EMPIRICAL $\Sigma - D$ RELATIONS AND MAIN GALACTIC RADIO LOOPS

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SUMMARY: This paper represents a review of the empirical Galactic and extragalactic $\Sigma - D$ relations (relations between the surface brightness $\Sigma$ and the diameter $D$) for supernova remnants (SNRs), with checking the connection of the main Galactic radio loops (Loop I, II, III and IV) with these relations. We present results which suggest, in another way, that the radio loops may have an SNR origin. The updated relations have been measured to have slopes, $\beta \approx 2$ in log-log space. The best $\Sigma - D$ relations for M31 and M33 galaxies were derived and these relations are shown to be flatter ($\beta < 2$) than those for Galactic SNRs alone. This result confirms that selection effects play an important role in data-sets made up of Galactic SNRs. A $\Sigma - D$ relation with 157 reliable calibrators (both Galactic and extragalactic) is derived. This relation also has a slope $\beta = 2.07 \pm 0.13$.

1. INTRODUCTION

In this paper, we present a review of the relation between the radio surface brightness $\Sigma$ and the diameter $D$ (so called $\Sigma - D$ relation) for supernova remnants (SNRs). The updated $\Sigma - D$ relations represent the basic points of our analysis. Also we check some old relations. The $\Sigma - D$ relations have been used to investigate the origin of the main Galactic radio loops (Loop I, II, III and IV). The $\Sigma - D$ relations were updated by adding data on the radio loops to the SNR data set. An extensive analysis of all Galactic and extragalactic $\Sigma - D$ relations is presented in Urošević (2000).

In Section 2, we present a short review of the theoretical, Galactic and extragalactic $\Sigma - D$ relations. In Section 3 the influence of the selection effects on the $\Sigma - D$ dependence is discussed. The short review of the main Galactic radio loops is given in Section 4 (emphasis is on the loops origin). Section 5 contains connection of the basic evolution theories of the SNRs’ radio emission with radio loops. The Section 6 contains analysis of two old and all updated $\Sigma - D$ relations. The discussion is in Section 7. The main conclusions derived from this paper are in Section 8.

2. THE $\Sigma - D$ RELATION

2.1. The Theoretical Relation

The relation between the surface brightness $\Sigma$ and the diameter $D$ (so called $\Sigma - D$ relation) is a convenient method for investigation of the radio brightness evolution of supernova remnants (SNRs). Shklovsky (1960a) theoretically analyzed synchrotron radiation of a spherical expanding nebula and the $\Sigma - D$ relation is the result of that theoretical
analysis. It has the form:

$$\Sigma = AD^{-\beta}. \quad (1)$$

This relation was analyzed theoretically also by Lequeux (1962), Poveda and Wolter (1968) and Kesteven (1968). An updated theoretical derivation of this relation for shell-like SNRs is reported by Duric and Searl (1986, hereafter D&S). The structure of the derivation is similar to Shklovsky’s, they adopted Bell’s (1978a, b) formulation of Fermi’s acceleration mechanism. Bell formulated a model in which acceleration of particles is produced by repeated scattering of particles across a shock front. Turbulence downstream from the shock (convection zone) provides the scattering in the upstream direction. As particles propagate upstream, ahead of the shock, they excite Alfvén waves if the shock is super-Alfvénic. They are scattered by the waves, and some find their way downstream again. The particles have finite probabilities of repeating the cycle, the probability being directly proportional to the particle velocity. This leads to a power-law distribution in energy for the accelerated particles and a number density which is a function of the shock velocity. The magnetic field model D&S used is based on the research of Gull (1973) and Fedorenko (1983). Gull proposed a model in which the ambient magnetic field is amplified in the convection zone and that it provides the environment in which relativistic electrons can radiate efficiently. Fedorenko formulated a model in which the magnetic field $B$ varies with $D$ according to $B \propto D^{-x}$, where $1.5 \leq x \leq 2$.

### 2.2. The Galactic Relation

Early observations suggested the existence of a $\Sigma - D$ relation in the form the Shklovsky theory had predicted. The first empirical $\Sigma - D$ relation was determined by Poveda and Wolter (1968). Using the $\Sigma - D$ relation Shklovsky (1960b) presented a way for determining the distances to SNRs as surface brightness is independent of the distance to the radio source. Milne (1970) derived an empirical $\Sigma - D$ relation and calculated distances to all observed SNRs in our Galaxy (97 in all).

Furthermore, the relation was the subject of many investigations in an attempt to determine precisely the specific set of calibrators and therefore achieve a better $\Sigma - D$ relation. The basic criterion for the choice of calibrators is a reliable distance to the remnant. Most studies done during the 1970s and the early 1980s are of this kind. Better observations enabled more precise calculations of the distances to the calibrators and their numbers increased. In this period Galactic $\Sigma - D$ relations were determined by: Downes (1971), Ilovaisky and Lequeux (1972), Wolter (1972), Berkhuijsen (1973), Clark and Caswell (1976), Sabin (1977), Milne (1979), Caswell and Lerche (1979), Göbel et al. (1981), Lozinskaya (1981), Sakhilov and Smirnov (1982). Critical analysis of this relation began with Allakhverdyev et al. (1983a, b) and continued with the research of Green (1984) and Allakhverdyev et al. (1986a, b). Inaccurate calculation of the distances to certain calibrators is the basic deficiency of the relations derived in this manner, i.e. there are not enough remnants with precisely calculated distances necessary for the derivation of the proper $\Sigma - D$ relation (Green 1984). Also, the interstellar medium where supernovae exploded must be taken into consideration. Allakhverdyev et al. (1983a, b; 1986a, b) showed that it makes sense to derive the relation only for the shell-like remnants. In this period, Huang and Thaddeus (1985) and Berkhuijsen (1986) worked on the $\Sigma - D$ relation. These studies generally have some specific particularities. From the beginning of the investigation of this relation, difference between theoretical and empirical results was established. A $\Sigma - D$ relations were not an interesting research topic in late 1980s and early 1990s. Research in this field was discontinued after the decade-long "attacks" on the $\Sigma - D$ relation. Once again, Green (1991) showed that calibrators are too scattered on the $\Sigma - D$ diagram so that no valid relation can be derived. However, this view was challenged by Case and Bhattacharya (1995, 1996, 1998). Their paper of 1998 is based on the calculations of new distances to the calibrators (37 of them), with the help of new galactic constants. The result was a much flatter slope for the $\Sigma - D$ relation. They further emphasized the inconsistency between the empirical and the theoretical relations. Case and Bhattacharya (1998, hereafter C&B) updated Galactic empirical $\Sigma - D$ relation and determined distances for all identified shell-like SNRs. At this moment, 225 galactic SNRs are known (Green 2000). Even though some aspects of the $\Sigma - D$ relation have not been completely explained yet, almost four decades after it was first published the study of the relation continues to evolve, both theoretically and empirically.

### 2.3. Extragalactic Relations

Constructions of the extragalactic $\Sigma - D$ relations are very useful because all calibrators are approximately at the same distance. Therefore the distance determination problem does not exist, if we know the distance to the galaxy. If we identify a SNR, that SNR becomes a calibrator. Furthermore, an extragalactic set of SNRs does not suffer from Malmquist bias (distance dependent selection effect). However, sensitivity becomes an issue and most extragalactic SNRs have therefore been detected in the nearby galaxies LMC, SMC, M31, M33.

The first empirical extragalactic $\Sigma - D$ relation was constructed by Mathewson and Clarke (1973) for the Large Magellanic Cloud (LMC) with 15 identified SNRs. This was followed by the work of Milne et al. (1980) with 19 SNRs, Mathewson et al. (1983) 31 SNRs (25 from the LMC and 6 from SMC), Mills et al. (1984) 38 SNRs (27 in the LMC and 11 SNRs in the SMC). The nearby spiral galaxies M31 and M33 were investigated by Berkhuijsen (1983). Braun and Walterbos (1993) identified 51 SNRs in M31 using VLA at 20 cm. Also, Duric et al. (1995) identified
53 SNRs in M33 using VLA. In general, the extragalactic relations were found to be flatter than the Galactic ones.

3. SELECTION EFFECTS

Data sets of Galactic SNRs suffer from strong Malmquist bias. Simply put, this means that intrinsically bright SNRs are favored because they are sampled from a larger spatial volume in any given flux limited survey. The result is a bias against low surface-brightness remnants such as highly evolved old SNRs. On the other hand, data sets made up of extragalactic SNRs do not suffer from Malmquist bias because all SNRs are at the same distance and are therefore sampled from the same volume. Though extragalactic data sets are generally better fitting relative to Galactic samples, they do suffer from other selection effects. These effects are connected with sensitivity, resolution and confusion leading to data sets that span a shorter range of diameters and surface brightness.

The Malmquist bias causes a surplus in the number of smaller and younger SNRs. As the radio loops are big (evolved) objects, they could moderate this selection effect if they are indeed old SNRs. Radio loops are important objects, and could be used as calibrators for valid $\Sigma - D$ relation. Therefore, the radio loops should on the $\Sigma - D$ relation for shell-type SNRs. It could be another way for showing of the SNR origin of the radio loops.

For the extragalactic sets, the loops-like objects are necessary, too. Therefore, we test extragalactic relations together with the radio loops. It is one way for moderation of the selection effect whose origin is in the radio telescope sensitivity (surface brightness limit). In the future, we should be able to try to detect the low brightness SNRs (loops-like objects) in the other galaxies, if we want to establish a valid set of calibrators and valid $\Sigma - D$ relation.

4. MAIN GALACTIC RADIO LOOPS

For more then three decades it has been known that some radio spurs can be joined into small circles. The set of spurs belonging to such a circle is referred to as a loop. By the early seventies four major galactic loops were recognized in this way. They can be seen clearly in all-sky radio continuum images. The discoveries and studies followed in the order: Loop I (Large et al. 1962, Haslam et al. 1964, Large et al. 1966, Salter 1970), Loop II (Large et al. 1962, Quigley and Haslam 1965, Salter 1970), Loop III (Quigley and Haslam 1965, Salter 1970) and Loop IV (Large et al. 1966, Salter 1970). The most precise determination of the parameters of these loops was performed by Salter (1970) and published in Berkhuizen et al. (1971). Salter used the best data available at the time, at 408 MHz, 404 MHz, 240 MHz and 178 MHz. A detailed review of the subject was also published by Salter (1983).

Although our understanding of these intriguing objects still contains a considerable number of loose ends and question marks, the supernova theory of their origin acquired an enhanced respectability thanks to the extensive observations of X-ray emission from Loop I. Discovery of analogous HI and X-ray loop features (e.g. Heiles 1979, Nousek et al. 1981, Egger and Aschenbach 1995) suggests that large shells may be a rather common and important feature of the interstellar medium. Such observations seem to be particularly relevant at the present time regarding the suggestions that old supernova remnants probably shape the character of a major fraction of the interstellar medium (McKee and Ostriker 1977).

The SNR radio loop hypothesis originated in the work by Brown et al. (1960). At the time, most discussions concerned the North Polar Spur (NPS) – the main part of Loop I – clearly visible in the earliest radio-continuum surveys. The first radio loop model supportive of the SNR hypothesis can be attributed to Berkhuizen et al. (1971). It is based upon the geometry of radio loops (follow a small circle on the sky) (Salter 1970); (later on confirmed in the paper by Milogradov-Turin and Urošević (1997)) through observations of intensity gradients in the brightest part of the spur, HI regions attached to the outer edges of the remnants, runaway stars and spectral indices typical of non-thermal objects. The previously mentioned characteristics are geometrical, observed in the radio range of the spectrum, except the spectral index (anticipated by theory).

These pioneering investigations led to many others that supported the SNR radio loops hypothesis (e.g. Salter 1983, Kosarev et al. 1994, Egger and Aschenbach 1995, Sembach et al. 1997). Naturally, there were other models which explained the origin of radio loops in totally different manner (e.g. Mathewson 1968, Sofue 1977, 1994). Mathewson (1968) considered the hypothesis of loop unification. He noted that the spur ridges and regions of strong radio polarization follow the "flow patterns" of the optical polarization vectors and concluded that the spurs and loops are tracers of the heliacal magnetic field structure of the local spiral arm. Sofue (1977) proposed a completely different theory for Loop I. This came about almost as a by-product of his ideas on the formation of the 3 kpc expanding spiral arm. He envisaged isotropic MHD waves from the Galactic center propagating through the halo and disk of the Galaxy and showed that these would converge with high efficiency into a ring in the disk. For his model, at $t > 10^5$ yr some 80% of the energy of the waves would converge in the disc at about 3.5 kpc from the centre, while the remainder would expand quasilaterally into the halo forming an immense shell structure. Sofue (1994) again explained NPS as an object of Galactic dimensions. The origin of NPS is a gigantic explosion that took place near the galactic center. A strong shock wave caused by an explosion expelled the shell into the halo, according to the model of Sofue (1994).
5. THE BASIC EVOLUTION THEORIES OF THE SNRs’ RADIO EMISSION AND THE RADIO LOOPS

There are two traditional evolution theories of the SNRs’ radio emission: the Shklovsky theory (1960a,b) and the van der Laan theory (1962a,b). The main difference between these two is the following: according to van der Laan, the SNR magnetic field is amplified by compression of the interstellar magnetic field (due to the interaction between the shock wave and the envelope ejected by the explosion) and the SNR radiates from the edge of the cloud whereas the magnetic field remains constant with the expansion of the remnant. Contrary to that opinion, the Shklovsky theory claims that the whole expanding sphere is radiating and the magnetic field (frozen in it) decreases with the square of radius. It is evident that the radio loops model (assuming that the loops are local SNRs) should be supported by van der Laan theory due to the shell-like remnant geometry and the constant magnetic field that should extend to greater dimensions easily. Spoonstra (1972, 1973) compared the parameters received from his polarization observations of the radio loops (also, showing the loops are nearby objects) with the parameters given by van der Laan theory and achieved a “reasonable” fit. Only at first glance, the updated DK&S theoretical interpretation of the initial Shklovsky theory does not explain the nature of the radio loops, however if we use updated empirical $\Sigma - D$ relations, with big uncertainty, loops could be explained as supernova remnants in the sense of McKee and Ostriker’s (1977) SNR model.

6. ANALYSIS AND RESULTS

In this Section, we update and analysis Galactic and extragalactic $\Sigma - D$ relations in two ways. First, we add radio loops to some old and updated relations to determine whether the loop properties are consistent with a SNR origin. Second, we compile the latest data and derive the M31, M33 and “master” relations.

The review of all Galactic and extragalactic $\Sigma - D$ relations, with variants of the original calibrators to which supplement loops or some other evolved SNRs are added, is given by Urošević (2000).

We have accepted surface brightness (at 1 GHz) and diameters (in pc) for four main radio loops from the Berkhuijsen (1986) study. All initial calibrators along with the radio loops, which define the following $\Sigma - D$ relations, are assumed to have equal statistical weight.

6.1. Some Old Relations

6.1.1. Test of the Ilovaisky and Lequeux (1972) Relation

Ilovaisky and Lequeux (1972) determined the set of the calibrators with precisely determined distances. For more details see Urošević (2000). They derived a $\Sigma - D$ relation with a steep slope. We use their 12 calibrators and obtain the relation:

$$\Sigma_{1GHz} = 2.09^{+2.80}_{-1.20} \times 10^{-15} D^{-3.79 \pm 0.31},$$

with a 94% fit quality. If we add the 4 main radio loops to the initial set of calibrators, the following relation is derived by the best fit method:

$$\Sigma_{1GHz} = 1.13^{+1.08}_{-0.55} \times 10^{-16} D^{-2.63 \pm 0.19},$$

with a 93% fit quality. This test was done by Berkhuijsen (1973). Original calibrators were supplemented with Loop I and the Origem loop. She concluded that test result is in accordance with the SNR hypothesis for the radio loops.

The $\Sigma - D$ diagrams associated with relations (2) and (3) are shown in Fig. 1.

The effect of adding all 4 Galactic loops to the Ilovaisky and Lequeux (1972) set of SNRs is to reduce the slope of the relation dramatically. It is clear that $\Sigma - D$ properties of the Galactic loops are inconsistent with this set of SNRs.

6.1.2. Test of the Huang and Thaddeus (1985) Relation

Huang and Thaddeus (1985) determined a homogeneous set of the shell-like SNRs associated with large molecular clouds or cloud complexes with well-determined distances. They derived the specific $\Sigma - D$ relation for remnants in a dense environment. They use a set of 12 SNRs. For these 12 SNRs we obtain the relation:

$$\Sigma_{1GHz} = 3.61^{+7.87}_{-2.47} \times 10^{-16} D^{-3.20 \pm 0.34},$$

with a 90% fit quality. If we add the 4 main radio loops to the initial set of SNRs, the following relation is derived:

$$\Sigma_{1GHz} = 4.25^{+6.71}_{-2.60} \times 10^{-17} D^{-2.48 \pm 0.24},$$

with a 88% fit quality. The $\Sigma - D$ diagrams associated with relations (4)and (5) are shown in Fig. 2.

This test shows that the radio loops are not consistent with the initial SNR set. But, this might be expected, because the radio loops are objects in a very thin density environment (e.g. Kosarev et al. 1994) - contrary to the SNRs near molecular clouds.

6.2. The Updated Galactic Relations

6.2.1. Test of the Relation Based on the Green’s (1991) Calibrators

Green (1991) determined the set of the calibrators (24 SNRs) with known distances. He concluded that the $\Sigma - D$ correlation is poor. Therefore, he did not derive $\Sigma - D$ relation. Using Green’s calibrators, we obtain the relation

$$\Sigma_{1GHz} = 1.61^{+2.28}_{-0.94} \times 10^{-17} D^{-2.16 \pm 0.30},$$

for more details see Urošević (2000). They
Fig. 1. The $\Sigma - D$ diagrams at a frequency of 1 GHz. Twelve Ilovaisky and Lequeux (1972) calibrators are represented by asterisks and the Galactic loops by circles.

Fig. 2. The $\Sigma - D$ diagrams at a frequency of 1 GHz. Twelve Huang and Thaddeus (1985) calibrators are represented by asterisks and the Galactic loops by circles.

with 71% fit quality. In comparison with Green's (1984) study, the fit quality is increased by 17%. If we add the 4 main radio loops to the initial set of Green's calibrators, the following relation is derived by the best fit method:

$$\Sigma_{1\text{GHz}} = 1.66^{+1.69}_{-0.84} \times 10^{-17} D^{-2.17\pm0.21}, \quad (7)$$

The resulting change in $\beta$ is negligible. This means that the addition of the loops does not qualitatively alter the slope of the original relation.
The Σ – D diagram associated with relation (6), as defined for Green’s calibrators, is shown in Fig. 3.

**Fig. 3.** The Σ – D diagram at a frequency of 1 GHz for 24 Green’s calibrators.

With the loops added, the fit quality of Green’s relation is increased by 10% (from 71% to 81%). Scatter is noticeable but basically explained by the fact that remnants are evolving in different interstellar media, they have different blast energies, unreliably calculated distances, and have both Type I and Type II origins (e.g. Dickel et al. 1993). It can be seen that Galactic loops are occupying the lower right area of the Σ – D diagram (see Fig. 4). Furthermore, the uncertainty in the parameters A and β are reduced with the loops added.

From this test, we conclude that the Σ – D dependence with radio loops exists with reasonable fit quality quotient. However all selection effects which affect this relation, precludes any strong conclusions. We may conclude, that the radio loops are in statistical agreement with Green’s Galactic SNR set.

### 6.2.2. Test of the C&B Relation

More recently an updated Galactic Σ – D relation was established by C&B. Thirty seven galactic shell-like remnants with reliably calculated distances were taken as calibrators. Whenever the kinematic method was required for determination of the distances to the calibrators, a rotation model based on the values of galactic constants \( R_\odot = 8.5 \text{ kpc} \) and \( V_\odot = 220 \text{ km/s} \) was used. Two Σ – D relations were derived. The first is related to all thirty seven remnants and the second to thirty six remnants (without Cas A which deviates greatly from the best-fit line). C&B considered the latter relation more representative since Cas A is unusual compared to other Galactic, shell-like remnants. They found a flatter slope \((\beta = 2.38 \pm 0.26)\) using the thirty-six calibrators, and concluded that this result is in good agreement with Σ – D relations for other galaxies.

If we add the Galactic loops to these 36 calibrators, the following relation is derived:

\[
\Sigma_{1\text{GHz}} = 1.31^{+1.46}_{-0.69} \times 10^{-17} D^{-2.23 \pm 0.20}, \quad (8)
\]

The change in \( \beta \) is greater than in the case of Green’s relation (equations 6 and 7) but is still within the statistical uncertainty of \( \beta \). The Σ – D diagram associated with relation (8) is shown in Fig. 5.

**Fig. 4.** The Σ – D diagram at a frequency of 1 GHz. Green’s calibrators are represented by asterisks and loops by circles.

With loops added, the fit quality for the updated C&B relation has risen by 5% (from 71% to 76%). Again, the loops are occupying the lower right area of Σ – D diagram (see Figs. 5 and 6). The uncertainties in the parameters A and β are both slightly reduced by adding the loops.

The C&B Σ – D diagram, for thirty six calibrators, with schemes of Galactic loops is shown in Fig. 6. Loop IV is on the best-fit line suggesting that it is an SNR given its Σ – D parameters. It is obvious from Fig. 6 that Loop I (diameter 230 pc) is closer to the best-fit line than Cas A (also shown). It is possible that Loop I is more normal remnant than Cas A, according to these criteria. In fact, all 4 Galactic loops may be considered to be SNRs by these criteria. Besides, Cas A is not the only remnant deviating from the fit line. Another five calibrator remnants are more distant from the fit line (W51, CTB37A, Kes67, CTB37B, and G349.7+0.2).

It means that for their brightness, these remnants have larger diameters than Loop I has for its brightness. Remnants SN1006, CTA1 and G156.2+5.7 are...
6.3. The Updated Extragalactic Relations

6.3.1. Test of the Relations for M31

Using the catalogue of Braun and Walterbos (1993), we derived three relations. Each relation was derived on the basis of the quoted quality of the data for the 51 detected SNRs in M31. The catalogue does not contain the spectral index information. Therefore, we have used an average spectral index of $\alpha = 0.5$ for the SNRs using the observed flux density at 1465 MHz. The diameters were obtained from the optical measurements (Braun and Walterbos 1993).

Using the 24 best radio identified SNRs with signal-to-noise ratios greater than 5, we obtained the relation at 1465 MHz in the form $\Sigma \propto D^{-1.68 \pm 0.26}$.

At 1 GHz this relation has the form:

$$\Sigma_{1\text{GHz}} = 2.14^{+3.12}_{-1.27} \times 10^{-18} D^{-1.68 \pm 0.26},$$

with a 66% fit quality.

If we add Galactic loops to these 24 calibrators, the following relation is derived:

$$\Sigma_{1\text{GHz}} = 3.83^{+3.38}_{-1.75} \times 10^{-18} D^{-1.86 \pm 0.17},$$

with a 83% fit quality.

The $\Sigma - D$ diagrams associated with relations (9) and (10) are shown in Fig. 7.

Using their 30 calibrators (previous 24 along with 6 low quality identified SNRs - 5 > signal-to-noise ratio in flux density > 3), we obtained relation at 1465 MHz in the form $\Sigma \propto D^{-1.69 \pm 0.27}$. At 1 GHz this relation has the form:

$$\Sigma_{1\text{GHz}} = 1.72^{+2.64}_{-1.04} \times 10^{-18} D^{-1.69 \pm 0.27},$$

with a 59% fit quality.

If we add Galactic loops to these 30 calibrators, the following relation is derived:

$$\Sigma_{1\text{GHz}} = 2.26^{+2.20}_{-1.12} \times 10^{-18} D^{-1.78 \pm 0.18},$$

with a 75% fit quality.

The $\Sigma - D$ diagrams associated with relations (11) and (12) are shown in Fig. 8.

Using their 51 calibrators (previous 30 along with 21 non detected SNRs), we obtained relation at 1465 MHz in form $\Sigma \propto D^{-1.82 \pm 0.28}$. At 1 GHz this relation has the form:

$$\Sigma_{1\text{GHz}} = 1.30^{+2.25}_{-0.83} \times 10^{-18} D^{-1.82 \pm 0.28},$$

with a 47% fit quality.

If we add loops to these 51 calibrators, the following relation is derived:

$$\Sigma_{1\text{GHz}} = 8.78^{+11.34}_{-4.95} \times 10^{-19} D^{-1.70 \pm 0.22},$$

with a 53% fit quality.

The $\Sigma - D$ diagrams associated with relations (13) and (14) are shown in Fig. 9.

more distant too, but from the left of the fit line, i.e. in the minimum diameter area.

Test result of the C&B relation, in the statistic manner, is supporting the SNR origin of the main radio loops - but without any strong conclusion.
Fig. 7. The $\Sigma - D$ diagrams at a frequency of 1 GHz. Twenty four M31 calibrators are represented by asterisks and the loops by circles.

Fig. 8. The $\Sigma - D$ diagrams at a frequency of 1 GHz. Thirty M31 calibrators are represented by asterisks and the loops by circles.

The best test for the M31 galaxy is the one of the first relation defined for 24 calibrators. The radio loops fit well the first and the second relation. The quotient variations $\Delta \beta = -0.18$ and $\Delta \beta = -0.09$ are easily noticeable as well as the fact that they correspond to the intervals predetermined by these quotient’s errors. This means that addition of the loops to other calibrators alters the slope of the original relations keeping them in the permitted ranges of values. The Loops II and IV have smaller diameters than the average diameters defined by relations (10) and (12). The Loop III (test of the first relation) is on the fit line. With loop addition, the fit quality for the test of the first relation is rising up by 17% (from 66% to 83%). The loops are occupying empty area of $\Sigma - D$ diagrams (see Figs. 7 and 8) and balancing the relations.
Fig. 9. The $\Sigma - D$ diagrams at a frequency of 1 GHz. Fifty one M31 calibrators are represented by asterisks and the loops by circles.

The third relation has a strong scattering. In Fig. 9 we could see SNRs with very low surface brightness (below the loops brightness) and with two times smaller diameters. These SNRs, probably, are not correctly identified objects in the radio part of spectrum. Therefore, they are not sufficiently good for researches of this kind. The radio flux densities are not determined precisely, because in the catalogue exists only lower limit of the flux densities (signal is not detected).

The addition of Galactic loops to the M31 data results in slightly steeper relations and reduced uncertainty in $\beta$. We again conclude that properties of the Galactic loops are consistent with those of M31 SNRs.
6.3.2. Test of the Relations for M33

Using the new sample of radio-selected and optically confirmed SNRs in M33 by Gordon et al. (1999), we derived three relations, again, for the 51 detected SNRs at the 1465 MHz. We adopted \( \alpha = 0.5 \) for 9 SNRs which had no spectral index information. From the flux density at 1465 MHz we have calculated the surface brightness at 1 GHz. The diameters are from the optical measurements (as in the case of M31 galaxy).

For the 36 remnants with reliable diameters and calculated spectral indices, the relation obtained at 1465 MHz is \( \Sigma \propto D^{-1.80 \pm 0.21} \). At 1 GHz, we obtain:

\[
\Sigma_{1\text{GHz}} = 5.13^{+6.02}_{-2.77} \times 10^{-18} D^{-1.82 \pm 0.22},
\]

with a 67% fit quality.

If we add the Galactic loops to the set of initial 36 M33 calibrators, the following relation is derived:

\[
\Sigma_{1\text{GHz}} = 1.06^{+0.79}_{-0.44} \times 10^{-17} D^{-2.02 \pm 0.16},
\]

with a 82% fit quality.

The \( \Sigma - D \) diagrams associated with relations (15) and (16) are shown in Fig. 10.

For the 36 remnants with reliable diameters and calculated spectral indices along with 7 remnants for which spectral indices were not calculated, the obtained relation at 1465 MHz has the form \( \Sigma \propto D^{-1.79 \pm 0.21} \). At 1 GHz, we obtain:

\[
\Sigma_{1\text{GHz}} = 4.33^{+4.87}_{-2.29} \times 10^{-18} D^{-1.81 \pm 0.21},
\]

with a 63% fit quality.

If we add the loops to the set of 43 calibrators, the following relation is derived:

\[
\Sigma_{1\text{GHz}} = 7.54^{+5.74}_{-3.26} \times 10^{-18} D^{-1.97 \pm 0.15},
\]

with a 79% fit quality.

The \( \Sigma - D \) diagrams associated with relations (17) and (18) are shown in Fig. 11.

Using the previous 43 along with the 8 SNRs with unreliable diameters, we obtained relation at 1465 MHz in the form \( \Sigma \propto D^{-1.74 \pm 0.19} \). At 1 GHz this relation has the form:

\[
\Sigma_{1\text{GHz}} = 3.50^{+3.48}_{-1.74} \times 10^{-18} D^{-1.76 \pm 0.19},
\]

with a 62% fit quality.

If we add the loops to the set of 51 calibrators, the following relation is derived:

\[
\Sigma_{1\text{GHz}} = 6.08^{+4.35}_{-2.54} \times 10^{-18} D^{-1.92 \pm 0.15},
\]

with a 77% fit quality.

The \( \Sigma - D \) diagrams associated with relations (19) and (20) are shown in Fig. 12.

![Fig. 11. The \( \Sigma - D \) diagrams at a frequency of 1 GHz. Forty three M33 calibrators are represented by asterisks and the loops by circles.](image-url)
The results of the previous three tests for the M33 galaxy are very similar to each other. For the M31 galaxy that is not the case (poor result of the third test). The set of SNRs from M33 galaxy is very homogeneous and good for further analysis. Radio loops fit well all of these relations. Again, quotient variations $\Delta \beta = -0.20, \Delta \beta = -0.16$ (the second and third test) are easily noticeable as well as the fact that they correspond to the intervals predetermined by these quotient’s errors. Like in previous case, this means that addition of the loops to other calibrators alters the slope of the original relations keeping them in the permitted ranges of values. The Loops II and IV have smaller diameters than the average ones defined by relations (16), (18) and (20). Loop III is on the fit lines (see Fig. 10, 11 and 12). With the loops addition, the fit quality increases approximately by 15% (from 67% to 82% – first test). Again, the loops are occupying empty area of the $\Sigma - D$ diagrams (see Fig. 10, 11 and 12) and balancing the relations.

Once again, it is evident that the addition of Galactic loops to the M33 data leads to a steeper relation and reduced uncertainty in $\beta$. The M33 data therefore support the possibility that the loops are indeed SNRs.

The results of the investigation of the $\Sigma - D$ relations in nearby galaxies M31 and M33 show that the loops represent, probably, some kind of old SNRs which are missing in the $\Sigma - D$ diagrams. The relations show very flat slope, $\beta < 2$ (without loops). We obtain relations with $\beta \approx 2$ including radio loops.

6.3.3. Test of the Relation for M82

The starburst galaxy M82 is very interesting object because it contains many young SNRs. Fifty SNRs were identified by Huang et al. (1994). All remnants are less than 6 pc in diameter. The same authors established the $\Sigma - D$ relation for these remnants at 8.4 GHz and they obtained $\beta = 3 \pm 0.3$. They used 39 remnants with precisely determined angular diameters and flux densities. Another relation for 28 calibrators with angular diameter less than or equal to the beam size was derived ($\beta = 3.6 \pm 0.4$). In this paper, we have chosen 11 calibrators (10 with smaller and 1 with larger angular diameter in comparison to the beam size). These 11 remnants have spectral indices calculated from observations (in range $0 < \alpha < 1$). For 11 remnants with reliable diameters and calculated spectral indices, the relation obtained at 8.4 GHz has the form $\Sigma \propto D^{-3.61 \pm 0.30}$. At 1 GHz, we obtain:

$$\Sigma_{1\text{GHz}} = 2.86^{+0.84}_{-0.65} \times 10^{-16} D^{-4.08 \pm 0.39},$$

(21)

with a 92% fit quality. If we add the loops to the set of 11 M82 calibrators, the following relation is derived:

$$\Sigma_{1\text{GHz}} = 2.11^{+0.65}_{-0.49} \times 10^{-15} D^{-3.12 \pm 0.10},$$

(22)

with a 99% fit quality.

The $\Sigma - D$ diagrams associated with relations (21) and (22) are shown in Fig. 13.

![Fig. 12. The $\Sigma - D$ diagrams at a frequency of 1 GHz. Fifty one M33 calibrators are represented by asterisks and the loops by circles.](image)
From this test we can see that the variation of the quotient $\beta$ is very great and loops statistically do not belong to the group of the initial calibrators. This result is expected, because remnants in M82 are very young and luminous. The diameters of the 11 calibrators are less than 3 pc (2 of them less than 1 pc). All 11 remnants are brighter (and with smaller diameters) than the remnant Cas A ($D = 5$ pc). The remnants in M82 galaxy are expanding in the high density environment (Huang et al. 1994). If the radio loops are very large remnants which are expanding in a very thin environment then they should not be similar to the M82 remnants. Previous test shows this statement. We may conclude that the evolution of the young SNR is faster than that of the old one. The theoretical relation derived by D&S shows this effect. Probably, remnants in M82 are in the first evolutionary, blast wave phase (free-expansion phase) and not in the adiabatic – Sedov phase (see Muxlow et al. 1994).

To sum up, the effect of adding Galactic loops to the M82 set of SNRs is to reduce slope of the relation dramatically. It is clear that properties of the Galactic loops are different from the those of young SNRs in M82. This is to be expected because as SNRs they are relatively old and therefore follow a $\Sigma - D$ relation that is representative of older SNRs. It is also evident that the $\Sigma - D$ relation for M82 is significantly steeper than the relations for M31 and M33. The difference in slope appears to be related to the fact that the SNRs in M82 are, on the average, significantly smaller (and presumably younger) compared to the SNRs in M31 and M33.

6.4. Combining the Galactic and Extragalactic Relations – the "Master" Relations

In this Section we will derive relations for the combination of Galactic and extragalactic SNRs. The first relation is established for 36 C&B Galactic calibrators (without Cas A remnant) along with calibrators from the Large (without 0505-679 SNR) and Small Magellanic Clouds. The calibrators from Magellanic Clouds are adopted from Berkhuijsen (1986). We use 29 calibrators from LMC and 11 from SMC, 30 radio detected calibrators from M31 (see section 6.3.1) and 51 from M33 galaxy (see section 6.3.2).

For 157 remnants from our Galaxy, LMC, SMC, M31 and M33 galaxies, the obtained "master" relation at 1 GHz has the form:

$$\Sigma_{1\text{GHz}} = 8.59^{+5.13}_{-3.21} \times 10^{-18} D^{-2.07 \pm 0.13}, \quad (23)$$

with a 61% fit quality.

If we add the Galactic loops to the set of 157 calibrators, the following relation is derived:

$$\Sigma_{1\text{GHz}} = 8.43^{+4.49}_{-2.93} \times 10^{-18} D^{-2.06 \pm 0.12}, \quad (24)$$

with a 65% fit quality.

The $\Sigma - D$ diagrams associated with relations (23) and (24) defined for 157 C&B, LMC, SMC, M31 and M33 calibrators without 4 main radio loops, and with loops are shown in Fig. 14.
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Fig. 14. The $\Sigma - D$ diagrams at a frequency of 1 GHz. The SNRs are represented by: asterisks (C&B), full dots (LMC & SMC), pluses (M31 & M33). Loops are represented by circles.

Fig. 15. The combined $\Sigma - D$ diagram at a frequency of 1 GHz. The SNRs are represented by: asterisks (C&B), full dots (LMC & SMC), pluses (M31 & M33), crosses (M82). Cas A remnant is drawn in. Loops are represented by circles.

Here it is obvious that more than a half of the remnants belong to M31 and M33 galaxies. Therefore the relation slope decreases to $\beta \approx 2$. Galactic remnants along with remnants from Magellanic Clouds define relation with $\beta = 2.4$ (C&B, Urošević 2000) even 81 remnants from M31 and M33 define relations with $\beta \approx 1.8$ (section 6.3.1 and 6.3.2). Therefore, on the average, we obtain relation with $\beta \approx 2$ (relation 23 and 24).

The second relation is constructed for 157 previous along with 11 M82 galaxy SNRs.

For 168 remnants from our Galaxy, LMC, SMC, M31, M33 and M82 galaxies, the obtained "master" relation at 1 GHz has the form:

$$\Sigma_{1\text{GHz}} = 3.47^{+1.42}_{-1.01} \times 10^{-16} D^{-3.10\pm0.10},$$  \hspace{1cm} (25)

with a 85% fit quality.

If we add the Galactic loops to the set of 168 calibrators, the following relation is derived:

$$\Sigma_{1\text{GHz}} = 2.76^{+1.10}_{-0.79} \times 10^{-16} D^{-3.02\pm0.10},$$  \hspace{1cm} (26)

with a 85% fit quality.

The $\Sigma - D$ diagram associated with relation (26) defined for 172 C&B, LMC, SMC, M31, M33 and M82 calibrators along with loops is shown in Fig. 15.

The previous tests support the SNR origin of the radio loops. With loops added, we get some smaller variation for quotient $\beta$ in comparison to the test of C&B relation. The SNRs from M31 and M33 galaxies reduced the slope to the $\beta \approx 2$ (relations 23 and 24). The SNRs from M82 galaxy dramatically alter quotients of the $\Sigma - D$ relation. This quotients changing is in accordance with our expectations (see Section 6.3.3).
The combined $\Sigma - D$ relation contains two distinct components. For diameters less than 10 pc the relation has a steep slope with $\beta \approx 4.1$ (relation 21) while for larger SNRs the slope is significantly shallower with $\beta \approx 2.1$ (relations 23 and 24). The Galactic loops are clearly a part of the shallower component.

7. DISCUSSION

The previous discussions for particular updated $\Sigma - D$ relations show that the large radio loops fit the $\Sigma - D$ relation and therefore could be consistent with large, evolved supernova remnants. The big loops diameters could not present a problem yet, because the expanded SNR in hot and thin environments could attain huge diameter and still be in the adiabatic phase (McKee and Ostriker 1977). This scenario agrees with the present knowledge on the Local Interstellar Medium (LISM) (Cox and Reynolds 1987; Bochkarev 1987, 1990; Frisch 1995, McKee 1998).

The best $\Sigma - D$ relations for extragalactic SNRs are derived. In the process demonstrated in this paper, beyond any doubt, M31 and M33 $\Sigma - D$ relations are flatter ($\beta < 2$) than those for the Galactic SNRs. This result confirms that selection effects play an important role in data-sets made up of the Galactic SNRs, especially the effect of Malmquist bias.

The most interesting test results are for M33 relation (see section 6.3.2). The fit quality is rising up by $\approx 15\%$. The loops are balancing the relation very well. This balancing is better than in the case of Galactic relations (for Green’s and C&B relations quality fit is rising up by 10% and 5%, for M33 $\approx 10\%$). In Figs. 10, 11 and 12, we can see “empty space” in the $\Sigma - D$ diagrams between M33 calibrators and radio loops. The Gordon et al. (1999) sample probably was not detected low brightness SNRs, because the M33 is sufficiently distant for the sensitive cut-off in $\Sigma \approx 10^{-21}$ W m$^{-2}$Hz$^{-1}$sr$^{-1}$. In future, with more advanced observational technique, we will be able to detect low brightness SNRs (loops-like) and probably $\beta$ will be increased to 2.

In addition, the Galactic $\Sigma - D$ relation is atypically steep relative to the “master” $\Sigma - D$ relation (not including the very young SNRs in M82). Again, this result supports the contention that the Galactic sample of SNRs is subject to severe selection effects and is, therefore, dubious concerning purposes of calculating $\Sigma - D$ relations. These results suggest that even the modestly steep relation of C&B may be too steep, possibly a result of Malmquist bias. These findings favor the interpretation of Green (1991).

The $\Sigma - D$ relation with the biggest number of the reliable calibrators (157) was derived. Calibrators are from our Galaxy, LMC, SMC, M31 and M33 galaxies. Value $\beta \approx 2$ is obtained (see Section 6.4). With the remnants from M82 galaxy, the number of the calibrators is increased to 168. The resulting relation has a steeper slope $\beta \approx 3$ (Section 6.4).

Furthermore, we investigated $\Sigma - D$ relation because we were interested in a clear separation of the two different classes—in a sense of the SNR evolution. These two classes were anticipated by D&S theory. Here, we try to “catch” this difference. Particularly important relation is M82 relation, because that galaxy is “full” of young remnants. Therefore, we extract from Huang et al. (1994) sample, only 11 remnants with the calculated spectral indices and determine $\beta$, which strongly depends on the spectral index (for passing from 8.4GHz to 1GHz relation).

The $\Sigma - D$ relation for M82 is anomalously deep relative to “master” relation and the SNR diameters are uniquely small compared to the other SNRs in the master sample. The measured slope for the smaller SNRs is $4.1 \pm 0.4$ while that of the remaining SNRs in the master relation is $2.1 \pm 0.1$. The change in slope for small diameter remnants is in the same sense and comparable in magnitude to that predicted by the theoretical $\Sigma - D$ relation of D&S. However, the actual values of the slope are different. The predicted values are $\beta \approx 5$ for small diameter remnants and $\beta \approx 3.5$ for the larger remnants, systematically steeper than is actually observed. Some of this discrepancy could be accounted for if the SNRs are expanding in widely differing media. In any case, the M82 result supports the previous observations that the SNRs in M82 are younger and follow a different evolutionary track in the $\Sigma - D$ plane.

The $\Sigma - D$ relation is still necessary for the distance determination to the Galactic SNRs. In our Galaxy, we could identify 225 SNRs (Green 2000). Some 59 SNRs have distances estimated in some different, more precise way. For 166 SNRs only valid is $\Sigma - D$ relation. Therefore, we must tend to determine better $\Sigma - D$ dependence and to overcome many problems connected with this relation. The SNRs distance determination is very important topic, because we should know Galaxy SNR distribution. It is necessary for many astrophysical topics (e.g. energetics of the interstellar medium dynamics and heating), tracers of the interstellar medium (interstellar clouds and magnetic field), star formation (matter compression, etc). Another one is, that the SNRs are the most active sources of the cosmic rays. If we know the distribution of the SNRs in our Galaxy, we shall know with higher degree the distribution of the Galactic cosmic rays. The radio loops are probably the most important sources of cosmic rays in the local interstellar environment, and they have a big influence on the cosmic rays reaching the Earth surface.

We think that the $\Sigma - D$ dependence exists, in spite of severe selection effects, better for the extragalactic than for the Galactic set of SNRs. The relation defined with Green’s (1991) calibrators with the radio loops has a good fit characteristics and could define a reasonable $\Sigma - D$ dependence.

If we compare tests in this paper, we deduce: (1) if in initial relation $\beta < 2$, after loops addition $\beta$ will increase and (2) if in initial relation $\beta > 2$, after loops addition $\beta$ will decrease. These test results are supported value $\beta \approx 2$ (e.g. Mills 1983; Mills et al. 1984; C&B; Urošević 2000). Theoretical $\Sigma - D$ relation does not predict $\beta = 2$ (e.g. from Shklovsky
model $\beta = 6$; from D&S model $2.75 \leq \beta \leq 3.5$). If we suppose spherically expanding SNR with constant luminosity $- L_{\nu}$ (or independent of the SNR diameter), the relation has the form:

$$\Sigma_{\nu} \propto L_{\nu}D^{-2}. \quad (27)$$

Theoretical $\Sigma - D$ relation in D&S form does not exist, if $\beta = 2$ in the empirical relation. Theoretical relation gets the "trivial" form (27). In future, with detection of greater number of the low brightness Galactic and especially extragalactic SNRs, theoretical relations will be much better interpreted. These results could explain whether or not theoretical $\Sigma - D$ relation exist.

8. CONCLUSION

(i) The addition of the Galactic loops to the relations extends the range of parameters to larger diameters. The absence of any significant change in the relations is consistent with the loops being large SNRs that represent more advanced stages of evolution. However, the small number of loops used and the lack of a clear understanding of the selection effects that affect them preclude any strong conclusions on this matter.

(ii) The addition of the Galactic radio loops moderates severe effects of Malmquist bias.

(iii) The addition of the Galactic loops pushes both the Galactic and extragalactic $\Sigma - D$ relations to a slope, $\beta = 2$. This suggests that SNRs with diameters greater than 10 pc form a $\Sigma - D$ relation in which $\Sigma \propto D^{-2}$. Such a relation is consistent with SNR luminosity that is statistically independent of $D$ (the trivial relation). This result is consistent with a scenario in which the SNRs evolve in widely differing media. Alternatives to the trivial relation will be explored in a future paper.

(iv) The "master" $\Sigma - D$ relations are derived. The SNRs are combined to the set of 172 calibrators (with 11 M82 SNRs and 4 radio loops). The "master" relation for 157 SNRs with diameters greater than 10 pc form the relation with $\beta = 2.07$.

(v) The Galactic $\Sigma - D$ relation is atypically steep relative to the "master" $\Sigma - D$ relation (not including the very young SNRs in M82). This result supports the contention that the Galactic sample of SNRs is subject to severe selection effect – Malmquist bias.

(vi) The change in slope for small diameter remnants from M82 is in the same sense and comparable in magnitude to that predicted by the theoretical $\Sigma - D$ relation of D&S.

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REFERENCES


Овај чланак приказује преглед емпиријских Галактичких и вангалаутичких $\Sigma - D$ релација (релација између површинског сјаја $\Sigma$ и дијаметра $D$) за остаци супернових звезда (SNRs), уз проверу повезаности главних Галактичких радио-петља (Петље I, II, III, IV) са овим релацијама.

Резултати добијени у овом раду, суперншу, на још један начин, да су радио-петље остаци супернових звезда. Најновије релације имају нагиб $\beta \approx 2$ у лог-лог скали. Најбоље $\Sigma - D$ релације за галаксије M31 и M33 имају ближе нагиб ($\beta < 2$) у односу на нашу Галаксију. Овај резултат потврђује да селекциони ефекти играју важну улогу у Галактичком узорку података. Релација изведена за 157 поуздано одређених калибритера има нагиб $\beta = 2.07 \pm 0.13$. 