TOTAL NUMBER ALBEDO AND AVERAGE COSINE OF THE POLAR ANGLE OF LOW-ENERGY PHOTONS REFLECTED FROM WATER

by

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The total number albedo and average cosine of the polar angle for water and initial photon energy range from 20 keV to 100 keV are presented in this paper. A water shield in the form of a thick, homogenous plate and perpendicular incidence of the monoenergetic photon beam are assumed. The results were obtained through Monte Carlo simulations of photon reflection by means of the MCNP computer code. Calculated values for the total number albedo were compared with data previously published and good agreement was confirmed. The dependence of the average cosine of the polar angle on energy is studied in detail. It has been found that the total average cosine of the polar angle has values in the narrow interval of 0.66-0.67, approximately corresponding to the reflection angle of 48°, and that it does not depend on the initial photon energy.

Key words: photon reflection, total number albedo, average cosine of polar angle, total average cosine of polar angle, Monte Carlo simulation, MCNP code, water

INTRODUCTION

Diverse modern applications of radiation, particularly in medical diagnostics and therapy, refer to thorough investigations of radiation interactions with matter (nuclear cross-sections, particle transport, and reflection) for photons in the energy domain bellow 100 keV. Especially in diagnostic radiology, scattered radiation from the patient's body is the main source of the medical team's exposure [1]. Thus, knowing the angular and energy distribution of the reflected radiation in the low-energy domain is of utmost practical interest. However, physical quantities that characterize photon reflection (differential and integral albedo coefficients) are not well known in this energy range, or at least, they have not been systhematically covered in existing literature [2, 3].

In the first part of this paper, values for the total number albedo obtained by Monte Carlo simulations of low-energy photon reflection from water, for initial photon energies from 20 keV to 100 keV in equal energy steps of 10 keV, are given. The results obtained by simulation, collected in ten equally wide energy intervals and nine equally wide polar angle intervals, additionally elaborated, were compared with referent literature data [2, 4]. In the second part of the paper, for initial photon energies of 40 keV, 60 keV, and 100 keV, energy distribution of back-scattered photons from water described by the average cosine of the reflected photons' polar angle and the total average cosine, integral of the previously mentioned quantity over energy, are presented and analysed. In all simulations, perpendicular incidence of penetrating photons is assumed.

Results presented here were obtained by the MCNP code [5]. They ilustrate only a part of an extensive research carried out in the last couple of years in the Vinča Institute of Nuclear Sciences in order to investigate low-energy photon reflection by analytical methods and numerical simulations. Some of the results have already been published [6, 7], while a more comprehensive review will be included into the Ph. D. thesis of one of the authors of this paper [8].

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TOTAL NUMBER ALBEDO

Functions which characterize photon reflection are defined under the assumption that a wide beam of directed monoenergetic radiation, described by the initial energy E_0 and polar angle θ_0, hits the boundary surface of the material and photons are reflected with different energies E in different directions described by the polar angle θ and azymuthal angle φ [6, 9]. Albedo coefficients defined as described are equivalent to the albedo coefficients determined by the narrow beam of the directed incident radiation.

The determination of the total number albedo is based on the Monte Carlo calculation of the double differential number albedo coefficient $a(E_0, \theta_0; E, \theta, \varphi)$. Actually, simulations provide the difference number albedo $a_N^{ji}(E_0, \theta_0)$ – double integral of the previously mentioned coefficient

$$a_N^{ji}(E_0,\theta_0) = \int_0^{2\pi} \mathrm{d}\varphi \int_{\Delta E_j} \mathrm{d}E \int_{\Delta\theta_i} a(E_0,\theta_0;E,\theta,\varphi)\sin\theta \mathrm{d}\theta \qquad (1)$$

In eq. (1), indexes j and i refer to the energy and angular intervals used to collect reflected photons. Based on $a_N^{ji}(E_0, \theta_0)$ and the total number albedo definition [8, 9]

$$a_N(E_0,\theta_0) = \int_0^{2\pi} \mathrm{d}\varphi \int_0^{E_0} \mathrm{d}E \int_0^{\pi/2} a(E_0,\theta_0;E,\theta,\varphi)\sin\theta \mathrm{d}\theta \qquad (2)$$

which represents total probability for photon reflection from the target material, the following equation can be written

$$a_N(E_0,\theta_0) = \sum_{j=1}^{10} \sum_{i=1}^{9} a_N^{ji}(E_0,\theta_0) \qquad (3)$$

The total number albedo $a_N(E_0, \theta_0)$ enables a semi-analytical reconstruction of the angular and energy distributions of the reflected photons from typical shielding materials by means of the procedure described in ref. [8]. Coefficient $a_N(E_0, \theta_0)$ depends of the initial photon energy E_0 and incident angle θ_0 only parametrically. As the results discussed here refer only to $\theta_0 = 0°$, parameter θ_0 is ommited from the list of arguments by the convention that $a_N(E_0) \equiv a_N(E_0, \theta_0 = 0°)$.

Water shield results

Values of the total number albedo calculated from the results of Monte Carlo simulations of photon reflection from water are presented in the second column of tab. 1. Energies of incident photons are chosen in the range of 20 keV to 100 keV. Photons reflected from the half-space water shield are collected in ten equally wide energy intervals (width of one energy group is $E_0/10$) and in nine equal intervals of the polar angle θ, each of them wide 10°. More detailed information on the numerical simulations performed, as well as basic information on the MCNP code, are given in refs. [5, 6]. The results published in the Mashkovich's Manuel [2], based on Berger's and Raso's paper [4], are shown in the third column of tab.1. Very good agreement between the results calculated from the MCNP simulation and literature data is evident throughout the entire energy range of up to 100 keV. Thus, regarding photon reflection from water, discrepancies of our MCNP-based results with the referent ones are less than 5% for the initial photon energy of 20 keV and around 2% for the initial energy of 100 keV. In the midle of the energy interval, the most important one for medical applications, the relative discrepancy of results for the initial photon energy of 50 keV is only 1%.

Table 1. Total number albedo for water

E_0 [keV]	Total number albedo	
	MCNP	Referent results [2, 4]
20	0.0478	0.050
30	0.128	
40	0.209	
50	0.273	0.276
60	0.319	
70	0.351	
80	0.373	
90	0.389	
100	0.400	0.391

AVERAGE COSINE OF THE POLAR ANGLE

The average cosine of the polar angle of reflected photons $\overline{\cos\theta}(E_0, \theta_0; E)$ defined by the differential spectral albedo $a(E_0, \theta_0; E, \theta)$ is given by

$$\overline{\cos\theta}(E_0,\theta_0;E) = \frac{\int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi/2} \cos\theta\, a(E_0,\theta_0;E,\theta,\varphi)\sin\theta \mathrm{d}\theta}{\int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi/2} a(E_0,\theta_0;E,\theta,\varphi)\sin\theta \mathrm{d}\theta} \qquad (4)$$

and can be calculated from the values of the difference number albedo $a_N^{ji}(E_0, \theta_0)$ determined by Monte Carlo simulations of photon reflection

$$\overline{\cos\theta}(E_0,\theta_0;E) = \frac{\sum_{i=1}^{9} \cos\bar{\theta}_i a_N^{ji}(E_0,\theta_0)}{\sum_{i=1}^{9} a_N^{ji}(E_0,\theta_0)} = \frac{\sum_{i=1}^{9} \cos\bar{\theta}_i a_N^{ji}(E_0,\theta_0)}{a_N(E_0,\theta_0;E)} \qquad (5)$$

Here, $\cos \overline{\theta}_i$ denotes the cosine of the value of the polar angle at the middle of the i-th angular interval. It is understood that energy E in the argument of function $\overline{\cos \theta}(E_0, \theta_0; E)$ belongs to the j-th energy interval. However, denominators of eqs. (4) and (5) actually present the spectral photon albedo $a_N(E_0, \theta_0; E)$. The quantity $\overline{\cos \theta}(E_0, \theta_0; E)$ denotes energy distribution of the average cosine of the polar angle for reflected photons initially directed towards the target surface under angle θ_0 and with the initial energy of E_0. As for the total number albedo, when the photon incidence happens at the right angle ($\theta_0 = 0°$), parameter θ_0 is ommited from the function argument and the average cosine of the polar angle can be denoted as $\overline{\cos \theta}(E_0; E)$.

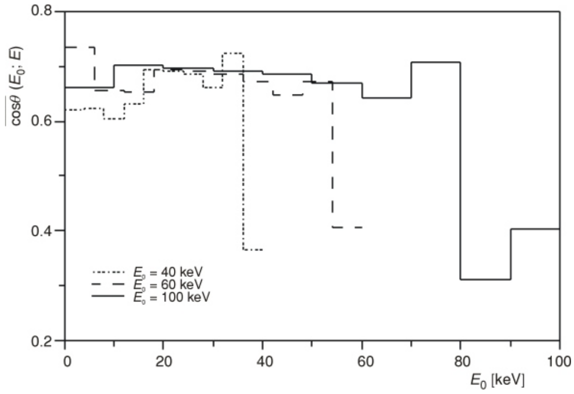

Figure 1. Energy dependence of the average cosine of the polar angle for photons reflected from water

Water shield results

The values of the average cosine of the polar angle for photons reflected from water, presented in tab. 2 have been obtained from the results of numerical simulations performed by the MCNP code. In this analysis, a set of simulation results have been used for initial energies of the incident photons of 40 keV, 60 keV, and 100 keV. Figure 1 shows the graphical representation of the data from tab. 2. Histograms of the function $\overline{\cos \theta}(E_0; E)$ for all three initial energies have a similar shape: from the left side of the peak that appears at energies slightly bellow the initial ones there is a wide energy range with approximately flat distribution and, after that, a second peak in the distribution can be observed for very low energies. More detailed explanation of the rather common distribution shape can be given by simple analysis of the transport process that causes the reflection of the photons from the target material.

Photons that hit the target surface at a right angle can be reflected after (a) only one scattering backward (scattering at an angle greather than 90°), (b) a small number of scatterings – several scatterings with a small angle deflection forward related to the initial photon direction, and (c) a larger number of scatterings.

If the photons are reflected after only one scattering backward, that will be due to scattering at an angle of about 140° on average, independently of the initial photon energy in the energy range up to 100 keV [8]. The corresponding polar angle for such photon reflection is $\theta = 40°$ with $\cos \theta = 0.766$. This points to the height of the first peak observed in the energy distribution of the function $\overline{\cos \theta}(E_0; E)$. Small differences for different initial photon energies come from the contribution of the photons reflected after a small number of scatterings. For the scattering at 140°, photon energy is reduced for an amount dependent on the initial energy and will be reflected by, approximately, the following energies: for $E_0 = 40$ keV with $E = 35$ keV, for $E_0 = 60$ keV with $E = 50$ keV, and for $E_0 = 100$ keV with $E = 75$ keV [8]. Peaks on the histograms appear exactly at energy intervals which cover the energies of the reflected photons.

Table 2. Average cosine of the polar angle for photons reflected from water

$E_0 = 40$ keV		$E_0 = 60$ keV		$E_0 = 100$ keV	
E [keV]	$\overline{\cos\theta}(E_0; E)$	E [keV]	$\overline{\cos\theta}(E_0; E)$	E [keV]	$\overline{\cos\theta}(E_0; E)$
2.0	0.622	3.0	0.735	5.0	0.661
6.0	0.623	9.0	0.656	15.0	0.704
10.0	0.604	15.0	0.654	25.0	0.697
14.0	0.631	21.0	0.695	35.0	0.691
18.0	0.693	27.0	0.692	45.0	0.685
22.0	0.691	33.0	0.685	55.0	0.669
26.0	0.686	39.0	0.672	65.0	0.644
30.0	0.660	45.0	0.647	75.0	0.707
34.0	0.723	51.0	0.672	85.0	0.311
38.0	0.365	57.0	0.407	95.0	0.402

If the photons are reflected through two or three scatterings at sharp angles, they can appear with outcoming polar angles θ that are greather than 40°, up to the value of 90°. Then the values of $\overline{\cos \theta}(E_0; E)$ are lower and on the histograms minimums of about 0.37 to 0.41 are observed. Outcoming energies of such photons are usually higher than the energies of the photons reflected by one backscatter [8]; thus, the minimum in the distribution appears closer to the initial photon energy.

The majority of reflected photons leave the target material after more subsequent scatterings in all directions. Due to this, angular photon distribution becomes practically isotropic, resulting in the constant value of the function $\overline{\cos \theta}(E_0; E)$. This effect is more explicit for higher initial photon energies, when reflection occurs after a considerably greater number of scatterings.

At the lowest energies, the second peak can be observed, this being more explicit for the initial energy of 60 keV and not so obvious for that of 100 keV. For low initial energies (case 40 keV), the peak is absent due to the domination of photoelectric absorption over the photon scattering process.

Total average cosine of the polar angle

By averaging function $\overline{\cos\theta}(E_0; E)$ over energy, the total average cosine of the polar angle $\overline{\cos\theta}(E_0)$ is obtained

$$\overline{\cos\theta}(E_0) = \frac{\sum_{j=1}^{10}\sum_{i=1}^{9}\cos\bar{\theta}_i a_N^{ji}(E_0,\theta_0)}{\sum_{j=1}^{10}\sum_{i=1}^{9} a_N^{ji}(E_0,\theta_0)} = \frac{\sum_{j=1}^{10}\sum_{i=1}^{9}\cos\bar{\theta}_i a_N^{ji}(E_0,\theta_0)}{a_N(E_0,\theta_0)} \qquad (6)$$

For water, this coefficient has values inside the narrow interval 0.66-0.67 for an initial energy range of up to 100 keV. These values correspond to the reflection angle $\theta \approx 48°$. It has been confirmed by further calculations that, for aluminum and iron in the same range of initial photon energies, maximal discrepancies of the photon reflection angle from the value given above are 1-2% [8].

CONCLUSION

The main contribution of this paper is the systematic analysis of low-energy photon reflection from the water shield, as well as a good agreement between the results of the Monte Carlo MCNP code and the referent literature data. Values obtained for total number albedo coefficients are reliable enough to be used as baseline data for determining angular and energy distributions of reflected photons. It has also been demonstrated that photons of initial energies of up to 100 keV which hit the water target at a right angle are reflected at approximatelly 48°, independently of initial photon energies. A proper radiation protection strategy for medical teams performing radiological diagnostics can be established on the basis of this.

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ТОТАЛАН БРОЈНИ АЛБЕДО И СРЕДЊИ КОСИНУС ПОЛАРНОГ УГЛА НИСКОЕНЕРГЕТСКИХ ФОТОНА РЕФЛЕКТОВАНИХ ОД ВОДЕ

У раду су приказани тоталан бројни албедо и средњи косинус поларног угла за воду и примарне фотоне у енергетској области од 20 keV до 100 keV. Претпостављено је да је водени штит у виду дебеле хомогене плоче и да сноп моноенергетских фотона упада под правим углом. Резултати су добијени Монте Карло симулацијама фотонске рефлексије помоћу МЦНП рачунарског програма. Израчунате вредности за тоталан бројни албедо упоређене су са најраније објављеним подацима и уочено је њихово добро слагање. Подробно је проучена енергетска зависност средњег косинуса поларног угла фотона. Нађено је да тоталан средњи косинус поларног угла има вредности у уском интервалу од 0.66 до 0.67, чему приближно одговара угао рефлексије од 48°, који не зависи од почетне енергије фотона.

Кључне речи: рефлексија фотона, тоталан бројни албедо, средњи косинус поларног угла, тоталан средњи косинус поларног угла, Монте Карло симулација, МЦНП програм, вода