OPTIMIZATION OF THE SELF-SUFFICIENT THORIUM FUEL CYCLE FOR CANDU POWER REACTORS

by

Boris R. BERGELSON\textsuperscript{1}, Alexander S. GERASIMOV\textsuperscript{1}, and Georgy V. TIKHOMIROV\textsuperscript{2}

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The results of optimization calculations for CANDU reactors operating in the thorium cycle are presented in this paper. Calculations were performed to validate the feasibility of operating a heavy-water thermal neutron power reactor in a self-sufficient thorium cycle. Two modes of operation were considered in the paper: the mode of preliminary accumulation of $^{233}$U in the reactor itself and the mode of operation in a self-sufficient cycle. For the mode of accumulation of $^{233}$U, it was assumed that enriched uranium or plutonium was used as additional fissile material to provide neutrons for $^{233}$U production. In the self-sufficient mode of operation, the mass and isotopic composition of heavy nuclei unloaded from the reactor should provide (after the removal of fission products) the value of the multiplication factor of the cell in the following cycle $K > 1$. Additionally, the task was to determine the geometry and composition of the cell for an acceptable burn up of $^{233}$U. The results obtained demonstrate that the realization of a self-sufficient thorium mode for a CANDU reactor is possible without using new technologies. The main features of the reactor ensuring a self-sufficient mode of operation are a good neutron balance and moving of fuel through the active core.

Key words: thorium cycle, CANDU reactor, self-sufficient mode

INTRODUCTION

The absence of isotope $^{233}$U in nature does not exclude the possibility of its use as a nuclear fuel. $^{233}$U is a product of radioactive decay of $^{233}$Pa which is formed by means of neutron capture by $^{232}$Th, with accompanying $\beta$-decay. Thereby, thorium, existing in nature in a single stable isotope $^{232}$Th, can be transformed using nuclear reactions into $^{233}$U, i.e., into the fuel of power reactors. Reserves of thorium in the world exceed those of uranium by several times. The costs of thorium mining are much lesser than those of uranium. This is due to approximately 100 times lower radiation danger in the process of mining thorium in comparison with that of mining uranium. The possibility of commercial production of $^{233}$U in nuclear reactors was considered numerous times in publications in connection with the thorium-uranium fuel cycle. However, even a partial substitution of an uranium-plutonium fuel cycle by a thorium-uranium cycle is connected with significant difficulties. One of the difficulties is related to the necessity of changing the process itself. However, there are also physical problems: the half-life of the decay of $^{233}$Pa in $^{233}$U, which is 27 days, greatly exceeds the analogous half-life of the decay of $^{239}$Np in $^{239}$Pu in the uranium-plutonium fuel cycle, while the cross-section of the capture of thermal neutrons in $^{232}$Th is 2.8 times higher than the analogous cross-section of $^{238}$U. The said disadvantages of the thorium fuel cycle were, seemingly, a reason that over the past years, thorium was considered solely as a raw material for feeding nuclear reactors operating in the uranium-plutonium fuel cycle [1]. The mixed mode allows us, more or less,
save natural uranium during the production of electricity. The possibility of operating a power reactor in a thorium-uranium fuel cycle without feed by fissile materials was considered in preceding publications, for instance in [1, 2]. However, this possibility was only declared, without a demonstration of concrete ways to achieve it.

The thorium-uranium fuel cycle, nevertheless, also has certain advantages. In particular, the number of secondary neutrons in fission of $^{233}$U by thermal neutrons is higher than for any other uranium isotope. This gives hope for the possibility of operating the reactor in a self-sufficient mode.

The results of calculations substantiating the possibility of realizing in practice the self-sufficient mode of operation of heavy-water power reactors of the CANDU-type were presented in paper [3]. It was assumed that the $^{233}$U required for stationary operation in the self-sufficient mode should be preliminarily accumulated in the CANDU reactor itself. This stage of transition in the self-sufficient mode of operation was referred to as the mode of accumulation of $^{233}$U. In this mode, the use $^{235}$U or $^{239}$Pu as the nuclear fuel producing neutrons for the accumulation of $^{233}$U was assumed. Taking into account that the parameters of the mode of accumulation of $^{233}$U and the self-sufficient mode of operation related to the time of accumulation of $^{233}$U, losses in electric power production and fuel burn up required more careful substantiation and optimization, we performed additional studies.

In this paper, the results of both optimization studies of the mode of accumulation of $^{233}$U and the self-sufficient mode of operation are presented. Parameters of the active core and the scheme of refueling of the current heavy-water power reactor HWPR (CANDU) with the heat power of 2776 MW [4] were used in said calculations. A complex of MCCOOR codes designed on the basis of MCNP, COUPLE, and ORIGEN-S codes was used for this purpose.

**THE MODE OF ACCUMULATION OF $^{233}$U**

The rate of formation of $^{233}$U in the unit of volume filled by $^{232}$Th and irradiated by the neutron flux $\phi_{Th}$, i.e. the number of nuclei of $^{233}$U forming per unit of time in the unit of volume equals

$$Q = Q_0 (1 - e^{-\lambda \tau}) = N_{Th} \sigma_{Th} \phi_{Th} (1 - e^{-\lambda \tau})$$  \hspace{1cm} (1)

where $Q_0$ is the rate of formation of $^{233}$U, $\tau$ – the time of irradiation, $\lambda$ – the decay constant of $^{233}$Th, $\sigma_{Th}$ – the cross-section of neutron capture in $^{232}$Th, and $N_{Th}$ – the number of nuclei of $^{232}$Th in the unit of volume. Under the condition that $\tau > 1/\lambda$, which corresponds to $\tau \geq 100$ days, the rate of formation of nuclei of $^{233}$U is equal to the rate of formation of nuclei of $^{233}$Pa.

For the evaluation of the influence of changing the geometry and composition of the channels of the active core of the reactor have on the rate of accumulation of $^{233}$Pa ($^{233}$U), a one-group multiplication factor for the cell of the active core, $K_{cell}$, was used

$$K_{cell} = \frac{v_5 \sigma_5^f N_5 \phi_5 + v_5 \sigma_5^s N_5 \phi_5}{(\sigma_5^f + \sigma_5^s) N_5 \phi_5 + \sum_l \sigma_l \phi_l N_l + q}$$  \hspace{1cm} (2)

where $\sigma$ is the microscopic cross-section, $\phi$ – the neutron flux, and $N$ – the number of nuclei in the unit of volume of the cell; indexes $l = 1, 2, 3, 4,$ and $5$ correspond to nuclei $^{235}$U, $^{232}$Th, and $^{233}$Pa fission products of $^{235}$U and fission products of $^{233}$U; $q$ is the rate of non-productive losses of neutrons as a result of absorbing in the oxygen, structural materials, and heavy water.

We can use eq. (2) for the determination of $Q_0$. Since at $\tau = 0$ the amounts of $^{233}$U, $^{233}$Pa, as well as fission products of $^{235}$U and $^{233}$U are equal to zero, in accordance with eq. (1), the rate of formation of $^{233}$Pa in the beginning of the process of accumulation is

$$Q_0 = C P_{cell} \left( \frac{v_5}{K_{cell}} - 1 - \frac{\sigma_5^f}{\sigma_5^f} \cdot \frac{\sigma_8^f}{\sigma_8^f} \cdot \frac{1 - e^{-\lambda \tau}}{e^{-\lambda \tau}} \right) - q$$  \hspace{1cm} (3)

In this equation, $C$ is the certain constant, $P_{cell} = C^{-1} \sigma_5^f \phi_5 N_5$ – the heat power of the cell of the active core, and $e$ – the enrichment of uranium used for the accumulation of $^{233}$U. According to eq. (3), the rate of accumulation of $^{233}$Pa, under the condition that $\tau > 1/\lambda$, is also the rate of the formation of $^{233}$U, do not depend directly on the neutron flux, but depend on it indirectly, via the power of the cell, $P_{cell}$. In order to increase $Q_0$, in the first place, one should tend to increase the power of the cell. The rate of accumulation of $^{233}$Pa also increases with the increase in the enrichment of the fuel, as far as it is accompanied with a decrease in the amount of $^{238}$U and the reduction of the non-productive losses of $q$ neutrons.

We considered 8 variants of cells of the heavy-water lattice of the reactor HWPR-881 of the CANDU-type [4] (a square elementary cell including 4 channels), as well as two variants of a light-water lattice of the VVER-1000- type reactor for comparison (tab. 1). The pitch of the lattice, geometric sizes of the channels, fuel assemblies, and fuel elements corresponded to the geometric characteristics of these reactors.

While calculating the power of the cell, maximum specific heat release in the fuel element for heavy-water variants of the cell was accepted to be equal to the value for the CANDU reactor, while for the light-water variant of the cell – equal to that of the VVER-1000 reactor. Therefore, the powers of the cells presented in the table in relative units are determined in relation to the
For all variants of heavy water cells, the heat load on the fuel is identical and equal to 23.7 MW/t. Fuel burn up in which \( Q = Q_0 \), equals 2.37 MWd/kg. Accordingly, for the light water lattice, the heat load on the fuel is 47 MW/t and fuel burn up 4.7 MWd/kg.

The data presented in tab. 2 demonstrate the following.

1. Changes in the enriched uranium by U-Pu fuel (variants 3, and 4) do not result in any significant changes in \( Q_0 \).

2. Changing the heavy water inside the channel with light water (variants 5 and 6) results in a loss in the rate of accumulation of \( ^{233}U \) by about 10%.

3. The maximum power of the cell, and consequently, the maximum power of the reactor at the beginning of the mode of accumulation, is achieved in variant 2. However, the rate of accumulation in this variant is, approximately, by 29% lesser than in variant 7.

4. The use of composite assemblies (fuel pins with target pins) allows us to obtain maximum values of \( Q_0 \) (variants 5 and 7).

5. The rate of accumulation of \( ^{233}U \) in the cell of the VVER-1000 reactor is, approximately, 32% lesser than that of variant 7 in the cells of the reactor HWPR-881 (CANDU).

In fact, the values of \( Q_0 \) for the different variants of the lattice presented in tab. 2 determine the relative rate of the formation of \( ^{233}U \) for \( \tau \geq 100 \) days after the beginning of the process of accumulation.

It is apparent that the data presented above are insufficient for determining the optimum variant of the cell. For further studies, variants 2, 3, 5, 7, and 8 were chosen. The results of kinetic calculations for a single cell are presented in figs. 1 and 2.

Under constant powers of the reactor, curves in these figures describe \( K_{\text{cell}}(\tau) \) and \( G(\tau) \) – dependencies of reactivity and the amount of accumulated \( ^{233}U \) (per unit of length of the cell) on the time of reactor operation \( \tau \). The numbers of these curves correspond to the above chosen variants of the cells. It should be noted

**Table 1. Composition of cells**

<table>
<thead>
<tr>
<th>Variant</th>
<th>Composition of cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 channels with fuel pins (UO₂)</td>
</tr>
<tr>
<td>2</td>
<td>3 channels with fuel pins (UO₂), and 1 channel with target pins (ThO₂)</td>
</tr>
<tr>
<td>3</td>
<td>2 channels with fuel pins (UO₂), and 2 channels with target pins (ThO₂)</td>
</tr>
<tr>
<td>4</td>
<td>2 channels with fuel pins (Pu), and 2 channels with target pins (ThO₂)</td>
</tr>
<tr>
<td>5</td>
<td>4 channels with fuel-target assemblies; each fuel-target assembly contains 18 fuel pins (UO₂) in the external layer and 19 target pins (ThO₂) in internal layers; the moderator is heavy water, while the coolant is light water</td>
</tr>
<tr>
<td>6</td>
<td>4 channels with fuel-target assemblies. Each fuel-target assembly contains 18 fuel pins (UO₂) in the external layer and 19 target pins (ThO₂) in internal layers; the moderator is heavy water, while the coolant is light water</td>
</tr>
<tr>
<td>7</td>
<td>4 channels with fuel-target assemblies; each fuel-target assembly contains 18 target pins (ThO₂) in the external layer and 19 fuel pins (UO₂) in internal layers; the moderator and coolant is heavy water</td>
</tr>
<tr>
<td>8</td>
<td>4 channels with fuel-target assemblies; each fuel-target assembly contains an uniform mix of 18 target pins (ThO₂) and 19 fuel pins (UO₂)</td>
</tr>
<tr>
<td></td>
<td>The cell of a VVER-1000 type reactor</td>
</tr>
<tr>
<td>9</td>
<td>1 channel with 317 fuel elements (UO₂)</td>
</tr>
<tr>
<td>10</td>
<td>1 channel with 193 fuel elements (UO₂), and 124 target elements (ThO₂)</td>
</tr>
</tbody>
</table>

Peak-to-average ratio for power distribution with a radius of the channel \( K_r \). The characteristics of the above listed cells were calculated by using the code complex MCCOOR for \( \tau = 0, \) as presented in tab. 2.

For purposes of comparison, the results of the enrichment of UO₂ for all variants of the cells was determined according to the condition of approximate equality of \( K_{\text{cell}} \) at \( \tau = 0, \) i. e. at the beginning of the mode of accumulation.

The data presented in tab. 2 demonstrate the following.

1. Changes in the enriched uranium by U-Pu fuel (variants 3, and 4) do not result in any significant changes in \( Q_0 \).

2. Changing the heavy water inside the channel with light water (variants 5 and 6) results in a loss in the rate of accumulation of \( ^{233}U \) by about 10%.

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Under constant powers of the reactor, curves in these figures describe \( K_{\text{cell}}(\tau) \) and \( G(\tau) \) – dependencies of reactivity and the amount of accumulated \( ^{233}U \) (per unit of length of the cell) on the time of reactor operation \( \tau \). The numbers of these curves correspond to the above chosen variants of the cells. It should be noted

**Table 2. Characteristics of cells**

<table>
<thead>
<tr>
<th>Variant of the cell</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power of the cell</td>
<td>1.0</td>
<td>0.71</td>
<td>0.38</td>
<td>0.35</td>
<td>0.56</td>
<td>0.56</td>
<td>0.5</td>
<td>0.43</td>
<td>1.0</td>
<td>0.56</td>
</tr>
<tr>
<td>Fuel enrichment, [%]</td>
<td>0.71</td>
<td>1.5</td>
<td>5.0</td>
<td>Pu*</td>
<td>2.3</td>
<td>2.3</td>
<td>5.0</td>
<td>3.2</td>
<td>3.6</td>
<td>3.6</td>
</tr>
<tr>
<td>Number of fuel pins in cell</td>
<td>148</td>
<td>111</td>
<td>74</td>
<td>74</td>
<td>72</td>
<td>76</td>
<td>76</td>
<td>76</td>
<td>317</td>
<td>193</td>
</tr>
<tr>
<td>Number of target pins in cell</td>
<td>0</td>
<td>37</td>
<td>74</td>
<td>74</td>
<td>76</td>
<td>76</td>
<td>72</td>
<td>72</td>
<td>0</td>
<td>124</td>
</tr>
<tr>
<td>Peak-to-average ratio, Kr</td>
<td>1.15</td>
<td>1.21</td>
<td>1.51</td>
<td>1.63</td>
<td>1.0</td>
<td>1.0</td>
<td>1.19</td>
<td>1.29</td>
<td>1.05</td>
<td>1.15</td>
</tr>
<tr>
<td>Non-productive capture of neutrons per one fission</td>
<td>0.14</td>
<td>0.10</td>
<td>0.09</td>
<td>0.10</td>
<td>0.10</td>
<td>0.21</td>
<td>0.10</td>
<td>0.10</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>Multiplication factor ( K_{\text{cell}} )</td>
<td>1.11</td>
<td>1.13</td>
<td>1.07</td>
<td>1.08</td>
<td>1.12</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>Rate of ( ^{235}Pa ) accumulation ( Q_0 ), respective units</td>
<td>0.27</td>
<td>0.29</td>
<td>0.29</td>
<td>0.32</td>
<td>0.29</td>
<td>0.38</td>
<td>0.29</td>
<td>0.29</td>
<td>0</td>
<td>0.26</td>
</tr>
</tbody>
</table>

*Note, that plutonium is fuel in fourth variant of the cell (see tab. 1). Therefore Pu symbol is put in the column instead of value of enrichment of uranium fuel.
that, for all variants, reactivity decreases sharply at the beginning of irradiation, because of the accumulation of $^{235}$Pa. The reactor falls into the “Protactinium pit”. For variants 3 and 7 (enrichment of $^{235}$U 5%), $K_{cell}$ increases over 900 and 600 days, correspondingly. This is explained by the fact that in these time intervals, the rate of absorption of neutrons in $^{232}$Th is higher than the rate of absorption in $^{235}$U because of the behavior of neutron distribution over the cell (variant 3) and because of the shielding of fuel pins by target pins in the assembly (variant 7). For variants 2 and 8 (enrichment of $^{235}$U 1.5% and 3.2%), $K_{cell}$ decreases in the initial short period of time and then stabilizes in the time intervals of 200 and 600 days. In variant 5 (enrichment of $^{235}$U 2.2%), the period of stabilization is absent. After $\tau = 50$ days, reactivity falls quickly and the rate of accumulation of $^{233}$U decreases slowly because of fission products accumulation. A significant fall of reactivity at the beginning of the process can be avoided for all variants of cells. For this purpose, the reactor should come to the nominal power in accordance with the special time dependence of power which allows to, at least partially, compensate for the accumulation of $^{233}$Pa nuclei and the delay in formation of $^{233}$U nuclei. However, these measures lead to losses in electric power production.

The results of our studies [1] demonstrated that, for heavy-water lattices with the $^{235}$U-$^{232}$Th fuel composition, a stationary self-sufficient mode can be realized with a balanced concentration of $^{233}$U equaling approximately 14 kg of $^{233}$U per one ton of thorium. If we apply this value to the cell of the HWPR-881 reactor consisting of 4 channels, the equilibrium amount of $^{231}$U will be 22.5 g of $^{235}$U per 1 cm of the lengths of the cell. In following calculations, we assumed that the self-sufficient mode in the HWPR-881 reactor is possible when the loading of $^{235}$U in the cell is not less than 23 g per 1 cm of the length of the active core. According to the curves in fig 2, the accumulation of the required amount of $^{235}$U in a single irradiation of targets can not be obtained because of the accumulation of fission products and the corresponding reduction of the rate of accumulation of $^{233}$U for all considered variants of cells. Therefore, we considered modes of $^{235}$U accumulation in which targets are extracted from the active core and replaced by new targets after the partial accumulation of $^{233}$U. In tab. 3, the results presented are for the reduced rate of accumulation of $^{233}$U, $Q_0$, by 10% and 20% with respect to the mode of tab. 2. In tab. 3, $\tau_1$ is a time interval between the reloading of the targets, $g$ is an amount of $^{235}$U per unit of target length at a moment of unloading from the active core, $\tau_2$ is the time of reactor operation required for accumulations of 23 g/cm of $^{233}$U, and $M$ is the total number of targets extracted from one cell for reprocessing in order to obtain 23 g/cm of $^{231}$U.

The data presented above allow the following conclusion.

For cell variants 5, 7, and 8, the speed of the moving of targets via the active core is hardly connected with the speed of moving of fuel elements. Therefore, the choice of a different time of irradiation for targets and fuel elements is practically excluded for these variants. In addition, the dismantling of the irradiated fuel-target assembly is necessary for the extraction of targets. These rather complex operations are beyond the framework of CANDU technologies.

Only in variants 2 and 3, the time of the moving of targets in channels and, hence, the time of their irradiation in the active core can be chosen independently from the time of the fuel cycle. When comparing variants 2 and 3, it should be taken into account that the power of the cell and, hence, the power of the reactor at the beginning of irradiation in variant 2, are about twice higher than in variant 3. Due to this obvious advantage, variant 2 is beyond comparison.

The data in tab. 3 also demonstrate that the reduction in the time of $^{235}$U accumulations for about 1.5 times is possible only by using the technologies for dismantling irradiated fuel-target assemblies. In this
Table 3. Parameters of the process of $^{233}$U accumulation

<table>
<thead>
<tr>
<th>Variant of the cell</th>
<th>Reduction of the rate of $^{233}$U accumulation by 10%</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$ [day]</td>
<td></td>
<td>177</td>
<td>253</td>
<td>260</td>
<td>187</td>
<td>287</td>
</tr>
<tr>
<td>$g$ [g/cm³]</td>
<td></td>
<td>1.7</td>
<td>2.7</td>
<td>3.2</td>
<td>2.7</td>
<td>3.2</td>
</tr>
<tr>
<td>$r_2$ [day]</td>
<td></td>
<td>2400</td>
<td>2150</td>
<td>1870</td>
<td>1600</td>
<td>2060</td>
</tr>
<tr>
<td>M</td>
<td></td>
<td>500</td>
<td>630</td>
<td>546</td>
<td>615</td>
<td>516</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variant of the cell</th>
<th>Reduction of the rate of $^{233}$U accumulation by 20%</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$ [day]</td>
<td></td>
<td>257</td>
<td>353</td>
<td>387</td>
<td>283</td>
<td>420</td>
</tr>
<tr>
<td>$g$ [g/cm³]</td>
<td></td>
<td>2.3</td>
<td>3.7</td>
<td>4.6</td>
<td>3.9</td>
<td>4.5</td>
</tr>
<tr>
<td>$r_2$ [day]</td>
<td></td>
<td>2570</td>
<td>2200</td>
<td>1920</td>
<td>1660</td>
<td>2140</td>
</tr>
<tr>
<td>M</td>
<td></td>
<td>370</td>
<td>460</td>
<td>380</td>
<td>422</td>
<td>370</td>
</tr>
</tbody>
</table>

SELF-SUFFICIENT MODE OF OPERATION

In the self-sufficient mode of operation, the amount and composition of heavy nuclei in the fuel assembly unloaded from the reactor must ensure a multiplication factor $K_{\text{cell}} > 1$ after the extraction of fission products, fabrication of new fuel assemblies and the loading of these assemblies in the reactor. The problem is to define the geometry and composition of the elementary cell, for which this condition is met under the acceptable burn up of $^{233}$U. The cell of the heavy water lattice of the HWPR-881 reactor was assumed as a prototype. Fuel and target assemblies were located in four channels situated on the corners of a square lattice with a step of 26.8 cm. The fuel assemblies were filled with the dioxide of thorium and $^{233}$U. At the beginning of the fuel cycle, target assemblies were filled with ThO$_2$ with a density of 10.6 g/cm$^3$. Variants of the composition and geometry of the cells of the active core presented in tab. 1 were studied.

The breeding ratio for the cell under constant power of the reactor and $\tau \gg 1/\lambda$ can be determined in accordance with eq. (1) as follows

$$BR = \sum_2^\infty \frac{\int [\sigma^c_2(E)N_2(r)\varphi(E,r)E]dEdr}{\int [\sigma^a_3(E)N_3(r)\varphi(E,r)E]dEdr}$$

where $\sigma^c_2$ is the cross-section of neutron capture in $^{232}$Th, $\sigma^a_3$ is the cross-section of neutron absorption (sum of cross-sections of capture and fission) in $^{233}$U, $N_2$, $N_3$ – the number of nuclei of $^{232}$Th and $^{233}$U in point $r$, and $\varphi(E,r)$ – the flux density of neutrons with energy $E$ in point $r$. Integrating in eq. (4) is realized over the area of the cell and over the whole range of the energy of neutrons. The application of eq. (2) for $K_{\text{cell}}$ in the case of the heavy water reactor working in the thorium-uranium fuel cycle allows us to obtain the relationship between $K_{\text{cell}}$ and $BR$

$$K_{\text{cell}} = \frac{v_f \beta}{1 + BR + q\beta}$$

where $\beta = \sum_2^\infty \varphi(E)E$, is the ratio of macroscopic cross-sections of fission and absorption in $^{233}$U for the cell, $q$ – the loss of neutrons in structure materials, heavy water, oxygen, $^{233}$Pu, and fission products accumulated to a moment $t$, in the case of one fission in $^{233}$U. For those variants of the elementary cell, in which $\beta \approx \text{const}$ and $q \approx \text{const}$, eq. (5) describes an unambiguous relationship between $K_{\text{cell}}$ and $BR$.

The results of calculations of the breeding ratio and multiplication factor with the use of specialized reactor codes without the application of eq. (4) for the three variants of the reactor cell are presented by dots in fig. 3. When turning from one variant of a cell to another, as well as when changing the content of $^{233}$U in ThO$_2$ or pitch of heavy water lattice, parameters $q$ and $\beta$ in eq. (5) do not change. Therefore, all calculated dots are described with good accuracy by curve 1 which is obtained from decision of eq. (5). An analogous decision of eq. (5) for the heavy water lattice of the HWPR-881 reactor with uranium-plutonium fuel is described by curve 2. This curve lies much below curve 1, because parameters $q$ and $\beta$ in eq. (5) for $^{233}$U are less than for $^{235}$U. In calculation of curve 2, fission of $^{238}$U by fast neutrons was taken into account. Comparison of curves 1 and 2 demonstrates that the realization of the self-sufficient mode ($BR > 1$, $K_{\text{cell}} > 1$) in practice is possible only in a heavy water reactor with thorium-uranium fuel.

Curves in fig. 4 describe analogous dependency for hypothetical variants of the light water VVER-1000 power reactor. Curve 1 is calculated for $^{233}$U-$^{232}$Th fuel, curve 2 – for $^{235}$U-$^{238}$U fuel. For both
variants, boron is absent in light water, the amount of fission products identical. Curve 3 is an analogue of curve 1 in the case where boron is introduced in light water. Comparison of curve 1 from fig. 3 with curve 1 from fig. 4 demonstrates that the reproduction of thorium fuel can be obtained in the VVER-1000 reactor as in the heavy water reactor. In this case, VVER-1000 must operate under small enrichment of fuel (small content of 233U in 232Th), when there is no need to use boron for compensating reactivity excess, that, in turn, requires rapid reloading of fuel with small burnup.

In fig. 3, the significant range of \( K_{\text{cell}} \) and \( BR \) is quadrant \( K_{\text{cell}} > 1, BR > 1 \). This quadrant is filled by gray color. The dots on curve 1 located in this quadrant correspond to variants of the geometry and initial composition of the cell of the active core in which the self-sufficient mode of operation can be realized.

We should note that the even distribution of \(^{233}\text{U}\) over all channels and fuel elements of the reactor can ensure such conditions only for the lattice pitch lesser than 20 cm, \textit{i.e.} for a harder spectrum of neutrons than in the HWPR-881 reactor. Two variants are prospective for the self-sufficient mode of operation: 1\textsuperscript{st} - four identical fuel assemblies in the cell with non-uniform distribution of \(^{233}\text{U}\) over fuel elements within the fuel assembly and 2\textsuperscript{nd} - three fuel assemblies with even distribution of \(^{233}\text{U}\) over the fuel elements and one target assembly containing \(\text{ThO}_2\). Variants with combined fuel assemblies containing fuel elements and target elements within the same fuel assembly were not considered. The first variant was discussed in paper [3]. It was demonstrated that fuel burn up in the self-sufficient mode is rather low for this variant. The second variant is more preferable. All four channels can have a different consumption of cooling water, while fuel assemblies and target assemblies can move through channels with different velocity.

For the evaluation of fuel burn up in the self-sufficient mode, an integral over all values of fuel burn up in fuel assemblies was used. In addition to the analogous formula used in paper [3], weight function \((\pi/2)\sin(\pi W/W_{\text{max}})\) was introduced into this integral. This weight function takes into account that all fuel assemblies with \(W = W_{\text{max}}\) are located at the output from the active core, while fuel assemblies with \(W = 0\) are located at the input of the active core.

\[
\alpha = \frac{\pi}{2} \int_{0}^{W_{\text{max}}} K_{\text{cell}}(W) \sin \frac{\pi W}{W_{\text{max}}} \frac{dW}{W_{\text{max}}} \quad (6)
\]

Parameter \(\alpha\) in this formula is defined by neutron absorption in control rods and in structure materials of the active core, as well as by the leakage of neutrons from the active core. For the HWPR-881 reactor, \(\alpha = 1.045\) for control rods inserted into the active core, and \(\alpha = 1.035\) for control rods extracted from the active core [1].

The solution of eq. (6) relative to \(W_{\text{max}}\) is possible only if information on the dependence on \(K_{\text{cell}}(W)\) is present. Curves in fig. 5 describe the dependence of the multiplication factor on burn up with different initial contents of \(^{233}\text{U}\) for the two above mentioned variants (1 and 2) of the cell. These data demonstrate that only in variant 2 the reproduction of \(^{233}\text{U}\) ensures an increase of reactivity for an increase of fuel burn up from 1 to 8 MW·d/kg. Maximum burn up increases with the increase of the initial content of \(^{233}\text{U}\) in the cell. The decision of eq. (6) for \(\alpha = 1.045\) and initial content of \(^{233}\text{U}\) in the cell 26.5 g/cm gives \(W_{\text{max}} \approx 24\) MW·d/kg. However, for the determination of real burn up in the self-sufficient mode, data on reactivity change are insufficient. Data on the amount of \(^{233}\text{U} + ^{233}\text{Pa}\) in the cell at the end of the fuel cycle are necessary. For the self-sufficient mode, this amount must be not less

![Figure 3. BR values vs. \(K_{\text{cell}}\) for heavy water CANDU reactor. Curve 1 – Th-U fuel cycle: • – cell variant 1 at different lattice pitch (28.6, 23.2, 20.6 cm), □ – cell variant 5, and Δ – cell variant 2 at different contents of 233U in thorium. Curve 2 – U-Pu fuel cycle](image)

![Figure 4. BR values vs. \(K_{\text{cell}}\) for light water VVER-1000 reactor. Curve 1 – Th-U fuel cycle and curve 2 – U-Pu fuel cycle without boron, curve 3 – Th-U fuel cycle with boron](image)
than the initial amount of $^{233}$U nuclei in the cell. As to other heavy isotopes of uranium, calculations confirm that the accumulation of $^{235}$U compensates for the influence of non-fissile uranium isotopes on reactivity [3].

The curves in fig. 6 demonstrate that for variant 2 under the same burn up, the amount of $^{233}$U at the end of the fuel cycle decreases with the increase in the initial content of $^{233}$U. Note that for the reactor with moving fuel assemblies, the initial content of $^{233}$U in the cell means an amount of $^{233}$U in fuel assemblies at the input of the active core. The amount of $^{233}$U in the cell at the end of the fuel cycle means content of $^{233}$U in fuel assemblies at the output from the active core.

The data from tab. 4 allow us to conclude the possibility of a self-sufficient mode in the HWPR-881 reactor in which the square cell of the active core consists of three channels with fuel assemblies and one channel with the target assembly. Fuel burn up under such an arrangement of the active core is ~8 MW·d/kg. This value corresponds to technologies accepted for the HWPR-881 reactor. Maximum burn up can be increased to 17 MW·d/kg, if $\alpha = 1.035$ is used instead of $\alpha = 1.045$. This would result in the need to reduce by 1% the minimum excess of reactivity in the HWPR-881 reactor by a corresponding reduction of neutron losses under their capture in the control rods, structure materials and leakage from the active core.

**CONCLUSION**

The results of calculation studies demonstrate the practical feasibility of a self-sufficient mode for the HWPR-881 reactor of the CANDU type operating in a thorium-uranium fuel cycle without a need to use new technologies. The main prerequisites for a self-sufficient mode of operation when these types of reactors are concerned are a good neutron balance in the active core and moving of fuel assemblies through the active core during the fuel cycle.

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ОПТИМИЗАЦИЈА САМОДОВОЉНОГ ТОРИЈУМОВОГ ГОРИВНОГ ЦИКЛУСА У КАНДУ ЕНЕРГЕТСКИМ РЕАКТОРИМА

У раду су приказани резултати оптимизације Канду реактора који раде са торијумовим циклусом. Прорачуни су спроведени ради процене изгледа за рад тешководних енергетских нуклеарних реактора на термичке неутроне, са торијумовим самодовољним циклусом. Размотрена су два облика рада: поступак претходне акумулације $^{233}\text{U}$ у самом реактору и начин рада са самодовољним циклусом. У поступку претходне акумулације $^{233}\text{U}$, претпостављено је да се користи обогаћени уран или плутонијум као додатни фisionи материјал који обезбеђује неутроне за производњу $^{233}\text{U}$. У самодовољном начину рада, масени и изотопски састав тешких језгара изнетих из реактора треба да обезбеди (по уклањању фisionих продуката) да вредност фактора умношивања ћелије у наредном циклусу буде $K > 1$. Додатни задатак је био да се одреди геометрија и састав ћелија за прихватљиво изгарање $^{235}\text{U}$. Добијени резултати показују да је могуће остварење торијумовог самодовољног поступка у Канду реактору без коришћења нових технолошких решења. Добар баланс неутрона и размештање горива по активном језгру, представљају главне црте реактора са осигураним самодовољним начином рада.

Кључне речи: торијумов циклус, Канду реактор, самодовољни начин рада