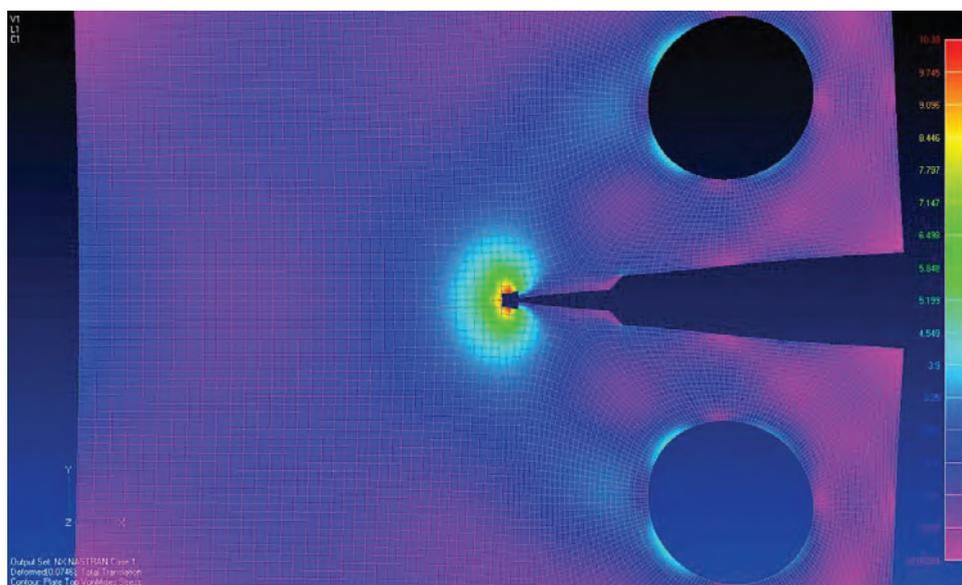
**Figure 1.** Geometry of Compact Tension specimen

The symbol B in equation (13) denotes the thickness of compact specimen and w is the distance between the applied force P and the left edge of the specimen (Fig.1). The

symbol  $a$  in equation (13) is the crack length measured from the line of the application of external load.

On the other hand, when using numerical approach, for determining the stress intensity factor Finite element method (FEM) is used.

A representation of the finite element analysis for CT specimen made of Al Alloy 2024 T351 ( $w = 0.075$  m,  $B = 0.010$  m) are shown in Figure 2. Figure 2 presents stress distribution at CT specimen for crack length  $a = 0.02625$  m. From the same figure it can be seen that for crack length  $a = 0.02625$  m (as a result of finite element analysis), the calculated maximum stress (for  $P_{\max} = 3300$  N and  $R = 0.1$ ) is  $10.39$  daN/mm<sup>2</sup>.



**Figure 2.** Stress distribution at the CT specimen ( $P_{\max} = 3300$  N and  $R = 0.1$ ) using finite element analysis.

Additionally, in this paper, the finite element analysis was used to investigate the plasticity-induced crack closure effects in the calculation of stress intensity factor range. So for stress distribution shown in Figure 2, the calculated stress intensity factor was  $K_{I\max} = 21.93$  daN mm<sup>-3/2</sup>. Furthermore, the same calculation of stress intensity factors were made for different external forces

## 6. THE EFFECTIVE STRESS INTENSITY FACTOR AND CRACK CLOSURE EFFECT

For the phenomenon of crack closure is known that it has a strong influence on fatigue crack growth<sup>11,12</sup>. Elber called this phenomenon plasticity-induced crack closure. Namely, if the crack has reached its current length through fatigue (cyclic loading), there would be a localized plasticity region formed at the crack tip and the wake of the crack. This localized plasticity in itself will generate residual stresses and play a role in crack closure.

Due to the fact that plasticity-induced crack closure phenomenon is included in crack growth analysis, it is necessary to correct relation for the effective stress intensity factor  $\Delta K_{eff}$  (Eq.(3) and Eq.(4)), i.e. to find adequate corrective factors. Since finite element analysis proved to be powerful tool<sup>12</sup> for determination of stress intensity factors, corrective factors were determined/introduced that include plasticity-induced crack closure effect.

When determining the stress intensity factor range, the ranging of the external force was from 3000 N to 14500 N. Namely, five different values from this range were used. For such defined range of load, as well as geometry of CT specimen ( $a = 0.030$  m,  $w=0.075$ m,  $B=0.010$  m) and type of material, after finite element analysis, it is possible to determine corrective factors for stress intensity factor range with including the effect of plasticity-induced crack closure. New corrective factors calculated on this way, for different approaches are listed in Table 2.

**Table 1 Corrective factors**

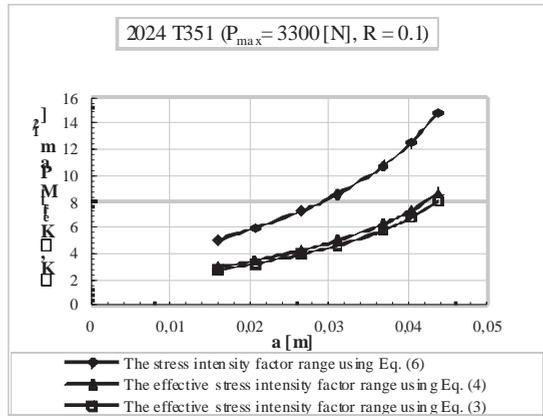
For equation	Corrective factor
$\Delta K_{eff} = (0.5 + 0.4 R) \Delta K$	0.926
$\Delta K_{eff} = (0.55 + 0.33 R + 0.12 R^2) \Delta K$	0.928

## 7. NUMERICAL RESULTS

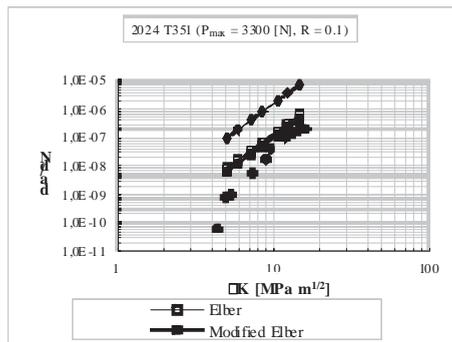
With introduced plasticity-induced crack closure effect, the validity of presented computation model for crack growth prediction could only be assessed through a comparison with experimental data which is the focus of this section. The subject of this work is improvement or modification of Elber's and Schjive's approaches and in examples that follow it is presented how important defined and introduced modification influences on the predicted fatigue crack life of structural components.

**7.1. Example 1a: Crack growth rate prediction of CT specimen subjected with constant amplitude loading**

This example considered crack growth rate and effective stress intensity factor calculation. The material used in this example is 2024 T351 Al Alloy, whose mechanical properties are:  $E = 74000 \text{ MPa}$ ;  $C = 1.51 \cdot 10^{-10}$ ,  $m = 4$ . The configuration of considered CT specimen is shown in Figure 1. Needed geometry parameters are:  $w = 0.075 \text{ m}$ ;  $B=0.010 \text{ m}$ ; and  $a=0.016 \text{ m}$ . The external cyclic loading is with constant amplitude (Load/force  $P_{\text{max}}=3300 \text{ N}$  and stress ratio  $R = 0.1$ ). Before starting the crack growth rate estimation it is necessary to determine the stress intensity factor and effective intensity factor for different values of crack length. In this example, for determination of the stress intensity factor range and effective stress intensity factor range were used equations (6), (3) and (4). The effective stress intensity factor as a function of crack length  $a$  (for different models: Elber, Schijve) are illustrated in Figure 3.



**Figure 3.** A crack length  $a$  versus the effective stress intensity factor range  $\Delta K_{\text{eff}}$  and stress intensity factor range  $\Delta K$ .



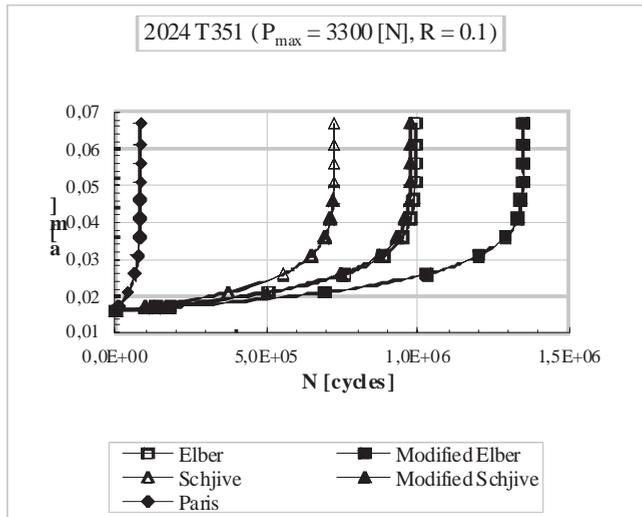
**Figure 4.** Fatigue crack growth rate as a function of stress intensity factor

Based on known characteristics of material, geometry and loading, calculated values of a crack growth rate using different models (Elber, Modified Elber, Schijve and Modified Schijve) are shown in Figure 4. At the same figures all predicted curves for crack growth rate are compared with experimental data<sup>14</sup>.

As observed from Figure 4, the estimated fatigue crack growth rates are in a good agreement with the experimental observations. Additionally, Figure 4 show that Paris’s model is very conservative, while Elber’s and Schijve’s models are less conservative when compared to experimental data. Defined improvements of Elber’s and Schijve’s models presented in this paper, including crack closure effect, provide better predicted values for fatigue crack growth rates. In addition, the best agreement between predicted fatigue crack growth rate and experimental data is obtained when using Modified Elber model.

**7.2. Example 1b: Crack growth life estimation of CT specimen subjected with constant amplitude loading**

In this example fatigue life prediction up to failure was considered. Structural element, material and the type of loading used here are the same as in example 1a. Using the fatigue parameters, according to the geometry of structural component and different fatigue growth models, enabled determination of the fatigue life to failure.



**Figure 5.** Crack growth analysis of CT specimen using different models.

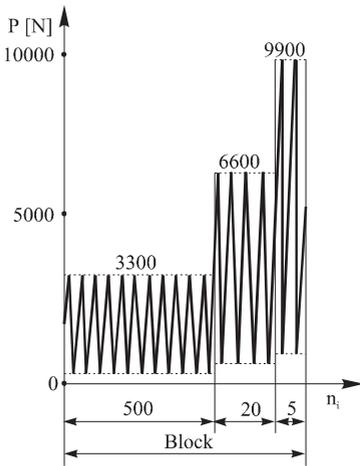
Actually, by using equations (1) or (2) (with (6), (3) or (4)) which were first integrated, the relations between crack length  $a$  and number of cycles to failure  $N$  were formulated. Predicted results using different models (Elber, Schijve, Modified Elber and Modified

Schjive) are shown in Figure 5 for external force  $P_{max} = 3300$  N. As it can be seen from Figure 5 improvements introduced for Elber's as well as Schjive's approaches have significant impact on predicted number of cycles to failure.

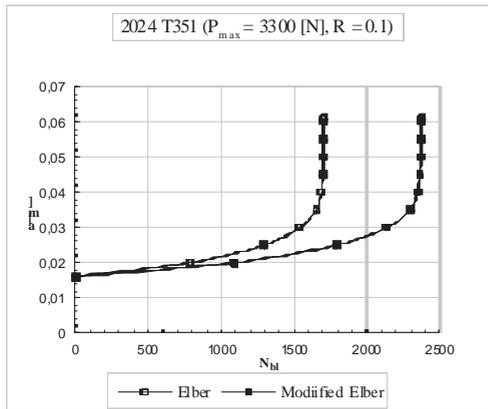
**7.3. Example 2: Crack growth estimation of CT specimen subjected load spectra**

Since that the structural components are usually subjected to load spectra, in this example fatigue crack growth prediction with including crack closure effect for CT specimen subjected load spectrum was carried out. From crack growth analysis in example 1 it can be concluded that Elber's and Modified Elber's approaches are more adequate for prediction of fatigue crack growth. (related to experimental data). That is the reason why they will be analyzed for crack growth prediction in this example, too.

Material used in this example is the same as previous. As a result of fatigue crack growth estimation, number of blocks to failure were obtained using equations (2), (6) and (3). For determination number of blocks to failure, equation (2) was first integrated. After integration, function between number of blocks  $N_{bl}$  and crack length  $a$  was determined.



**Figure 6.** Load spectrum ( $R = 0.1$ )



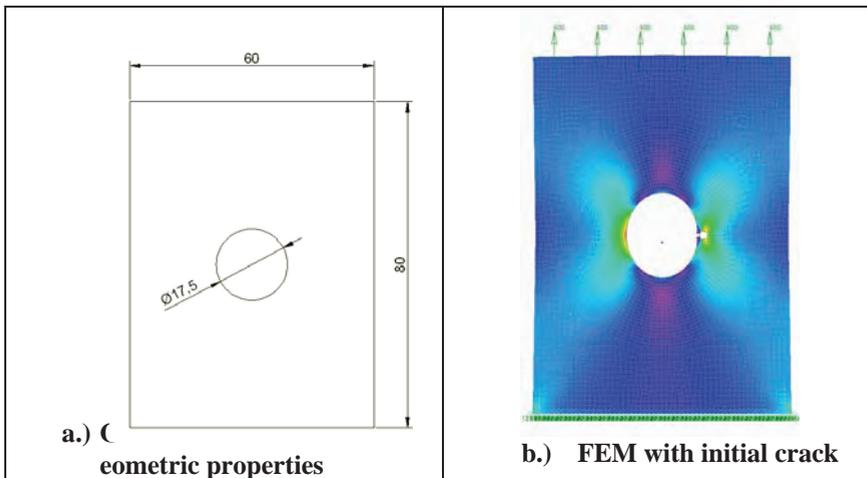
**Fig. 7.** Crack growth analysis of CT specimen subject to load spectra

Figure 7 shows a plot of the estimated number of blocks to failure versus a crack length  $a$ , for Elber and Modified Elber approaches for load spectrum (Fig.6). Conclusion from Figure 7 for fatigue crack growth prediction in the case of load spectrum (Fig.6), is that the effect of plasticity-induced crack closure has significant effect on number of blocks to failure. For load spectrum presented in Figure 6 calculated number of blocks to failure are listed in Table 2.

**Table 2** Comparison of number of blocks to failure for CT specimen ( $P_{max} = 3300$  N,  $R = 0.1$ ).

	$N_{bl}$	$\Delta$ [%]
Elber	1704	28.16
Modified Elber	2372	

Comparison of number of blocks to failure, presented in Table 2, shows that introduced modification that include effect of plasticity-induced crack closure, has been increased the value of predicted number of blocks to failure around 30% for considered load spectrum (Fig. 6).



**Fig. 8** Structural component with hole and initial crack under load spectrum

**7.4. Example 3: Crack growth analysis of plate with a hole under load spectrum**

Here is considered specimen (aluminum 2024 T4) with central hole under load spectrum, Fig 8a ( $w=60$  mm,  $r=8.75$  mm,  $t=6$ mm). Forman crack growth model (5) is used. Finite element model, with initial crack  $a_0$  is used to determine stress intensity factors  $K_I$ . The complete fatigue crack growth prediction, using in-house software, are shown in Table 3 and Fig. 9.

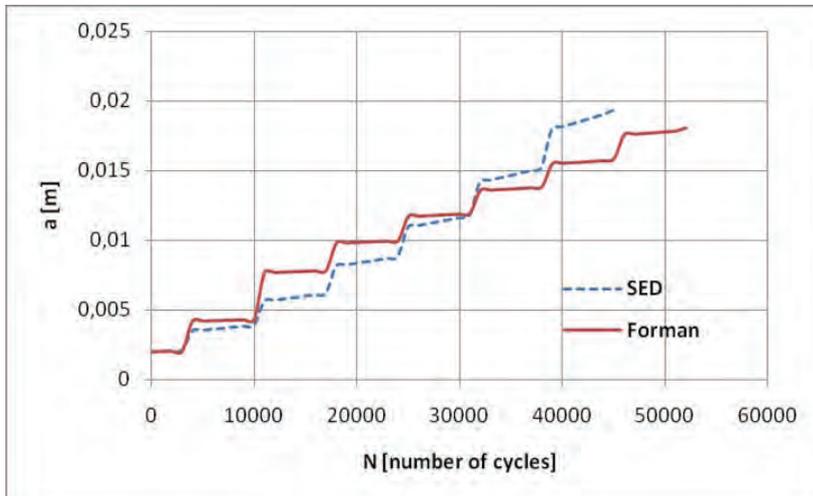
In Table 3:  $C_f$ ,  $n_f$  are Forman’s constants,  $a_c$  is critical crack growth length,  $N_1$  to  $N_{13}$  are number of cycles at load levels within load spectrum.

**Table 3:** Crack growth prediction of specimen with hole under load spectrum

Ulazni podaci				Korak	Finoca stampe	Izaberite zakon širenja:	Brisi		
nf	3,45	Cf	0,000000000434	10	1800	Forman	Izracunaj		
Karakteristike materijala				NUkupno	a	da/dN	KI	DeltaSigma	BrojacBlokova
a0	0,002	E	70430	0	0,002	0	10,90872305	56,3324	0
ac	0,02	SigmaF	764	1800	0,002020447	0,0000000	10,93487646	56,3324	0
w	0,06	EpsilonF'	0,334	3600	0,002868501	0,0000037	32,65258141	156,523	0
t	0,005	n'	0,098	5400	0,004218692	0,0000000	12,45024805	56,332	0
r	0,00875	ln'	3,067	7200	0,004254664	0,0000000	12,46361279	56,3324	1
		Psi	0,95152	9000	0,004290805	0,0000000	12,47676509	56,3324	1
		Kth0	7	10800	0,007613777	0,0000004	26,21720209	111,055	1
		KIc	37	12600	0,007718955	0,0000000	13,31957794	56,332	1
				14400	0,007767462	0,0000000	13,32939311	56,3324	2
				16200	0,007816230	0,0000000	13,33913555	56,3324	2
				18000	0,009799163	0,0000000	13,73502042	56,332	2
				19800	0,009855121	0,0000000	13,74636134	56,332	2
				21600	0,009911294	0,0000000	13,75785863	56,3324	3
				23400	0,009967684	0,0000000	13,76931792	56,3324	3
				25200	0,011724511	0,0000000	14,13494040	56,332	3
				27000	0,011788487	0,0000000	14,14859191	56,332	3
				28800	0,011852756	0,0000000	14,16243002	56,3324	4
				30600	0,011917320	0,0000000	14,17625463	56,3324	4
				32400	0,013604021	0,0000000	14,54540134	56,332	4
				34200	0,013677311	0,0000000	14,56177778	56,332	4
				36000	0,013750998	0,0000000	14,57837241	56,3324	5
				37800	0,013825087	0,0000000	14,59497944	56,3324	5
				39600	0,015534476	0,0000000	14,98456379	56,332	5
				41400	0,015619114	0,0000000	15,00415049	56,332	5
				43200	0,015704296	0,0000000	15,02399450	56,3324	6
				45000	0,015879858	0,0000013	29,69899725	111,055	6
				46800	0,017639345	0,0000000	15,47798944	56,332	6
				48600	0,017738705	0,0000000	15,50155750	56,332	6
				50400	0,017838828	0,0000000	15,52543744	56,3324	7
				52200	0,018531384	0,0000021	43,59665386	156,523	7
				*					

SigmaMax	Min	N1	N2	N3	N4	N5	N6	N7	N8	N9	N10	N11	N12	N13
81,6814	25,349	3040												
122,321	11,266	200												
167,789	11,266	189												
211,245	11,266	43												
245,7	-10,059	15												
298,559	-10,059	3												
316,666	-41,848	1												
298,559	-10,059	3												
254,7	-10,059	15												
211,245	11,266	43												
167,789	11,266	189												
122,321	11,266	200												
81,681	25,349	3040												



**Fig. 9** Crack growth prediction of cracked plate with central hole

## 8. CONCLUSIONS

In this paper improvement of Elber's and Schijve's models for prediction of fatigue crack growth life are recommended. Improvement i.e. modification of Elber's as well as Schijve's model was result of plasticity-induced crack closure effect in fatigue crack growth analysis.

Based on the results of the finite element simulations and the direct comparisons with experimental results, the following conclusions are presented:

Calculated fatigue crack growth rates which were obtained using Paris law are very conservative related to experimental data. So strict conservative result are obtain due to the fact that in Paris equation stress ratio was not included. Much less conservative data were shown in predictions obtained using Elber's and Schijve's approaches;

To include the stress ratio effect Forman's crack growth model is used here, together with Elber's crack closure model;

Finite element method is powerful and useful tool for analysis of plasticity-induced crack closure effect;

Comparison of closure levels between the FE model and experimental results revealed excellent agreement for all tests

By introducing the plasticity-induced crack closure effect in crack growth analysis, the predicted fatigue life can be significantly modified as well as number of blocks to failure, and with it, the high quality of crack growth estimation of cracked structural component could be improved.

Presented computation results are shown that crack growth method based on strain energy density approach is in a good agreement with conventional Forman's approach.

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**POBOLJŠAN PRORAČUNSKI METOD PROCENE PREOSTALOG VEKA  
ELEMENTA KONSTRUKCIJA****Stevan M. Maksimović<sup>\*</sup>, Katarina S. Maksimović<sup>\*\*</sup>**

U radu se razmatraju numeričke metode i procedure za analizu širenja prskotina kod strukturalnih elemenata sa inicijalnim oštećenjima u vidu prskotina. Proračunski metod bazira na proceni preostalog veka koristeći metod gustine energije deformacije (GED). Bazirano na teoriji gustine energije deformacije razvijen je model za za analizu širenja prskotine i procene preostalog veka strukturalnih elemenata za prskotine tipa moda I. Model je zasnovan na zakonu širenja prskotine koji bazira na korišćenju malociklusnih zamornih karakteristika materijala. Pažnja je usmerena na analize širenja prskotina pri opštem spektru opterećenja. Značajan uticaj plastifikacije oko vrha prskotine ima na širenje prskotine. Da bi se dobio efikasan i pouzdan proračunski model u radu je razmatran uticaj plasifikacije oko vrha prskotine na zatvaranje prskotine. Korišćenje gustine energije deformacije predstavlja sa svoje strane efikasan metod za analizu širenja prskotine kod strukturalnih elemenata sa inicijalnim oštećenjima u vidu prskotine. Metod gustine energije deformacije je pogodan sa aspekta inženjerske primene jer ne zahteva dodatne dinamičke karakteristike materijala (za čije bi određivanje bila potrebna dodatna ispitivanja) već koristi samo malociklusne zamorne karakteristike materijala kakve se koriste i za problem procene veka do pojave inicijalnog oštećenja. Precizno određivanje zatvaranja prskotine zbog plastifikacije oko njenog vrha predstavljao je kompleksan problem istraživanja tokom poslednjih godina. Ovaj fenomen je istraživan preko numeričkih i eksperimentalnih metoda. Metod konačnih elemenata (FEM) se pokazao kao pouzdan alat<sup>1,6</sup> za analizu širenja prskotine gde su bili uključeni i efekti zatvaranja vrha prskotine. Proračunski rezultati su upoređeni sa raspoloživim eksperimentalnim rezultatima.

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