MEASUREMENTS OF RADIO FREQUENT CAVITY VOLTAGES BY X-RAY SPECTRUM MEASUREMENTS

by

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This paper deals with X-ray spectrum measurement as a method for the measurement of radio frequent cavity voltage and the theory of X-ray spectrum calculation. Experimental results at 72 MHz for three different values of the radio frequent power of ACCEL K250 superconducting cyclotron are being presented.

Key words: radio frequent system, cavity, X-ray spectrum

INTRODUCTION

The peak voltage reached within an radio frequent (RF) cavity is one of the most important parameters of the cavity. Such voltage can be difficult to measure, since the introduction of probes generally changes electromagnetic field distributions and alters the voltage being measured. Electrons, which have been accelerated by the dee-voltage in a gap between the dee (dee is a part of a cyclotron; *i. e*., high voltage electrode) and a part of the liner (grounded electrode), may generate X-rays when they hit the opposite electrode. On impact, some of these electrons, accelerated to almost amplitude energy, generate X-rays with energies close to their amplitude energy. These X-rays form the upper limit of the X-ray spectrum. This upper limit in the X-ray spectrum gives the amplitude value of the dee voltage. If we pick up the spectrum with good statistics, the precision tends to be satisfactory. The X-rays are unaffected by local electromagnetic fields and can easily escape through cavity walls and be detected outside. The problem is to find the right spot in the accelerating system for the X-ray detector to be placed. Due to the high level of the magnetic field, all horizontal gaps are excluded because the radius of the curvature of the electrons is very small (bellow 1 mm). If the complex spatial magnetic field distribution does not cause an essential change in the trajectories of the electrons, no X-rays can be observed in the median plane. Thus, the appropriate spot for the X-ray detector to be placed is the position in which the magnetic field is perpendicular to the dee surface and the opposite surface of the liner [1]. In this case, it may be considered that the electrons are moving perpendicular to the electrodes and along the magnetic field lines. This configuration enables us to assume a simple model for studying the motion of electrons between two parallel electrodes without a magnetic field. Four penetrations in the iron joke in the valleys are available for this purpose. The diameter of each drilled hollow is 14 mm and they are placed at 300 mm from the center of the ACCEL K250 machine [2].

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THEORY

Energy gain of electrons in the RF field between liner and dee

The following simplified model of electron motion [1] has been considered:

- that the electron emitted by a microprotrusion starts with zero energy,
- that it is moving along the magnetic field lines perpendicular to the electrode surface, and
- that the RF electric field is homogenous.

Under these assumptions, we may draw the scheme in fig. 1 and write the equation of motion in the form of

$$\frac{\mathrm{d}p}{\mathrm{d}t} \quad eE(t) \tag{1}$$

where p is the momentum of the electron, e its charge and $E(t)$ the electric field.

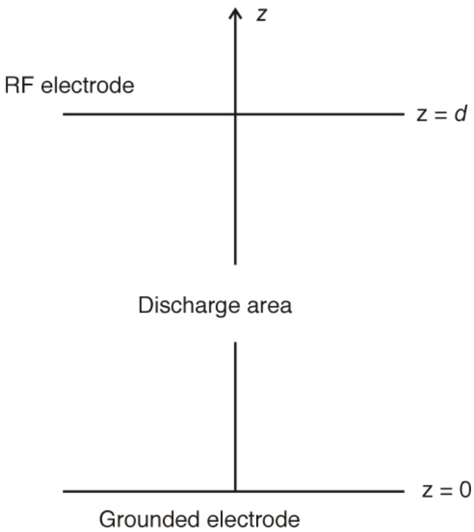

Figure 1. Schema of electron motion. The marks in this figure are explained in the text

The electric field varies with time as the applied voltage $V(t)$

$$V(t) \quad V_m \cos(\omega t \quad \phi_0) \tag{2}$$

$$E(t) \quad \frac{V(t)}{d} \tag{3}$$

where ω is the angular RF frequency $\omega \quad 2\pi f_{\mathrm{RF}}$, and ϕ_0 is the initial phase.

Integrating eq. (1) with the assumption that the momentum at the starting moment $t = 0$ is zero, we get:

$$p(t) \quad e\frac{V_m}{\omega d} \quad [\sin(\omega t \quad \phi_0) \quad \sin\phi_0] \tag{4}$$

Knowing the momentum, we find the only z component of the velocity

$$\frac{\mathrm{d}z}{\mathrm{d}t} \quad \frac{cp(t)}{\sqrt{p^2(t) \quad (m_0 c)^2}} \tag{5}$$

where m_0 is the rest-mass of an electron and c is the velocity of light.

At the moment $t = 0$, the electron is in the position $z = 0$ and, after a time of flight t, reaches the RF high-voltage plate spaced at a distance d (see fig. 1)

$$d \quad \frac{1}{\omega} \int_0^{\phi} \frac{cp(\phi)}{\sqrt{p^2(\phi) \quad (m_0 c)^2}} \mathrm{d}\phi \tag{6}$$

where $\phi = \omega t$ and

$$p(\phi) \quad e\frac{V_m}{\omega d}[\sin(\phi \quad \phi_0) \quad \sin\phi_0] \tag{7}$$

Having found, by using numerical integration (6), phase ϕ_k at which the electron passed the distance d for a given initial phase ϕ_0, we can calculate the gained energy which, expressed in units of electron-volts, will be

$$E_k(\phi_k) \quad \frac{1}{e} \quad \sqrt{p^2(\phi_k)c^2 \quad E_0^2} \quad E_0 \tag{8}$$

where $E_0 = m_0 c^2$ is the rest energy.

The limits of the initial phase are determined by the driving force and momentum gain. The lower limit comes from the sign of the driving force at the moment $t = 0$ in eq. (1). It has to be directed in the positive z-direction hence, $\phi_{0,\min} =$ = 90 . If ϕ_0 90 , the electron has no acceleration. The upper limit is determined by zero momentum gain and depends on the time of flight (expressed in the phase of arrival of the ϕ_k). The position of the electron versus phase ϕ for the two values of the initial phase of ϕ_0 is shown in fig. 2. The upper limit of the initial phase ϕ_0 is approximately 215 (when the amplitude dee voltage amounts to V_m =80 kV, d = 174 mm and when frequency f_{RF} = 72 MHz).

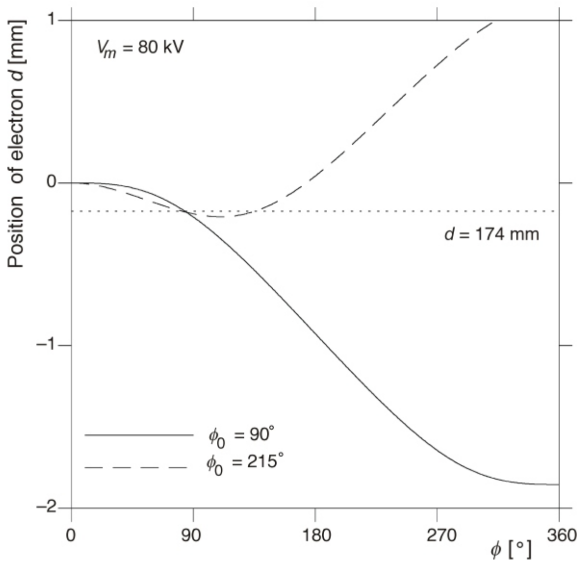

Figure 2. Position ef electron versus phase ϕ for the two values of the initial phase ϕ_0

X-ray spectra calculations

Ideally, the calculation of an X-ray spectrum from a RF cavity should take into account the angular dependence of the X-ray photons relative to their generating electrons. But, because it adds great complexity to the calculation and because it is, often, not quite clear what the distribution of the angles of the electrons themselves are, the present calculation is restricted to the integrated-over-angle spectra.

The integrated-over-angle X-ray spectrum as a function of photon energy E_{ph} for a fixed electron energy E_{el} can be written as [3]:

$$\frac{\mathrm{d}Y}{\mathrm{d}E_{ph}(E_{el}, E_{ph})} = \frac{N_A}{A}\int_0^{x_k} \frac{\mathrm{d}\sigma}{\mathrm{d}E_{ph}[E_{el}(x), E_{ph}]}\mathrm{d}x \quad (9)$$

where $\mathrm{d}Y/\mathrm{d}E_{ph}$ is the number of X-ray photons per unit energy interval per electron, N_A is Avogadro's number, A is the atomic weight of the material in which the electron slows down and stops, $\mathrm{d}\sigma/\mathrm{d}E_{ph}$ is the inegrated-over-angle bremsstrahlung cross-section, $E(x)$ is the electron energy after the electron has traveled a distance x (measured in g/cm^2) in the material, x_k is the thickness of the material at which the electron energy has reduced from E_{el} to E_{ph}.

Electron energy after penetrating a distance x through the material is given by [4]

$$E(x) = [E_{el}^{(1-D)} - (1-d)cx]^{\frac{1}{1-D}} \quad (10)$$

where $c = 5.85 \times 10^4$ (in units consistent with E_{el} being in keV) and $D = -0.664$.

The bremsstrahlung cross-section $\mathrm{d}\sigma/\mathrm{d}E_{ph}$ has been tabulated by Pratt *et al.* in [5], and the tabulated numbers may be represented by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_{ph}} = \frac{a\left(1 - b\dfrac{E_{ph}}{E_{el}}\right)}{E_{ph}\dfrac{\beta^2}{Z^2}} \quad (11)$$

where $a = 10$ mb $= 10 \times 10^{-27}$ cm^2, $b = 0.5$, β is the velocity of the electron as a function of the velocity of light.

The thickness of the material x_k at which the electron energy has reduced from E_{el} to E_{ph} is given by [4]

$$x_k(E_{el}, E_{ph}) = \frac{E_{el}^{(1-D)} - E_{ph}^{(1-D)}}{c(1-D)} \quad (12)$$

Since in an RF cavity the voltages are oscillating sinusoidally, the X-ray spectrum above must be integrated over a sinusoidally varying electron energy, giving

$$\frac{\mathrm{d}Y}{\mathrm{d}E_{ph}(E_{0,el}, E_{ph})} = \frac{2}{\pi}\int \frac{\mathrm{d}Y}{\mathrm{d}E_{ph}[E_{0,el}\sin\theta(x), E_{ph}]}\mathrm{d}\theta \quad (13)$$

where the integral extends from $\sin^{-1}(E_{ph}/E_{0,el})$ to $\pi/2$ and $E_{0,el}$ is the peak value of the oscillating voltage.

X-ray spectra can also be calculated by using a very simple and convenient approximate expression given by Kramers [6]

$$\frac{\mathrm{d}Y_{Kramers}}{\mathrm{d}E_{ph}} = \frac{Z \cdot 10^{-3}}{511}\,\frac{E_{el} - E_{ph}}{E_{ph}} \quad (14)$$

where Z is the atomic number of the material.

Only the end point energy of the X-ray spectrum gives information of the dee voltage. Also, the X-ray detector has a limited count rate. Due to these reasons, the X-ray detector must be shielded from the low energy photons. This can be done by inserting thin layers of metal between the dee and the detector.

The mass attenuation coefficient μ/ρ is the basic quantity used in calculations of the penetration and energy deposition by photons (X-ray, γ-ray, bremsstrahlung) in biological, shielding and other materials. These coefficients are defined and treated in [6].

A narrow beam of mono energetic photons with the intensity of I_0, penetrating a layer of material with mass thickness x and density ρ, emerges with intensity I given by the exponential attenuation law

$$I = I_0 e^{-\frac{\mu}{\rho}x} \quad (15)$$

where μ/ρ is the mass attenuation coefficient which depends on photon energy and characteristics of the shielding material.

The values of the mass attenuation coefficient for different materials and photon energies can be obtained from [7].

If the X-ray spectra calculated by eqs. (13) or (14) is multiplied by $\exp -[\mu/\rho(E_{ph})]x$ where μ/ρ [cm^2/g] is the mass attenuation coefficient for photon absorption for 2 mm thickness of aluminum and copper [6, 7], the resulting spectrum is presented in fig. 3. This is the spectrum arriving at the detector entrance.

It can be seen that, while the two spectra (Duke's and Kramers' calculations) are similar at

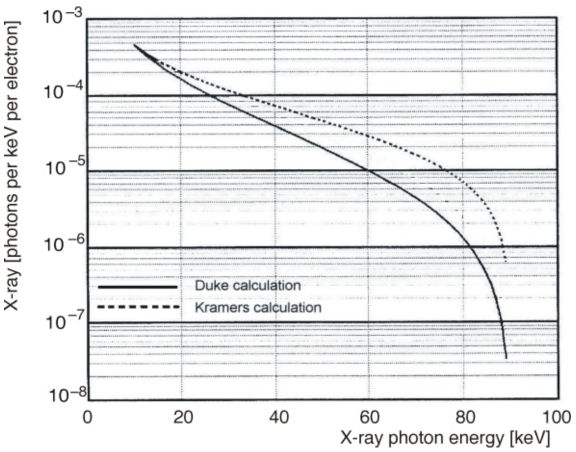

Figure 3. Number of X-ray photons per unit energy interval per electron

low energies, they increasingly diverge as the photon energy increases, up to an order of magnitude.

Figure 3 demonstrates that the X-ray yield is strongly decreasing with photon energy.

EXPERIMENTAL RESULTS

The X-ray spectrum was measured using a SPEAR X-ray detector with a 5 5 5 mm^3 CdZnTe crystal operating at room temperature. The measuring system has been developed by ACCEL Instruments [2]. The detector including a pre amplifier is mounted in a cylindrical housing (ϕ 12.8 mm, L = 90 mm). Its small size allows for the detector to be mounted inside the penetration through the joke (ϕ 14 mm) [2] behind and close to the liner (to maximize detector intensity) which makes detector calibration easier. Detector resolution for the model used has been approximated with a Gaussian. The exact FWHM (Full Width Half Maximum) was established using the 26.340 and 59.536 keV lines from a small ^{241}Am source [8]. By moving the detector closer to or further away from the liner, the intensity (detector count rate) can be optimized.

Before measurements, the energy calibration of the detector must be done. The detector is taken from its normal position in the pole cap and positioned in front of a ^{241}Am radiation source emitting photons of 26.340 and 59.536 keV. With the shaping parameters and amplification factor set, the spectrum is measured.

The second step is determining the FWHM of the spectrum at the 59.536 keV peak.

The RF cavity was excited at three different RF powers: of 120 kW, 130 kW, and 135 kW. The RF frequency was kept constant at 72 MHz. The measured spectra in the case of 135 RF power and dee No. 3 is illustrated in fig. 4 by a full line. The open circle represents the calculated X-ray spectra.

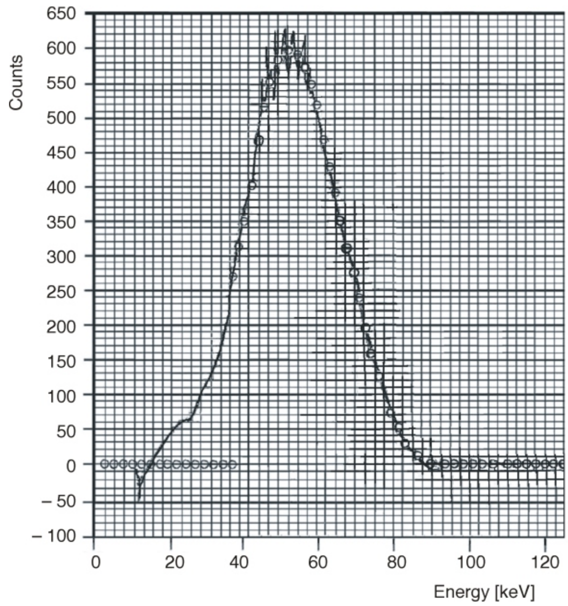

Figure 4. Comparison of measured and calculated X-ray spectra for dee No. 3 in the case of 135 kW RF power for the ACCEL K250 superconducting cyclotron. The RF frequency is 72 MHz

The dee fitted voltages for all 4 dees and for the three different RF powers are presented in fig. 5.

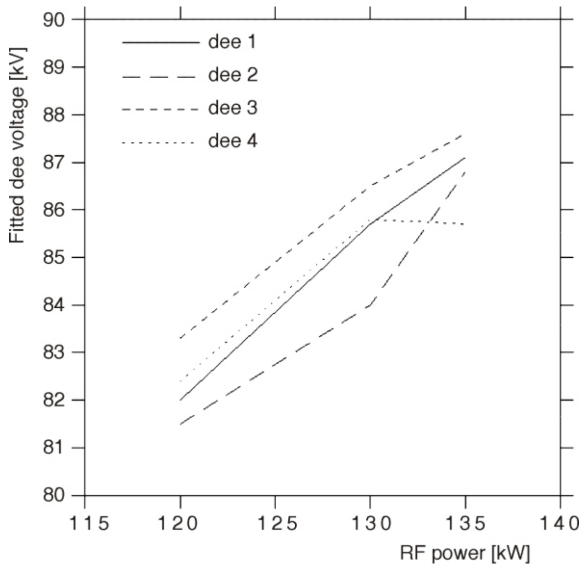

Figure 5. RF voltage amplitude measuring results for four dees for the tree different values of the RF power

CONCLUSIONS

The voltage of the RF cavity of the ACCEL K250 superconducting cyclotron measured by X-ray spectrum is accurate and reliable because it is a non-invasive technique (no variation of the RF cavity resonant property was induced by the measuring probe). Moreover, X-rays generated by the RF electric field can be detected at a suitable position. According to error analysis, the RF voltage measurement error of the X-ray spectrum method is around 1 kV.

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Драган ТОПРЕК

МЕРЕЊЕ НАПОНА РАДИО-ФРЕКВЕНТНОГ РЕЗОНАТОРА МЕТОДОМ МЕРЕЊА СПЕКТРА X-ЗРАКА

У овом раду је описана метода мерења напона дуаната радио-фреквентног система мерењем спектра X-зрака. Теорија прорачуна спектра X-зрака такође је укључена у овом раду. Приказани су експериментални резултати у случају фреквенције резонатора од 72 MHz и у случају три различите вредности снаге радио-фреквентног система ACCEL K250 суперпроводног циклотрона.

Кључне речи: радио-фреквентни систем, резонатор, спектар X-зрака